G. Parisi: RANDOMNESS AS A SOURCE OF MASSLESS PARTICLES

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RANDOMNESS AS A SOURCE OF MASSLESS PARTICLES

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It is well known that massless particles arise naturally from symmetry principles, e.g. Goldstone bosons, gauge fields, $\gamma_5$ invariance, ... In this talk I speculate that maybe there is a new unforeseen mechanism which could generate massless particles.

Let us construct a very simple method to illustrate this mechanism. We assume that space-time can be divided into flat regions (which will be called points) connected by regions where the structure of space-time is very twisted (bridges or links). We suppose that the points and the links can be organized in such a way as to form a lattice (or an amorphous solid): the links should connect only points which are not too far separated. A macroscopically flat space-time structure can be introduced.

Let us consider a quantum field theory on this space: a self-interacting scalar field $\phi$. We assume that the field $\phi$ will be practically constant inside each point: we denote by $\phi_i$ the value of the field at the point $i$; the net effect of the bridges is to couple the fields defined on different bonds. The final Lagrangian is

$$\mathcal{L} = \sum_i \left( \frac{1}{2} m^2 \phi_i^2 + g \phi_i^4 + \sum_k J_{ik} (\phi_i - \phi_k)^2 \right)$$

Equation (1) is the starting point of this analysis; the reader should note that in the derivation of Eq. (1) he (she) is completely free to substitute for the gravitational interaction I have used, any other interaction he (she) may prefer.
It is evident that if all the $J_{ik}$ are positive and $m^2$ is negative the symmetry $(Z_2) \phi \leftrightarrow -\phi$ is spontaneously broken, but Goldstone bosons are not present as the symmetry group is discrete. Let us consider a more complex case in which the $J_{ik}$ take randomly the values $\pm 1$.

Normally we would think that the distribution of the $J_{ik}$ is influenced by their interaction with the field $\phi_i$. Let us assume that this effect is very small and can be neglected; we can visualize the $J_{ik}$ as semi-classical macroscopic variables which evolve according to the internal (ergodic) dynamics and are not influenced by the interaction with a quantum field.

We consider now the mean value of the propagators:

$$G^{(1)}(i) = \frac{\sum_k \langle \phi_{i+k} \phi_k \rangle}{\sum_k 1}$$

$$G^{(2)}(i) = \frac{\sum_k \langle \phi_{i+k} \phi_k^2 \rangle}{\sum_k 1}$$

The sum over $k$ is needed because the system is not translationally invariant. Although $\langle \phi_{i+k} \phi_k \rangle$ depends on the particular realization of the $J_{ik}$, it is believed that with probability one in the infinite volume limit the mean propagators will be equal for all the choices of the $J_{ik}$ made according to the same probability law (central limit theorem).

The invariance of the model under the local gauge transformation ($\phi_i \rightarrow \phi_i$, $J_{ik} \rightarrow -J_{ik}$) implies that $G^{(1)}(i) = 0$, $i \neq 0$. The only information is contained in the $G^{(2)}(i)$.

It is not clear to me if this model is in contradiction with well-known facts, e.g. energy conservation is only a statistical law; my aim is to point out that when $m^2$ is negative, one finds in a rather unexpected way that the $G^{(2)}(i)$ is long range, i.e. its Fourier transform $\tilde{G}^{(2)}(k^2)$ contains an infinite number of poles with an accumulation point at zero:

$$\tilde{G}^{(2)}(k^2) \sim \sum_{n=0}^{\infty} \frac{C_n}{(k^2 + \mu_n^2)} : \mu_n \rightarrow 0 \text{ as } n \rightarrow \infty$$
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We have succeeded in generating in a natural way an infinite number of particles from only one field. The reader expert in solid state physics will find this note rather uninteresting: our construction reproduces the definition of a quenched spin glass\textsuperscript{1}. Equation (3) has been suggested in Ref. 2 and it is based on the approach of Ref. 3. The arguments leading to Eq. (3) are rather involved and they will not be reproduced here: unfortunately a simple clear-cut physical explanation of the phenomenon is still lacking.

The content of this paper can be summarized as follows: in recent years, solid state physicists have started to study systems with quenched disorder: they have discovered that in particular cases long range correlations may be present (i.e. massless particles in the field theory language). I think that physicists working in the theory of elementary particles should be aware of this effect and study if and how it can be incorporated into a realistic theory.

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REFERENCES