A. Turrin: MEZEI NEUTRON SPIN ECHO CONFIGURATION AS A SPIN-FLIPPING DEVICE.
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Abstract.

Hayter's matrix description of neutron-spin dynamics predicts extraordinarily high flipping efficiencies for the conventional Mezei neutron spin echo configuration, when used as a nonadiabatic spin-flipping device.

In the present letter we discuss the principle of operation of the Mezei spin echo configuration\(^1\). To do this we develop its theory based on Hayter matrix technique\(^2\) and conclude that this configuration can be used as a very efficient nonadiabatic spin flipper for polychromatic polarized neutron beams. In other words we show that there is no requirement for an exact one half Larmor precession in each coil, even under white beam conditions.
A sketch of the spin echo configuration, as originally suggested by Mezei, is shown in Fig. 1 (or and Fig. 4 of Ref. 1).

![Diagram of the Mezei spin echo configuration](image)

**FIG. 1 -** Schematic representation of the Mezei spin echo configuration. The arrows indicate the direction of the guide field, $\vec{H}_0$. $d$ is the thickness of each coil.

Here, the beam travels in the $y$-direction. The direction of the guide field, $\vec{H}_0$, parallel to the $z$-direction, is reversed at the point halfway between the two coils C1 and C2. Such a sudden reversal may be accomplished by the current carrying sheet method.\(^3\), \(^4\)

Coils C1 and C2 are equal in thickness and produce equal magnetic fields $\vec{H}_1 = \vec{H}_2 = \vec{H}_c$, which are taken to point perpendicular to the plane of the figure (i.e. both antiparallel to the $x$-direction, that points upwards).

Used as a spin flipper, this configuration is in the "flipper on" state when the coils C1 and C2 are in off (i.e. $H_c = 0$). Therefore, this state of the device calls for no comment from here downwards.

To calculate the flipper efficiency, $f$, under the tuning conditions that will make the polarization vector be parallel to the reversed guide field ($-\vec{H}_0$) after the sequence is passed through ("flipper off" state) we follow Hayter's treatment and give here below the transfer matrix $\hat{M}$ connecting the polarization vector $\vec{P}_i = (0, 0, 1)$ before and
\( \vec{P}_f = (P_x, P_y, P_z) \) after passage through the sequence indicated in Fig. 1.

To start, let

\[
c = \gamma \omega_L (>0), \quad s = \frac{\Delta}{\omega_L}, \quad \omega_L = (\omega^2 + \Delta^2)^{1/2}
\]

(1)

and

\[
C = \cos \psi, \quad S = \sin \psi,
\]

(2)

where \( \omega = \gamma H_c \) is the Rabi precession frequency,

\( \Delta = \gamma H_o \) is the free precession frequency about the guide field

\( H_o (\Delta \geq 0 \text{ in coil C1, C2, respectively}), \omega_L \) is the Larmor precession frequency, and

\[
\psi = \omega_L d/v
\]

(3)

is the Larmor precession angle suffered by the particle after the coil is passed. Here, \( \gamma \) is the neutron gyromagnetic ratio, \( v \) is the neutron velocity. \( d \) is the thickness of each coil.

The transfer matrix \( \hat{M} \) is given by the product

\[
\hat{M} = \hat{T}_- \hat{T}_+,
\]

(4)

where (see Eq.(12) of Ref. 2)

\[
\hat{T}_- = \begin{vmatrix}
c^2 + s^2/C & -sS & -sc(1-C) \\
sS & C & cS \\
-sc(1-C) & -cS & s^2 + c^2C
\end{vmatrix}
\]

(5)

and where \( \hat{T}_+ \) can be expressed quickly by replacing in matrix \( \hat{T}_- \) the
\( +s \) by \( +s \). Here, we assume that there are no asymmetries in the device.

On carrying out the multiplication indicated in Eq. (4), one obtains, after a fair amount of algebraic manipulations, for \( M_{33} \)

\[
M_{33} = 2(s^2 + c^2 C)^2 - 1.
\]  \( \text{(6)} \)

We now impose the well known tuning conditions that will make the polarization vector of a neutron having velocity \( v_o \) (wavelength \( \lambda_o \)) be parallel to the reversed guide field \((-\bar{H}_o\)) after the sequence is passed through ("flipper off" state).

As it is well known, these conditions are expressed by

\[
\omega = |\Delta| = \chi
\]  \( \text{(7)} \)

and

\[
\sqrt{2} \chi \frac{d}{v_o} = \pi,
\]  \( \text{(8)} \)

so that Eq. (3) can be written in the form

\[
\Psi = \pi \lambda' \lambda_o,
\]  \( \text{(9)} \)

where \( \lambda \) is the wavelength of an off-momentum neutron of the distribution, which we take centered at \( \lambda_o \).

It follows for \( M_{33} \)

\[
M_{33} = (1/2) (1 + C)^2 - 1 \equiv C - (1/2) S^2.
\]  \( \text{(10)} \)

Assuming now a spectrum of rectangular form of width \( \Delta \lambda \), centered at \( \lambda_o \), we calculate the corresponding spectral average

\[
\langle M_{33} \rangle = 1 - 2f
\]  \( \text{(11)} \)
of $M_{33}$ and obtain for the flipper efficiency $f$

$$f = (1/2) \left[ \frac{5}{4} - \frac{(1/4) \sin (\delta \pi) / (\delta \pi)}{\sin (\delta \pi') / (\delta \pi')} \right],$$

(12)

where $\delta = \Delta \lambda / \lambda_0$.

The assumption we have made concerning the form of the spectrum is only roughly correct, but one thing is easily demonstrated: for $\delta < 0.5$, we have $f > 0.9956$, and this figure corresponds to an extraordinarily high efficiency. Such a result gets better if one works with more realistic spectral distributions.

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REFERENCES.

1) F. Mezei; Z. Physik 255, 146 (1972).
4) T. J. L. Jones and W. G. Williams; Nuclear Instr. and Meth. 152, 463 (1978).