E. Etim and L. Schulke: VECTOR MESON–PROTON CROSS SECTIONS FROM DEEP INELASTIC SCATTERING DATA
VECTOR MESON - PROTON CROSS SECTIONS FROM DEEP INELASTIC SCATTERING DATA

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ABSTRACT

Contrary to naive scaling, vector meson-proton total cross sections are not proportional, at high collision energies, to the geometrical sizes of the vector mesons concerned. The Ansatz \((m^2 + M^2)\sigma_{\gamma p} = \text{const.}\), where \(M\) is the proton mass, is found to be sufficient to produce cross section ratios

\[ \sigma_{\gamma p} : \sigma_{\omega p} : \sigma_{\rho p} : \sigma_{\phi p} = 7.1 : 7.1 : 5.5 : 1 \]

in agreement with experiment, if this were all one wanted. Actually the constant in this modified scaling law can be fixed by using the generalised vector meson dominance model to relate \(\sigma_{\rho p}\) to the virtual Compton cross section \(\sigma_{\gamma p}(\nu, Q^2)\) and hence to the deep inelastic scattering data. Consequently, \(\sigma_{\gamma p}\) can be determined in absolute value. There are important consistency conditions which this scheme has to and does effectively satisfy.

1. INTRODUCTION

Given the proton-proton total cross section \(\sigma_{pp}\), the quark model allows one to estimate the absolute value of the rho meson-proton cross section \(\sigma_{\rho p}\) as follows

\[ \sigma_{\rho p} = \frac{2}{3} \sigma_{pp} \quad . \quad (1) \]
This estimate is quite good; it is essentially a quark-counting rule. The vector meson dominance model too offers some arguments on how to estimate vector meson-proton cross sections. It postulates scaling, that is

$$\sigma_{\gamma p} = \frac{A_0}{m_v^2}$$

(2)

where $A_0$ is a constant and $m_v$ the vector meson mass. For $\gamma = \varphi, \omega, \rho$ one gets from eq.(2) the ratios

$$\sigma_{\varphi p} : \sigma_{\omega p} : \sigma_{\rho p} = 1.8 : 1.7 : 1.$$  

(3)

These too compare impressively well with the experimental ratios(1)

$$\sigma_{\varphi p} : \sigma_{\omega p} : \sigma_{\rho p} = 2 : 2 : 1-1.25.$$  

(4)

However, if this scaling law is extended to apply also to the $\psi$-meson one finds

$$\sigma_{\varphi p} : \sigma_{\omega p} : \sigma_{\rho p} : \sigma_{\psi p} = 16.2 : 15.6 : 9.2 : 1$$

(5)

which do no longer agree with the experimental ratios(1)

$$\sigma_{\varphi p} : \sigma_{\omega p} : \sigma_{\rho p} : \sigma_{\psi p} = 8 : 8 : 4-5 : 1.$$  

(6)

Eq.(2) thus predicts too small a value for the $\psi$-p cross section $\sigma_{\psi p}$.

Some time ago, Gunion and Soper(2) calculated corrections to the naive scaling law in eq.(2) and found improvements in the cross section ratios in the direction of agreement with experiment. In their calculation they combined simple vector meson dominance (VMD) phenomenology with quantum chromodynamics (QCD) applied directly to hadron-hadron scattering.

We shall present in this paper a different VMD model which improves on eq.(2) and exploits asymptotic QCD predictions in the well studied sector of lepton-hadron scattering. Unlike in the simple VMD model (viz. eq.(2)) which predicts only cross section ratios, the present model is able to predict the absolute values of the vector meson-proton cross sections. We define this model and discuss its predictions in Sect. 2. Section 3 concludes the paper with some comments.
2. \( \sigma_{\gamma p} \) FROM DEEP INELASTIC SCATTERING DATA

Let us start by recalling the definition of the proton structure function, \( F_2(x,Q^2) \), in terms of the virtual Compton cross section, \( \sigma_{\gamma p}(x,Q^2) \), off the proton, that is:

\[
F_2(x,Q^2) = \frac{Q^2}{4\pi^2 a} \frac{Q^2(1-x)}{Q^2+4M^2 x^2} \sigma_{\gamma p}(x,Q^2)
\]  

(7)

where \( x = Q^2/2M\nu \), \( M \) the proton mass and \( \nu \) the energy of the virtual photon in the proton rest frame. For \( Q^2 \to \infty \) one gets from eq.(7), in leading order (alternatively in the quark parton model)

\[
\int_0^1 dx F_2(x,Q^2) \xrightarrow{Q^2 \to \infty} \frac{1}{N_f} \sum_i q_i^2
\]

(8)

where \( q_i \) is the charge of the quark of flavour \( i \) and \( N_f \) is the total number of flavours. Equation (8) is all that we shall need of the QCD prediction for \( \sigma_{\gamma p}(x,Q^2) \) in what follows. Note that eq.(8) is a theoretical prediction of the quark parton model. Experimentally the value of the integral over \( F_2(x,Q^2) \) is about a factor 2 smaller than given by eq.(8).

Next consider the \( Q^2 \)-dependent cross section \( \sigma_{\gamma p}(Q^2) \) defined by the following \( x \)-average

\[
\frac{Q^2}{4\pi^2 a} \sigma_{\gamma p}(Q^2) = \int_0^1 dx F_2(x,Q^2).
\]

(9)

In terms of \( \sigma_{\gamma p}(x,Q^2) \) eq.(9) reads

\[
\sigma_{\gamma p}(Q^2) = Q^2 \int_0^1 dx \frac{1-x}{Q^2+4M^2 x^2} \sigma_{\gamma p}(x,Q^2).
\]

(10)

Clearly, from eq.(10),

\[
\sigma_{\gamma p}(Q^2) \neq \sigma_{\gamma p}(x=0,Q^2).
\]

(11)

The generalized vector meson dominance model (GVMD)\(^\dagger\) has a formula for

\[
\bar{\sigma}_{\gamma p}(Q^2) = \sigma_{\gamma p}(x=0,Q^2).
\]

(12)

It reads

\[
\bar{\sigma}_{\gamma p}(Q^2) = \sum_i \bar{\sigma}_{\gamma p}^i(Q^2),
\]

(13.a)
\[
\sigma^1_{\gamma p}(Q^2) = \sum_{n=0}^{\infty} \frac{e m^n_{n_i}^2}{f_{n_i}} \frac{\sigma_{n_i p}(m^2_{n_i})}{(m^2_{n_i} + Q^2)^2} \tag{13.b}
\]

where the \(i\)-th sum runs over flavours while the \(n\)-th sum runs over an infinite number of vector mesons, \(V_{n_i}\), of mass \(m_{n_i}\) made up of the quark-antiquark pair of flavour \(i\). For fixed \(Q^2\), the \(x=0\) limit in \(\sigma_{\gamma p}(x, Q^2)\) corresponds to the infinite energy limit \((\nu \to \infty)\) in the cross section \(\sigma_{V_{n_i} p}(\nu, m^2_{n_i})\). The vector meson-proton cross section in this limit has been written simply as \(\sigma_{V_{n_i} p}(m^2_{n_i})\) in eq.(13.b). \(e m^n_{n_i} / f_{n_i}\) is the coupling of \(V_{n_i}\) to the photon. One makes progress with eq.(13.b) by eliminating this coupling in favour of the imaginary part of the photon vacuum polarisation amplitude \(\Pi(s)\), through the definitions

\[
\begin{align*}
\text{Im} \Pi(s) &= \sum_i \text{Im} \Pi_i(s), \\
\text{Im} \Pi_i(s) &= \pi \sum_{n=0}^{\infty} \frac{e^2 m^n_{n_i}^2}{f_{n_i}^2} \delta(s - m^2_{n_i}).
\end{align*}
\tag{14.a}
\tag{14.b}
\]

Eq.(14.b) gives, upon locally integrating about \(s=m^2_{n_i}\),

\[
\frac{e^2 m^n_{n_i}^2}{f_{n_i}^2} = \frac{1}{\pi} \int_{m^2_{n_i} - \Delta^2_{n_i}}^{m^2_{n_i} + \Delta^2_{n_i}} ds \text{Im} \Pi_i(s). \tag{15}
\]

\(\Delta^2_{n_i}\) is a small but undetermined mass squared interval about \(s=m^2_{n_i}\). For the purpose of this paper it will not be necessary to fix \(\Delta^2_{n_i}\). In fact substituting from eq.(15) into eq.(13.b) and recalling the definition of the Riemann integral one finds

\[
\sigma^1_{\gamma p}(Q^2) = \frac{1}{\pi} \int_0^{\infty} ds \text{Im} \Pi_i(s) \frac{\sigma_{V_{n_i} p}(s)}{(s + Q^2)^2} s \sigma_{V_{n_i} p}(s). \tag{16}
\]

We have replaced \(\sigma_{V_{n_i} p}(m^2_{n_i})\) by \(\sigma_{V_{n_i} p}(s)\) in the continuum limit \((m^2_{n_i} \to s)\).

We now explicitly assume that \(\sigma_{V_{n_i} p}(s)\) is given, not by eq.(2), but by the closely related scaling law

\[
\sigma_{V_{n_i} p}(s) = \frac{A}{s + M^2} \tag{17}
\]

with the constant \(A\) independent of flavour. For the vector meson-proton cross section ratios, the Ansatz (17) yields
\[ q_{\gamma p} : \omega_{\gamma p} : \sigma_{\gamma p} : \sigma_{\gamma p} = 7.1 : 7.1 : 5.5 : 1 \] (18)

in fairly good agreement with the experimental ratios in eq.(6).

We now want to determine the constant A. To this end substitute from eq.(17) into eq.(16) and obtain

\[ \frac{1}{\gamma_{p}}(Q^2) = \frac{\alpha}{Q^2-M^2} \left[ Q^2 \Pi_{1}'(-Q^2) + M^2 \frac{\Pi_{1}(-Q^2)-\Pi_{1}(-M^2)}{Q^2-M^2} \right] \] (19)

where the photon vacuum polarization amplitude \( \Pi(p^2) \) is given by the dispersion integral

\[ \Pi_{1}(p^2) = \frac{\mathcal{E}}{\pi} \int_{0}^{\infty} ds \frac{\text{Im} \Pi_{1}(s)}{s(s-p^2)} \] (20)

and \( \Pi_{1}'(p^2) = d \Pi_{1}(p^2)/dp^2 \).

The leading large \( p^2 \) behaviour of \( \Pi_{1}(p^2) \) is known in the quark parton model(5).

\[ \Pi_{1}(p^2) \quad \stackrel{p^2 \to \infty}{\longrightarrow} \quad \frac{e_{q_i}^2}{4\pi^2} \ln(-\frac{2}{4M_i^2}). \] (21)

\( M_i \) is the quark mass.

Hence from eqs.(19) and (21)

\[ \frac{1}{\gamma_{p}}(Q^2) \quad \stackrel{Q^2 \to \infty}{\longrightarrow} \quad \frac{a_{q_i}^2}{\pi} \frac{A}{Q^2}, \] (22)

\[ \sigma_{\gamma p}(Q^2) = \sum_{q_i} \frac{1}{\gamma_{p}}(Q^2) \quad \stackrel{Q^2 \to \infty}{\longrightarrow} \quad \frac{a_{\sum q_i}^2}{\pi} \frac{1}{Q^2}. \] (23)

On the other hand from eqs.(8) and (9) the large \( Q^2 \) behaviour of \( \sigma_{\gamma p}(Q^2) \) is given by

\[ \sigma_{\gamma p}(Q^2) \quad \stackrel{Q^2 \to \infty}{\longrightarrow} \quad \frac{4\pi^2(a_{\sum q_i}^2/4)}{N_f}. \] (24)

According to eq.(11), \( \sigma_{\gamma p}(Q^2) \neq \sigma_{\gamma p}(Q^2) \). For fixed \( Q^2 \), \( F_2(x, Q^2) \), or equivalently \( \sigma_{\gamma p}(x, Q^2) \), is a decreasing function of \( x \), with its maximum value at \( x=0 \) and a (kinematical) zero at \( x=1 \). From eq.(10) this means that \( \sigma_{\gamma p}(Q^2) < \sigma_{\gamma p}(Q^2) \). Let us assume for simplicity that

\[ \sigma_{\gamma p}(Q^2) \quad \frac{1}{B} \sigma_{\gamma p}(Q^2). \] (25)
with \( B > 1 \) a constant independent of \( Q^2 \). From eqs.(10) and (25) one gets:

\[
\frac{1}{B} = \int_0^1 dx \frac{F_2(x,Q^2)}{F_2(0,Q^2)} = \int_0^1 dx (1-x) \frac{\sigma_{\gamma p}(x,Q^2)}{\sigma_{\gamma p}(0,Q^2)}.
\]

(26)

Now substitute eqs.(23) and (24) into eq.(25) in the limit \( Q^2 \to \infty \). One gets in this way

\[
A = \frac{4\pi^2 B}{N_f}.
\]

(27)

To determine the constant \( B \) we note that at \( Q^2 = 0 \) the simple VMD model for \( \sigma_{\gamma p}(Q^2) \) is roughly in agreement with experiment as the numbers here below show:

\[
\sigma_{\gamma p}(Q^2 = 0) \bigg|_{\text{expt}} \approx (99.9 \pm 1.6) \mu b ,
\]

(28.a)

\[
\sigma_{\gamma p}(Q^2 = 0) \bigg|_{\text{VMD}} \approx \frac{11}{9} \frac{4\pi a}{r_0^2} \sigma_{\gamma p} \approx 95 \mu b .
\]

(28.b)

The factor \( 11/9 \) takes into account the \( \varrho, \omega, \) and \( \varphi \) contributions. Moreover we have used \( r_0^2/4\pi \approx 2.5 \), \( \sigma_{\gamma p} = 2/3 \sigma_{pp} \approx 26.7 \) mb, and \( 1/r_0^2 : 1/r_\omega : 1/r_\varphi = 1 : 1/9 : 2/9 \); \( \sigma_{\varphi p} \approx \sigma_{\omega p} \approx 2\sigma_{\gamma p} \).

Thus if one combines eqs.(17), (18), (25) and (28) one finds

\[
\frac{11}{9} \frac{4\pi a}{r_0^2} \frac{A}{m_q^2 + M^2} \approx - \frac{A}{BM^2} \sum_i \pi_1 (-M_i^2) =
\]

\[
\approx \frac{A}{BM^2 \pi} \sum_{i=u,d,s} q_i^2 \ln \left( \frac{M_i^2}{4M_1^2} \right).
\]

(29)

In the last step in eq.(29) we have used eq.(21). Eq.(29) fixes \( B \)

\[
B = \frac{9}{11} \frac{r_0^2}{4\pi^2} \left( 1 + \frac{M_u^2}{M^2} \right) \sum_{i=u,d,s} q_i^2 \ln \left( \frac{M_i^2}{4M_1^2} \right) \approx 6.2
\]

(30)

where we have taken \( M_u = M_d = 5 \) MeV, \( M_s = 2M_u = 10 \) MeV and \( r_0^2/4\pi \approx 2.5 \).

Substituting from eqs.(30) and (27) into eq.(17) and setting \( N_f = 6 \), we get, as a check of consistency of this approach,

\[
\sigma_{\gamma p} = \frac{4\pi^3 B}{N_f} \frac{1}{m_0^2 + M^2} \approx 31 \text{ mb}
\]

(31)

(*) To get eq.(26) we have, for simplicity, neglected \( 4M^2 x^2 \) with respect to \( Q^2 \).
as against $\sigma_{pp}^{2/3} \sigma_{pp} \approx 26.7 \text{ mb}$ obtainable from the quark model (cf. eq.(1)). Thus with the Ansatz in eq.(17), vector meson-proton total cross sections can be determined in absolute value by using the generalized vector meson dominance model to relate them to deep inelastic scattering data. These data are needed for both $Q^2=0$, in order to fix the constant $B$ (see eqs.(28) and (29)) and asymptotically for $Q^2 \rightarrow \infty$ to fix the ratio $A/B$. Simple VMD phenomenology is important for the former set of data while leading QCD behaviour is relevant to the latter. The generalised vector meson dominance model has the virtue of unifying these two behaviours ($Q^2=0$ and $Q^2 \rightarrow \infty$) and interpolating smoothly between them.

3. COMMENTS AND CONCLUSIONS

Concerning the results of the last section a few comments are in order:

i) The large $\varphi$-p and $\psi$-p cross sections obtained from eqs.(17), (27) and (30) indicate extremely large contributions of sea quarks within the proton. This fact contradicts naive expectations and arguments based on the Zweig rule. For $\gamma$-p and $V$-p scattering, where quark-antiquark pairs are involved, couplings to valence quarks, in the proton, do not exhaust all the possibilities for the interaction. Sea quarks within the proton actually offer many more possibilities which can lead to the observed large $\varphi$-p and $\psi$-p cross sections. Nature seems to have actually taken advantage of these possibilities.

ii) In evaluating the constant $A$, it was sufficient to know the values of $\sigma_{pp}(Q^2)$ at $Q^2=0$ and $Q^2 \rightarrow \infty$ only. The GVMD interpolates between these limits. To see this interpolation explicitly as a function of $Q^2$ let us approximate $\text{Im} \Pi_1(s)$ in eq.(16) by the quark model asymptotic expression

$$\text{Im} \Pi_1(s) = \frac{a q_i^2}{\pi} \Theta(s-4M_1^2).$$ (32)

From eqs.(19), (20) and (25) one gets

$$\sigma_{pp}^{\varphi}(Q^2) = \frac{4a_i^2 q_i^2}{N_f} \cdot \frac{1}{Q^2-M_2^2} \left[ \frac{Q^2}{Q^2+4M_1^2} + \frac{M_2^2}{Q^2-M_2^2} \ln \left( \frac{Q^2-M_2^2}{Q^2+4M_1^2} \right) \right].$$ (33)

For simplicity we have put $B=N$, and shall comment briefly on this position below. $\sigma_{pp}^{\varphi}(Q^2)$, as given by eq.(33), is plotted against $Q^2$ in Fig.1 for $i=u,d,s,c,b$. We use $M_1=M_2=5 \text{ MeV}$; $M_3=2M_1=10 \text{ MeV}$; $M_4=4.2 \text{ GeV}$. The plot of
\( \sigma_{\gamma p}(Q^2) \), that is the sum over \( i=u,d,s,c,b \), is also shown.

**FIG. 1** - The total \( \gamma p \) cross section \( \sigma_{\gamma p}(Q^2) \) (full curve) as a function of \( Q^2 \), and its various flavour contributions (dashed curves).

Note from eqs. (7) and (26) that

\[
F_2(0, Q^2) = \frac{N_f}{4\pi^2} Q^2 \sigma_{\gamma p}(Q^2) \tag{34}
\]

is an increasing function of \( Q^2 \). It vanishes at \( Q^2 = 0 \) and tends asymptotically to

\[
F_2(0, Q^2) \rightarrow \sum_1^\infty q_i^2 Q^2 \rightarrow \infty. \tag{35}
\]

This behaviour of \( F_2(0, Q^2) \) is consistent with experiment also in orders of magnitude (7). It is a pattern of violation of scaling common to many models.

[1] The value of the constant \( B \approx 6.2 \) is remarkably close to the value of \( N_f \approx 6 \). This value is in fact quite stable, for we get practically the same number for \( B \) if, instead of the symmetry relations \( 1/f_0^2 : 1/f_\omega^2 : 1/f_\phi^2 = 1 : 1/9 : 2/9 \), we used directly the experimental values \( f_0^2/4\pi \approx 2.5 \); \( f_\omega^2/4\pi \approx 18.3 \); \( f_\phi^2/4\pi \approx 13.3 \). We stress however that \( B = \frac{\sigma_{\gamma p}(Q^2)}{\sigma_{\gamma p}(Q^2)} \), in-
dependent of $Q^2$, is an approximation. From eq.(26), its immediate consequence is that $F_2(x, Q^2)$ may be factorized as

$$F_2(x, Q^2) = F_2(0, Q^2) f(x)$$

(36)
on equivalently

$$\sigma_{\gamma p}(x, Q^2) = \sigma_{\gamma p}(0, Q^2) f(x)$$

(36')

where the function $f(x)$ satisfies

$$f(0) = 1; \quad f(1) = 0; \quad \int_0^1 dx f(x) = \frac{1}{B}.$$  \hspace{1cm} (37)

The function $f(x) = (1-x)^{B-1}$ satisfies these conditions but it is obviously not the only one.

It is suggestive in the present model to identify the constant $B$ with $N_f$. The numerical coincidence, $B - N_f = 6$, helps of course. So does the fact, noted under (i) above, that contrary to the intimations of the Zweig rule, $\gamma$-$p$ and $V$-$p$ scattering seem to sample the quark sea within the proton. In fact, if one defines $f_i(x) = f(x)$ as the probability to find a $\bar{q}q$ pair of flavour $i$ and energy fraction $x$ in the proton, then the integral in eq.(38) becomes

$$\sum_{i=1}^{N_f} \int_0^1 dx f_i(x) = \frac{N_f}{B}$$

(38)

and is unity for $B = N_f$.

In summary we conclude that:

a) Vector meson-proton total cross sections are not exactly proportional to the geometrical sizes of the vector mesons concerned, that is $m_v^2 \sigma_{\gamma p} \neq \neq A_0$, with $A_0$ independent of flavour. The slightly modified scaling law $(m_v^2 + M^2) \sigma_{\gamma p} = \text{const}$, where $M$ is the mass of the proton, gives cross section ratios in agreement with experiment for $V = q, \omega, \phi, \Psi$. By using the generalized vector meson dominance model to relate $\sigma_{\gamma p}$ to $\sigma_{\gamma p}(V, Q^2)$, in the limit $V \rightarrow \infty$, it is possible to determine the constant in the modified scaling law. Vector meson-proton total cross sections can thus be predicted in absolute value. The scheme which emerges is consistent.

b) The GVMD provides a smooth interpolation for the cross section $\sigma_{\gamma p}(Q^2)$ between $Q^2 = 0$, where simple VMD gives good results, and $Q^2 \rightarrow \infty$, where QCD holds and simple VMD gives too small a cross section $(\sigma_{\gamma p}(Q^2)_{\text{VMD}} \rightarrow 1/Q^4)$.
This interpolation is displayed as a function of $Q^2$ in Fig.1. There is strong similarity between the contributions of $\omega, \varphi$ and $\Psi$ families to $\sigma_{\gamma p}(Q^2)$ in this figure and their corresponding contributions to $\sigma_{\gamma \gamma}(Q^2)$ obtained in the same model\(^8\).

REFERENCES


