R. Barbini and G. Vignola: LELA: A FREE ELECTRON LASER EXPERIMENT IN ADONE

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1 – Introduction.

In the last few years there has been an increasing interest on the study of free electron lasers. The experiments performed by MADEY and coworkers [1] with the Stanford Superconducting Linac showed that it is possible to achieve both amplification of the coherent radiation and laser oscillation [2] by sending an electron beam through the periodic field of a magnetic undulator.

Tunability over a wide wavelength range (0.1–20 μm), sharp linewidth (Δλ/λ = 10⁻²–10⁻⁴) output power and efficiency make this type of laser an attractive tool for research in many fields such as photochemistry, molecular physics, solid state physics, etc.

From the theoretical point of view various approaches have been attempted in studying the interaction between electrons and radiation field, viz: quantum and classical theories, single particle and many particles (Maxwell–Boltzmann) pictures.

While we refer, for more details, to the extensive review articles of Pellegrini [3–5] on this argument, we just recall here that all theoretical treatments lead to the experimentally verified small signal gain formula, while disagreements appear on other aspects of the FEL (large signal, saturation). In particular, since all experimental information comes from Madey’s experiments only, the discussion is still open on which is the best electron accelerator to be used in connection with the FEL (storage ring, linac, microtron or else). However, the interaction between radiation and the recirculated e⁻ beam in a storage ring is, from the experimental point of view, an open problem worth of accurate investigation: we therefore propose to build a free electron laser device (LELA) by installing a suitable magnetic undulator on a straight section of the storage ring Adone.

The main goals of the LELA experiment are to collect informations on:
— amplification of radiation with the aid of an external ‘seed’ laser (Argon laser λ = 5145 Å);
— wavelength and optical gain as functions of electron energy and undulator magnetic field;
— transient behaviour of the laser radiation;
— steady-state interaction between laser radiation and stored electrons (i.e. optical gain vs. electron energy, maximum extractable optical power, optical spectrum).
2 - Fel and storage ring parameters.

It is proposed to install a transverse undulator on the Adone straight section no. 11. An optical cavity can then be built by adding a mirror at each end of the straight section, or an external laser beam can be sent along the s.s. axis to interact with the electron beam and the undulator field (see Fig. 1).

![Diagram of amplification and oscillation experiments](image)

Fig. 1 - (a) The amplification experiment; (b) the oscillation experiment.

In order to minimize the laser beam losses in the optical cavity (oscillator experiment) and to avoid head-on collisions between photons and electrons, it is convenient to operate Adone with three electron bunches and to adjust the optical cavity length to half the distance between two consecutive bunches: a single photon bunch will then travel inside the optical cavity and will meet one of the electron bunches once per every round trip.

The spontaneous radiation wavelength, observed on the undulator axis ($\equiv y$ axis) is given by

$$\lambda = \frac{\lambda_q}{2\gamma^2} \left(1 + K^2\right)$$

where $\lambda_q$ is the undulator period and $\gamma = E/m_0c^2$.

For a pure cosine-like vertical magnetic field on axis

$$B_z = B_0 \cos \frac{2\pi}{\lambda_q} y$$
the parameter $K$ is given by

$$K = \frac{e B_0 \lambda_d}{\sqrt{2} \frac{\pi m_e c^2}} \approx 6.8 \; B_0 \; (\text{kG}) \; \lambda_d \; (\text{m}).$$

By defining the R.M.S. magnetic field on axis by

$$\bar{B} = \left[ \frac{1}{\lambda_d} \int_0^{\lambda_d} |B_z(y)| \, dy \right]^{1/2}$$

one can also write

$$K = 9.33 \; \bar{B} \; (\text{kG}) \; \lambda_d \; (\text{m}).$$

According to Eq. (1), the wavelength can be tuned by varying either the electron energy or the undulator magnetic field or both.

In order to avoid all unnecessary technical complications, we choose to operate in the visible wavelength region with magnetic fields that can be achieved with standard magnet technology. The electron energy should be the highest possible in order to achieve the highest possible peak current.

**Table 1 — Basic FEL parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undulator period</td>
<td>$\lambda = 11.6 ; \text{cm}$</td>
</tr>
<tr>
<td>Number of periods</td>
<td>$N = 20$</td>
</tr>
<tr>
<td>Undulator length</td>
<td>$L_w = 2.32 ; \text{m}$</td>
</tr>
<tr>
<td>Homogeneous broadening</td>
<td>$\frac{\Delta \lambda}{\lambda} = \frac{\lambda_d}{2 L_w} = 2.5 %$</td>
</tr>
<tr>
<td>RMS magnetic field on axis</td>
<td>$\bar{B} = 3153 ; \text{G}$</td>
</tr>
<tr>
<td>Electron energy</td>
<td>$E = 610 ; \text{MeV}$</td>
</tr>
<tr>
<td>Radiation wavelength</td>
<td>$\lambda = 5145 ; \lambda$</td>
</tr>
<tr>
<td>Optical cavity length</td>
<td>$L = 17.5 ; \text{m}$</td>
</tr>
</tbody>
</table>

The undulator period should be designed so as to accommodate the maximum number of periods in the fixed length of the Adone straight section ($\simeq 2.5 \; \text{m}$). On the other hand the achievable magnetic field on axis will depend both on the undulator period and the gap height [7–9]. Finally electron energy, undulator period and magnetic field are connected by the wavelength Eq. (1). Taking into account the above constraints and using the results of magnetic field calculations [6–9] we end up with the basic FEL parameters listed in Table 1.

The FEL gain has been demonstrated [11,1] to be inversely proportional to the
square of the total spontaneous radiation linewidth, which, in turn, is made up of two contributions, homogeneous and inhomogeneous broadening, adding quadratically. We require the inhomogeneous broadening to contribute a negligible amount to the sum.

This places upper limits on the $e^-$ beam angular divergence and energy spread. It also requires that the off energy $\eta$ function vanishes in the section where the undulator is mounted. By increasing the number of independent $qdp$ families from 2 to 4, Adone can be made into a six-period machine with $\eta$ vanishing in alternative straights. The main machine parameters are listed in Table 2.

### Table 2 – Main machine parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron energy</td>
<td>$E = 610$ MeV</td>
</tr>
<tr>
<td>Momentum compaction</td>
<td>$\sigma_z = 1.36 \times 10^{-3}$</td>
</tr>
<tr>
<td>Fractional energy spread</td>
<td>$\sigma_\theta = 2.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>Invariant</td>
<td>$\langle H \rangle = 0.38$ m</td>
</tr>
<tr>
<td>Radial emittance (off coupling)</td>
<td>$A_x = 0.25$ mm $\times$ mrad</td>
</tr>
<tr>
<td>Energy loss in bending magnets</td>
<td>$U_b = 2.45$ keV/turn</td>
</tr>
<tr>
<td>Energy loss in undulator</td>
<td>$U_\nu = 109$ eV/pass</td>
</tr>
<tr>
<td>Radial betatron tune</td>
<td>$c_x = 5.15$</td>
</tr>
<tr>
<td>Vertical betatron tune</td>
<td>$c_z = 3.15$</td>
</tr>
<tr>
<td>Radial natural chromaticity</td>
<td>$C_x = -1.61$</td>
</tr>
<tr>
<td>Vertical natural chromaticity</td>
<td>$C_z = -1.06$</td>
</tr>
<tr>
<td>Damping partition numbers</td>
<td>$J_x = 2$ $J_z = J_x = 1$</td>
</tr>
<tr>
<td>Damping times</td>
<td>$\tau = 174/J D$ msec</td>
</tr>
<tr>
<td>Revolution frequency</td>
<td>$f_s = 2.856$ MHz</td>
</tr>
<tr>
<td>RF frequency</td>
<td>$f_{RF} = 51.4$ MHz</td>
</tr>
<tr>
<td>Harmonic number</td>
<td>$h = 18$</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>$n_b = 3$</td>
</tr>
<tr>
<td>1 RF cavity : RF peak voltage</td>
<td>$V_{RF} = 300$ kV</td>
</tr>
<tr>
<td>RF acceptance</td>
<td>$\varepsilon_{RF} = 3.58$ %</td>
</tr>
<tr>
<td>2 RF cavities : RF peak voltage</td>
<td>$V_{RF} = 600$ kV</td>
</tr>
<tr>
<td>RF acceptance</td>
<td>$\varepsilon_{RF} = 5.06$ %</td>
</tr>
</tbody>
</table>

3 – Small signal gain and bunch lengthening.

The FEL small signal gain per pass in the homogeneous broadening regime and for a monochromatic $e$ beam reads [4,5]

$$
\varepsilon_0 = -32 \sqrt{2} \pi^2 \lambda_{\nu}^{3/2} \lambda_\varphi^{1/2} \frac{K^2}{(1 + K^2)^{3/2}} \frac{I_p}{I_A} \frac{N^3}{\Sigma L} f(x)
$$

where

$I_A = \frac{e \gamma_0}{r_0} = 17,000$ A, \hspace{1cm} $\gamma_0 =$ working energy (in unit of $m_0c^2$).
\[ I_p = \text{peak current/bunch}, \quad \gamma_R = \text{resonance energy} = \left[ \frac{\lambda_0}{\lambda} (1 + K^2) \right]^{1/2}. \]

\[ x = 4 \pi N \frac{\gamma_0 - \gamma_R}{\gamma_R}, \quad f(x) = -\frac{1}{x^3} \left[ \cos x - 1 + \frac{1}{2} x \sin x \right]. \]

The function \( f(x) \) is proportional to the derivative of the spontaneous emission lineshape (sinc \( x/2 \)) (homogeneously broadened). By assuming its maximum value \((-0.0675 \text{ for } x = 2.6056)\) and taking for the E.M. beam cross section the value \( \Sigma_L = L_m \lambda_0^2 / \bar{\lambda} \) (see Eq. (17)), together with the parameters listed in Table 1, the gain can be written

\begin{equation}
(7) \quad g_0 = 6.7 \times 10^{-1} I_p(A) \approx 3 \times 10^{-3} \frac{i(mA)}{\sigma_r(cm)}
\end{equation}

where \( I_p(A) \) is the peak current/bunch \( (I_p = ci/2.35\sigma_h f_0) \), \( i(mA) \) is the mean current/bunch and \( \sigma_r \) is the R.M.S. bunch length.

\[ \langle a \rangle \]

\[ \langle b \rangle \]

\[ \text{NATURAL } \sigma_y \]

\[ \text{WITH ANOMALOUS LENGTHENING} \]

\[ V_{RF} = 300 \text{ kV} \]

\[ \text{WITH ANOMALOUS LENGTHENING} \]

\[ V_{RF} = 600 \text{ kV} \]

**Fig. 2** - \( \langle a \rangle \) Expected gain vs. current in the ring with (lower curve) and without (upper curve) anomalous lengthening for \( V_{RF} = 300 \text{ kV} \); \( \langle b \rangle \) idem for \( V_{RF} = 600 \text{ kV} \).

For the radiation build-up to take place, the optical gain per pass must exceed the cavity losses. Diffraction losses are negligible (see below) while mirror absorption
and transmissivity can reasonable be kept below a total of 4 \%. We require, by consequence, that the gain be raised to the level of percent, which can be accomplished with mean currents of some tens mA/bunch and \( \sigma_y \) of the order of cm (see [10]): at our working energy (610 MeV) the anomalous bunch lengthening phenomenon must therefore be properly taken in account.

Both according to the anomalous lengthening model of CHAO and GAREYTE [12] and to the experimental results obtained in Adone [13], \( \sigma_y \) can be written as [19]

\[
\sigma_y (\text{cm}) \approx 29 \left[ \frac{I (\text{mA})}{V_{RF} (\text{kV})} \right]^{37} ;
\]

accordingly

\[
\delta_0 \approx 10^{-4} I (\text{mA})^{43} V_{RF} (\text{kV})^{37}
\]

Fig. 2a and 2b show the gain curves for the two cases \( V_{RF} = 300 \) kV and \( V_{RF} = 600 \) kV.

We conclude that the anomalous lengthening significantly affects the gain. Accurate measurements at the energy and currents considered, with the new RF system, will have to be performed. Our present thinking is that the Chao–Gareyte type extrapolation gives as upper limit to the anomalous lengthening and, consequently, a lower limit to the gain.

4 – Electron beam lifetimes.

E-beam lifetimes are evaluated by taking into account the single and multiple Touschek effect, the vacuum chamber aperture and the RF acceptance.

4.1 – Touschek effect.

By using a computer code developed by WANG [18] we obtained, for \( I = 50 \) mA/bunch, the Touschek lifetimes (including multiple Coulomb scattering and anomalous lengthening)

\[
\tau_T = 449 \text{ hours} \quad (V_{RF} = 300 \text{ kV}), \quad \tau_T = 1056 \text{ hours} \quad (V_{RF} = 600 \text{ kV}).
\]

4.2 – Vacuum chamber aperture.

We recall [11] that the total radial spread can be written as

\[
\sigma_{z}^{2} = \sigma_{z\beta}^{2} + \sigma_{z\epsilon}^{2},
\]

where

\[
\sigma_{z\beta}^{2} = \sigma_{p}^{2} \langle H \rangle \text{ mag} \frac{J_{x}}{J_{x}} \beta_{z} \frac{1}{1 + z^{2}}
\]
is the betatron contribution ($\chi^2$ is the coupling coefficient for betatron oscillations) and $\sigma^2_{xe}$ is the energy spread contribution.

Though nothing can be said on the transient laser behaviour, we can reasonably take that if a steady state is to be reached, then the R.M.S. electron energy spread should attain the equilibrium value

\[
\frac{2 \Delta \gamma}{\gamma} = \frac{1}{2 N},
\]

so that

\[
\sigma_{xe} \approx \eta \frac{\Delta \gamma}{\gamma} \approx \eta \frac{1}{4 N},
\]

which would be the major contribution to the total spread[13], occurring where $\eta \neq 0$.

In this case the beam lifetime is [15]

\[
\tau_{ge} = \tau_x e^{(d \sigma_{xe})^2} \left( \frac{\sigma_{xe}}{d} \right)^2
\]

where $d$ is the minimum halfwidth of the vacuum chamber in Adone. With $d = 7$ cm, $\tau_x = 174$ msec and $\eta = 2$ m, we get: $\tau_{ge} \approx 56$ sec.

4.3 - RF acceptance.

With similar arguments we can write the beam lifetime for energy oscillations, by assuming a gaussian energy spread distribution function, as

\[
\tau_{ge} = \tau_x e^{\left( \frac{\Delta \gamma}{\gamma} \right)^2 \left( \frac{\Delta \gamma}{\gamma_{RF}} \right)^2}
\]

With $\tau_x = 87$ msec we have $\tau_{ge} \approx 38$ sec (1 RF cavity), $\tau_{ge} \approx 19$ hours (2 RF cavities).

We observe that lifetimes under 4.2 and 4.3 are calculated by assuming gaussian e$^-$ beam distributions. Actually, in a steady state laser operation, electrons should have (4) an harmonic oscillator energy distribution whose tails are more sharply cut than those of gaussian distributions. This should possibly lead to an increase of lifetimes.

5 - Optical parameters.

According to Eq. (6) the FEL gain is inversely proportional to the optical mode cross section $\Sigma_L$ in the interaction region, provided the electron beam is fully contained within the laser beam ($\Sigma_e < \Sigma_L$).
The laser beam cross section $\Sigma_L$ is given by

\begin{equation}
\Sigma_L = \frac{1}{L_w^2} \int_{-L_w^2/2}^{L_w^2/2} \Sigma_L(y)dy = \frac{2\pi w_0^2}{L_w^2} \int_0^{L_w^2/2} \left[ 1 + \left( \frac{\lambda y}{\pi w_0^2} \right) \right] dy
\end{equation}

where $L_w$ is the length of the interaction region and $w_0$ is the beam waist for a TEM$_{00}$ gaussian mode.

The optimum beam waist can be found by minimizing the mode cross section which implies

\begin{equation}
\tilde{\Sigma}_L = \frac{L_w \lambda}{\sqrt{3}}, \quad w_0 = \sqrt{\frac{L_w \lambda}{\pi \sqrt{3}}}.
\end{equation}

For $\lambda = 5145$ Å (our experiment) the optimum beam waist becomes $w_0 \approx 0.35$ mm.

5.1 – The amplification experiment.

A possible layout of the amplification experiment is sketched in Fig. 1a, where distances and focal lengths are designed to achieve $w_0 = 0.35$ mm at the undulator mid-point [17].

5.2 – The oscillator experiment.

The optical cavity parameters must also be chosen so as to produce a waist $w_0 = 0.35$ mm at the center of the interaction region for a wavelength $\lambda = 5154$ Å.

In Fig. 1b $M_i$ are concave mirrors with curvature radii $R_i$.

In Table 3 we show the optical cavity parameters. $w_0$ are the beam sizes on the mirrors and the corresponding Fresnel number $N = a^2/d\lambda$ is calculated assuming the mirror size $a$ is equal to the largest $w_0$.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
$d_1$(m) & $d_2$(m) & $w_0$(mm) & $w_1$(mm) & $w_2$(mm) & $R_1$(m) & $R_2$(m) & $N$ \\
\hline
8.75 & 8.75 & 0.35 & 4.14 & 4.14 & 8.61 & 8.61 & 2.2 \\
\hline
\end{tabular}
\end{table}

\( \left( 1 - \frac{d_1 + d_2}{R_1} \right) \left( 1 - \frac{d_1 + d_4}{R_2} \right) = 0.973 \)
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[16] Bruck, H.: ibidem, Chapter XXX and XXXI.


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[16] Bruck, H.: ibidem, Chapter XXX and XXXI.

