S. Date and A. Nakamura: A DEPENDENCE OF LARGE-TRANSVERSE AND MASSIVE-MUON-PAIR PRODUCTIONS: EFFECTS OF MULTI-QUARK CLUSTERS ON HIGH ENERGY REACTIONS OFF NUCLEAR TARGETS
A DEPENDENCE OF LARGE-TRANSVERSE AND MASSIVE-MUON-PAIR PRODUCTIONS: EFFECTS OF MULTI-QUARK CLUSTERS ON HIGH ENERGY REACTIONS OFF NUCLEAR TARGETS

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ABSTRACT

Assuming the existence of multi-quark clusters in nucleus, we investigate large transverse momentum production in hadron-nucleus and alpha-alpha collisions and massive muon pair production off nuclear targets in the framework of hard scattering model. The probability of multi-quark clusters in nucleus is parametrized in a simple form and the modified Kuti-Weiskopf model is employed to determine the parton distributions in the clusters. The approach yields results which are in reasonable accord with the data. The folklore that multi-quark clusters in nuclei are rejected by muon pair experiments is misleading.

1. - INTRODUCTION

Particle production at large transverse momentum and massive lepton pair production on nuclear targets attract much interest because they offer a chance to study the behavior of constituents inside nuclear matter. Due to a small distance between the nucleons in the nucleus, it is doubtful whether these processes can be described in terms of quasi-free nucleons and free hadrons passing through them. Actually, the experimental results on the inclusive cross
section at large transverse momentum are contrary to the naive picture: the cross section is proportional to $A^\alpha$, with $\alpha > 1$, rather than $A^1$, where $A$ is the nuclear mass number. Fig. 3 shows the data\(^1\) of the power $\alpha$ as the function of the $p_T$ for $\pi^\pm$ production in proton-nucleus collisions at $\sqrt{s} = 27.4$ GeV.

In the framework of the quark-parton model, two theoretical ideas have been proposed to explain this what is called abnormal nuclear enhancement of large $p_T$ production; One is based on the multiple scattering picture. Another is on the existence of multi-quark states in a nucleus. Several authors\(^2\) have studied the effects of multiple scattering of a parton, which is kicked out at a hard collision, off the other nucleons inside the nucleus. The maximum momentum of a parton in a 31-quark state\(^3\) is $i$ times as large as that of a parton in a usual nucleon; they may be high momentum partons in nucleus. The idea of coherent fluctuation of nuclear matter was first used by Blokhintsev\(^4\) and the quark-parton modifications of this idea proposed by Krzywicki\(^5\) and Efremov\(^6\) are applied to large $p_T$ production\(^5,7,8\) and backward production processes\(^6,9\), though the large $p_T$ process is treated in a rough manner.

From the present theoretical understanding of hadron physics, we cannot reject nor accept as a matter of course these two possibilities. There is no reliable estimate how long a hard parton can run through\(^10\). And we cannot calculate the probability of finding multi-quark states in nucleus though there is a guess based on classical picture.

It is believed, however, that the latter failed because the existence of multi-quark states in nucleus will lead to abnormal nuclear enhancement of massive lepton pair production, which is contradict to the experimental results. This is a misunderstanding. Partons which contribute mainly to the present experimental data of large $p_T$ reaction and lepton pair production are in different kinematical regions. See Fig. 10.

Recent analysis of electromagnetic form factors of deuteron, $^3$He and $^4$He showed that the derivation from the prediction of standard nuclear physics at large momentum transfer regions can be well explained by taking into account multi-quark configurations in these nuclei\(^11,12\). These configurations in elastic form factors will affect parton distribution functions through Drell-Yan-West relation\(^13,14\).

In this paper we will present the quantitative analysis of large $p_T$ and lepton pair production processes off nuclear targets and the first measurement of large $p_T$ process in high energy alpha-alpha collisions under the assumption of the existence of multi-quark clusters in nuclei. We will show that the observed mass number dependence for massive muon pair production is consistent with that of large $p_T$ production. The consistency between the probability of multi-quark configurations in light nuclei using in our analysis and that of elastic form factors will be discussed.
2. HARD SCATTERING MODEL WITH MULTI-QUARK CLUSTERS

The basis of our calculation is the simple hard scattering model. We write the differential cross section for large $p_T$ processes\(^{(15)}\) in collisions of particles $A$ and $B$

\[
\frac{d^2\sigma}{dp_T^2} = \sum_{a,b,c} \frac{1}{z_c} \frac{1}{\pi} \delta(\mathbf{p}_T - \mathbf{x}_a + \mathbf{x}_b) \frac{d\sigma}{d\Omega} + \frac{d\sigma}{d\Omega},
\]

and for massive lepton pair production\(^{(13)}\)

\[
\frac{d^2\sigma}{dx_F dM^2} = \frac{4\pi\alpha^2}{9M^2s} \sum_{a,b,c} x^2 \int dx_a dx_b \frac{G_a(x_a)}{x_a} \frac{G_b(x_b)}{x_b} \delta(x_a x_b - \frac{M^2}{s}) \delta(x_F - x_a + x_b),
\]

where $a, b$ and $c$ denote $u, d, \bar{u}, \bar{d}$ and $\bar{s}$ quarks. For the quark-quark scattering cross sections $d\sigma/d\Omega$ and $d\sigma/d\Omega$, and the decay function $D_c(z)$, we borrow the phenomenological form of Field and Feynman\(^{(15)}\). We also use their form for the quark distribution functions $G_{a/A}(x)$ when $A$ is a nucleon. For the pion structure functions, we neglect sea quarks and used the form

\[
G_{a/\pi}(x) = 0.9 x^{-1/2} (1 - x)^{1.27}
\]

where $a$ denotes valence quarks in a pion, which was obtained from the measured muon pair data by the Chicago-Illinois-Princeton group\(^{(16)}\). Here we neglect $Q^2$ dependence of $D_c(z)$ and $G_{a/A}(x)$ because inclusion of scale violations will not change our conclusions drastically. We also neglect an internal transverse momentum of constituents.

The most essential ingredient of the model considered here is the distribution function $G_{a/A}(x)$ in a nucleus. We do not regard this distribution as a collection of $A$ nucleons. When a nucleus is probed through a large momentum transfer process, we can take its snapshot of very short space time. There is a possibility that, in such very short time, several nucleons overlap each other within a hard core radius. It is difficult to treat these states in terms of the present nuclear physics where only nucleons are considered as degrees of freedom. Rather it is simple and better to manipulate these states in terms of quark physics.

The parton distribution function of a nucleus is assumed to have the form

\[
G_{a/A}(x) = \sum_{i=1}^{A} n_i^A \tilde{G}_{a/3i}(x),
\]

where $x$ is momentum fraction of a parton $a$ which is normalized to the mean momentum of one nucleon in the nucleus $A$. The quantity $n_i^A$ stands for the expectation value of the number of $3i$-quark clusters in a nucleus. The parton distribution in the $3i$-quark cluster, $\tilde{G}_{a/3i}(x)$, satisfies the following conditions:
\[ \sum_a \int_0^1 x \tilde{G}_{a/3l}(x) \, dx = 1, \]  
\[ \sum_a \int_0^1 \tilde{G}^v_{a/3l}(x) \, dx = 3l, \]  
where \( l = 1, 2, \ldots, A \), and in the first equation runs over all flavors and gluon and \( \tilde{G}_{a/3l}(x) \) is the contribution from valence quarks. Note that the upper limit of these integrals is not 1 but 1.

We have the condition for \( n_i^A \):

\[ \sum_{i=1}^A n_i^A = A. \]  

We parametrize \( n_i^A \) in the form of the simplest probability distribution:

\[ n_i^A = \binom{A}{i} (1-p(A))^{A-1} (1-p(A))^{A-1}, \quad 0 \leq p(A) \leq 1. \]

We treat \( p(A) \) as parameters and investigate whether many experimental data can be explained consistently.

3. DISTRIBUTIONS OF PARTONS IN MULTI-QUARK CLUSTERS

An explicit form is needed for \( \tilde{G}_{a/3l}(x) \). We cannot, of course, extract this function from experimental data. So far there have been two trials to calculate this \( x \)-dependence using quark models\(^{(5,8)} \). However we cannot adopt their results here. In the pioneer work of Krzywicki\(^{(5)} \), he treats a whole nucleus as one multi-quark cluster. This is a too simple and, as the author said, toy model. Wakamatsu\(^{(8)} \) neglects sea quarks which contribute largely to the Drell-Yan processes. Moreover his results do not continue smoothly to the case of \( l = 1 \), i.e. nucleons.

Here we use an extension of the modified Kuti-Weisskopf model\(^{(17)} \). This model has been exhaustively investigated in the case of usual hadrons and many parameters in the model have been determined in these analyses. The distributions calculated below are consistent with the counting rule and the result reduces to that of Field and Feynman when \( l = 1 \).

In the framework of the modified Kuti-Weisskopf model\(^{(17)} \), the exclusive probability density function for a system with \( m \) valence quarks, 2\( n_a \) sea quarks of the flavours \( a \) and \( \bar{a} \), and \( n_g \) gluons is given by:
\[ p_{n_a}^m, n_g (x_1, \ldots, x_m, y_a, \ldots, y_a, z_1, \ldots, z_n_g) = N \prod_{a} \frac{\text{C}_a^{2n_a}}{(2n_a)!} \frac{\text{C}_g^{n_g}}{n_g!} \delta(1 - \Sigma_{i=1}^m x_i - \Sigma_{a=1}^n y_{a} - \Sigma_{j=1}^n z_j) \prod_{j=1}^m \sqrt{\frac{v_j}{x_j^2 + \mu^2/P^2}} \prod_{a} \frac{s_a(y_{a})}{\prod_{j=1}^m \sqrt{\frac{s_a(y_{a})}{v_{a}^2 + \mu^2/P^2}}}, \]

where \( N \) is the normalization factor, \( \text{C}_a \) and \( \text{C}_g \) are the statistical factors for sea quarks of the flavors \( a \) and \( \bar{a} \) and gluons, respectively. The index \( a \) runs over \( u, d \) and \( s, \mu \) is the effective mass of the partons and \( P \) is the total momentum of the system. The physical quantities of interest are calculated in the limit \( P \to \infty \). In the expression (9), we have indistinguishable sea quarks and anti-sea quarks labeled by the same flavor index \( a \). We parametrize the function \( v_j(x) \) as

\[ v_u(x) = v_d(x) = x^\delta(1 + \beta x + \gamma x^2), \]

and put

\[ s_a(y) = g(z) = 1. \]

From eqs. (9)-(11) one obtains, with the help of standar Laplace transformation techniques, the valence quark distributions,

\[ G_{u/m}^V(x) = (1 + \frac{Z}{A}) \frac{1}{3} V_m(x), \]

\[ G_{d/m}^V(x) = (2 - \frac{Z}{A}) \frac{1}{3} V_m(x), \]

where \( Z \) denotes number of protons in the nucleus \( A \) and

\[ V_m(x) = x^{-1}(1 + \beta x + \gamma x^2)(1 - x)^{C+(m-1)\delta-1}\Phi_m^{-a}(x), \]

and the sea parton distributions,

\[ C_{a/m}^s(x) = \frac{1}{2} C_{a} x^{-1}(1 - x)^{C+m\delta-1}\Phi_m^m(x), \]

for sea quarks \((a = u, d, s, \bar{u}, \bar{d}, \text{or} \bar{s})\) and

\[ G_{g/m}^s(x) = C_g x^{-1}(1 - x)^{C+m\delta-1}\Phi_m^m(x), \]

for gluons. Here \( C = \frac{1}{2} \sum_a C_a + C_g \) and the functions \( \Phi_m^{-a}(x) \) and \( \Phi_m^m(x) \) are given by

\[ \Phi_m^{-a}(x) = \frac{1}{\Gamma(\delta) Y_m^{m-1}(1-x)}, \quad \Phi_m^m(x) = \frac{1}{Y_m^m(1-x)}, \]
where

$$Y^m(y) = \sum_{\mu=0}^{m} \sum_{\nu=0}^{\mu} \delta^{\mu}(\gamma+1)(\mu+\nu) \Gamma^{-1}(m\delta + C + \mu + \nu). \tag{18}$$

These functions approach finite constant when $x \to 1$. The distribution function for partons of type $a$ in a $3l$-quark cluster is now given by

$$G_{a/3l}(x) = G_{a/3l}^V(x) + C_{a/3l}^s(x). \tag{19}$$

Renormalizing the momentum fraction to the mean momentum of one nucleon in the cluster, we get

$$C_{a/3l}(x) = \frac{1}{1} G_{a/3l}^V(x), \quad 0 \leq x \leq 1. \tag{20}$$

The parameters of the distribution functions (12-15) are determined as follows. First, when $m = 3$, we put these values so that the distributions of Field and Feynman (15) are approximately reproduced (18);

$$\delta = 0.5, \quad \beta = 5.0, \quad \gamma = 0.0, \quad C_u = C_d = 0.24, \quad C_s = 0.18, \quad C_g = 2.84. \tag{21}$$

The distributions with these parameters and calculated values for $\nu W_2^{SP}$ are shown in Fig. 1a and 1b, respectively, with those of Field and Feynman.

**FIG. 1** - a) Distribution functions for valence $u$ and $d$ quarks within a proton. The solid curves are calculated from the modified Kuti-Weisskopf model with parameters given in Eq. (21). The dashed curves are valence distribution functions of Field and Feynman (15). b) Structure function $\nu W_2^{SP}(x)$ for electron-proton scattering. Solid and dashed curves are the same as in Fig. 1a.
Next, we assume $\beta$, $\nu$, $C_a$'s and $C_{g}$ are independent of $m$ and consider $m$ dependence of $\delta$. The behavior of the valence quark distributions at $x \to 1$ is

$$G_{a/m}^V(x) \sim (1 - x)^{C + (m-1)\delta - 1},$$

while the constituent counting rule claims that elastic form factor near $x = 1$ has the form\( (1 - x)^{2m-b} \).

Using the Drell-Yan-West relation\( (13, 14) \), we require these two behaviors to coincide;

$$\delta = \frac{2(m - 1) - C + 3 - b}{m - 1}.$$  \hfill (22)

Because $\delta = 0.5$ for $m = 3$, we obtain $b = 2, 5$.

Using the parameters given in (21) and (22), we can calculate the distribution functions of a multi-quark system with arbitrary number of valence quarks. Note that there are no further parameters to be adjusted. In Fig. 2, we show the quantity $x\tilde{G}_a^V(x) = \sum_{a=u,d,s} x\tilde{G}_a^V(x)$ for $i = 1, 2$ and 3. The momentum fraction carried by gluons is depressed for $i > 1$ due to the increase of $\delta$.

4. - COMPARISON WITH EXPERIMENTAL DATA

To fix the parameters $p(A)$ in Eq. (8), we calculate the differential cross sections for large $p_T \pi^+$ production in 400 GeV proton collisions on p, d, Be, Ti and W targets and fit the $A$ dependence of the cross sections to the data obtained by Chicago-Princeton group\( (1) \). The $A$ dependence of the cross sections is expressed by the quantity $\alpha$:

$$\alpha = \ln \left[ \frac{d^3\sigma_{A_1}}{dp^3} / \frac{d^3\sigma_{A_2}}{dp^3} \right] / \ln(A_1/A_2),$$  \hfill (23)
where $A_1$ and $A_2$ are atomic weights of target nuclei. From the fit to the values of $a$ obtained from the data\(^{(1)}\) for p-p and p-W collisions, we fixed p(W) = 0.0025 (See Fig. 3). The calculated cross sections for d, Be and Ti targets are divided by that for W target and the values of p(d), p(Be) and p(Ti) are adjusted to fit the data. Our fits are shown in Fig. 3.

**FIG. 3** - Values of $a$ versus $p_T$ for $\pi^+$ production in 400 GeV proton-nucleus collisions. The two targets, which are expressed as $A_1$ and $A_2$ in Eq. (23), are indicated below the data. The curves are calculated values for p(W) = 0.0025 (solid curve), p(d) = 0.065 (solid curve), p(d) = 0.040 (dashed curve), p(Be) = 0.022 (solid curve), p(Be) = 0.015 (dashed curve), p(Ti) = 0.0090 (solid curve) and p(Ti) = 0.0060 (dashed curve). The data are obtained from Ref. (1).

In order to calculate cross sections for other targets, we have investigated the $A$ dependence of the values of p(A). As shown in Fig. 4, p(A) can be fitted by

$$ p(A) = 0.085 A^{-0.67}. \quad (24) $$

In the following analysis, we use this form to evaluate $n_A^A$ and investigate whether it is consistent with experimental data on other nuclear targets and on other processes, i.e. massive muon pair production. The resulting predictions for $n_A^A$ for several values of A are shown in Fig. 5. In Fig. 6, we show calculations for quark distributions.

**FIG. 4** - Values of p(d), p(Be), p(Ti) and p(W) which are corresponding to the curves in Fig. 3. The solid line is fit to these values by Eq. (24).
FIG. 5 - i-dependence of expectation values of the number of 3i-quark clusters within a nucleus divided by the mass number A. Eq. (24) is adopted for p(A) which is need to evaluate these values. Black circles are results for A = 10, black triangles are for A = 50, black squares are for A = 100 and open circles for A = 200.

within a nucleus A summed over all flavors:

\[ xG_A(x) = \sum_{a=u,d,s,\bar{u},\bar{d},\bar{s}} xG_{a/A}(x). \]

In Fig. 7, we compared \( \alpha \) for 200 GeV p-p and p-Al collisions with the experimental data (20). The calculated curve well reproduces the data.

FIG. 6 - Flavor summed quark distribution functions within a nucleus of A = 10 (solid curve), A = 50 (dashed curve), A = 100 (dashed-dotted curve) and A = 200 (dotted curve), divided by each A.

FIG. 7 - Values of \( \alpha \) versus \( p_T \) for \( \pi^0 \) production in 200 GeV p-p and p-Al collisions. The solid curve is the result of calculations explained in the text. The data are taken from Ref. (20).
The A dependence of the differential cross sections for massive muon pair production in 225 GeV $\pi^-$ collisions on C, Cu and W targets are compared with the experimental data\(^{(21)}\) in Figs. 8a and 8b. The calculated values are consistent with the data. Our calculations, of course, reproduce the experimental power-law dependence of the cross sections, $\sigma \propto A^\alpha$.

In Fig. 9, we made a same comparison for massive muon pair production in 400 GeV proton collisions on Be and Pt targets. The agreement between the calculated values and the data\(^{(22)}\) is good.

One might wonder why the abnormal nuclear enhancement ($\alpha > 1$) observed in large $p_T$ reactions seems to be absent in massive muon pair production processes. In our calculation, this can be explained by the difference in kinematical regions of partons. Ranges of momentum fraction $x$ of partons which are in the target nucleus and contribute to the cross sections are displayed in Fig. 10. For large $p_T$ production cross sections of proton-nucleus, partons momentum fraction $x$, which is in the range given by intersection of the shaded region in Fig. 10 and the line of $x_T = 2p_T/\sqrt{s} = \text{const.}$, are integrated in Eq. (1). In this integration, main contributions arise from $x > x_T$. On the other hand, as shown in Fig. 10, values and ranges of $x$ for massive muon pair production are rather small compared with large
$p_T$ production. From Fig. 6, it is easily understood that contributions from enhanced fast quarks in large nuclei are smaller for massive muon pair production than for large $p_T$ production.

Finally, we have also analyzed recent experimental results on alpha-alpha collisions at CERN-ISR\textsuperscript{[23]}. Quark distributions in both a target alpha particle and a beam alpha particle are calculated according to Eq. (4) and by use of Eq. (24). The resulting prediction on the values of the power $\alpha$ in $(4 \times 4)^\alpha$ are shown in Fig. 11a. The calculated values are in agreement with the data. To see consistency of our calculations, we compared values of $\alpha$ for alpha-proton and proton-proton collisions with the data\textsuperscript{[23]} in Fig. 11b.

**FIG. 10** - Kinematical regions of partons for large $p_T$ and massive muon pair production. $x$ is momentum fraction of partons within a target nucleus $A$ and normalized by mean momentum of one nucleon in the nucleus. The shaded region is corresponding to the large $p_T$ experiments of Ref. (1), the black region and the dotted region are corresponding to massive muon pair production experiments of Ref. (22) and Ref. (21), respectively.

**FIG. 11** - Values of $\alpha$ versus $p_T$ for $\pi^0$ production in: a) alpha-alpha and proton-proton collisions at $\sqrt{s} = 30.6$ GeV, and b) alpha-alpha and proton-proton collisions at $\sqrt{s} = 44$ GeV. The solid curves are the results of calculations. The data are obtained from Fig. 1 in Ref. (23).
5. - DISCUSSION

In this paper we have emphasized that the existence of multi-quark states in nuclear matter is not denied by the present experimental data of lepton pair production off nuclear targets. Indeed we can explain both these data and abnormal nuclear enhancement of large \( p_T \) processes on the assumption of multi-quark clusters in nuclei. As far as we know, there was no quantitative analysis of lepton pair production off nuclear targets on these assumptions.

It is interesting that the parameters \( p(A) \) fitted here are very consistent with those which were obtained in the analysis of electromagnetic form factors of \( d, \ ^3\text{He} \) and \( ^4\text{He} \). To see this, we calculate \( n_1 \), \( n_3 \) and \( n_4 \) from the existence probabilities of 3i-quark clusters given in Ref. (11) and fit it to the form (8) by means of \( p(A) \). Then we get 

\[
 p(d) = 0.05, \ 0.014 \leq p(\ ^3\text{He}) \leq 0.040 \text{ and } 0.021 \leq p(\ ^4\text{He}) \leq 0.052.
\]

Those values are quite consistent with our fit given by Eq. (24). It is doubtful whether, in these high energy and large momentum transfer processes, the ingredients of a nucleus are only quasi-free nucleons.

In order to confirm, or reject, these multi-quark clusters in nuclei, direct measurement of nuclear structure functions is required, i.e. deep inelastic scattering of lepton-nucleus at \( x \geq 0.5 \). These experiments might reveal unexpected features of nuclei to us.

Though it is not a decisive one, simpler test is to search for the large \( p_T \) particles produced out of a kinematical region, i.e. \( x_T \sim 1 \). This experiment is impossible at more than several hundreds GeV because the kinematical limit of \( p_T \) is large and the cross section is very small. At less than scores GeV, however, the cross section is measurable. For example, at 4.3 GeV, the kinematical bound of \( p_T \) is 1.03 GeV/c and the cross section is roughly estimated to be \( 1 \mu \text{b} \). If we can find particles out of the kinematical region, they will be an evidence of "high density of energy in nuclear matter" (24).

Besides the hypothetical existence of multi-quark clusters discussed here, there is another interesting possibility in large \( p_T \) production and deep inelastic scattering off nuclear targets: Jets which are produced at a hard collisions may travel in nuclear matter. Though we used in this paper the same decay functions of jets as in the case of nucleon targets, recent experiments (20) suggests that a form of the decay function of jet depends on nuclear mass number. We will discuss this problem in a forthcoming paper.

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REFERENCES AND NOTES


(3) - We assume that multi-quark states in nucleus are made of 3l valence quarks where l is an integer, i.e. they have an integer baryon number.


(7) - E, Illam and Y. Zarmi, Bielefeld preprint BI-TP 76/28 (1976).


(18) - We do not distinguish distributions for valence u and d quarks out of normalizations to reduce the number of parameters. The values of C_q's and C_g are borrowed from Ref. (17).

(19) - Though the simple counting rule requires b = 3, this is contradicted to the d quark distribution in a proton.


