F. Balestra (x), L. Busso (x), I. V. Falomkin (o), R. Garfagnini (x), C. Guaraldo, R. Masch (o), F. Nichitiu (o), G. Piragino (x), G. B. Pontecorvo (o), Yu. A. Shcherbakov (o) and R. Scrimaglio: ON THE ENERGY DEPENDENCE OF \((\pi^-, \^12\text{C})\) BACKWARD SCATTERING CROSS SECTION.

The difference between \(\sigma_{\text{tot}}(\pi^-)\) and \(\sigma_{\text{tot}}(\pi^+)\) for zero isospin nuclei, shown by the experimental data on the total cross section of \(\pi^+\) and \(\pi^-\) on nuclei, obtained at CERN\(^{(1)}\) and at Rutherford Laboratory\(^{(2)}\), is a new interesting aspect of pion-nucleus scattering. This effect seem to be caused by the Coulomb distortion (Coulomb barrier effects, Coulomb trajectory distortion)\(^{(3, 1)}\); however, the different shape of the neutrons and protons densities may produce the same effect\(^{(4, 5)}\).

The \(\pi^\pm\)-nucleus total cross sections ratio, depends upon the relative densities of protons and neutrons and complements the data obtained from electron scattering and \(\mu\)-mesic atoms experiments. In fact, these electromagnetic probes, providing only charge and proton distribution, do not yield informations on neutron distribution. As pointed out in ref. \(^{(4)}\), the \(\sigma_{\pi^+}/\sigma_{\pi^-}\) ratio provides a constraint on the neutron distribution, giving useful informations about the nuclear surface. Moreover, the study of the energy dependence of the cross sections and of the ratios can verify the validity of the optical model utilized.

The differential elastic scattering cross sections at backward angles are also sensitive to the nuclear densities\(^{(4)}\). The Coulomb distortion effects are not strongly dependent upon energy and therefore the energy-dependence of the backward differential cross section is useful to understand the details of nuclear structure.

Furthermore, the knowledge of the backward scattering differential cross section gives useful informations for the phase shift analysis. In fact, in ref. \(^{(6)}\) it has been studied the sensitivity at 180\(^{\circ}\) of a few partial waves analysis of \((\pi^-, \^12\text{C})\) elastic scattering at 80 MeV and it has been shown that the low partial waves are strongly dependent upon the value of the 180\(^{\circ}\) elastic scattering cross section.

In this work we compare the results, obtained measuring the back
ward elastic scattering differential cross section at the Frascati Laboratory in the energy region \(55 \div 95\ \text{MeV}\)\(^7\), with the predictions of two recent developments of the Kisslinger optical potential\(^8\), proposed by Mach\(^9\) and by Cannata et al.\(^10\). These potentials differ from the originally proposed gradient potential by containing an additional term proportional to the Laplacian of the nuclear density. This term, because of its surface-peaked nature, mainly affects the large-angle scattering. Mach obtained an additional surface-term taking into account the Fermi motion of target nucleons. In configuration space, his potential has the form:

\[
V_M(r) = B(E) \varrho(r) + C(E) \left[ -\nabla^2 \varrho(r) \cdot \nabla + \frac{A - 1}{A} \frac{\mathcal{M}_R}{M} \frac{1}{2} \nabla^2 \varrho(r) \right]
\]

where \(M\) is the target nucleon mass and \(\mathcal{M}_R\) is the relativistic analog of the pion-nucleus reduced mass as deduced in ref.\(^9\).

The correction due to the inclusion of the Fermi motion had been estimated\(^11\) considering the nucleus to be a Fermi gas; but the average nucleon momentum was taken equal to zero for a nuclear ground state, obtaining only a quadratic Fermi motion term \(p_F^2\) to add to the original Kisslinger potential\(^8\).

Kisslinger and Tabakin\(^12\) obtained the same \(\nabla^2 \varrho\) term (but without the one-half coefficient as in Mach's) through a Lorentz transformation of the original gradient potential and taking into consideration the proper threshold behaviour of the individual partial waves. Their analysis was performed in the resonance region, where this potential shows a good agreement with the experimental points of Binon et al.\(^13\).

Cannata et al.\(^10\) introducing a non-vanishing linear contribution of the nucleon momenta which, on the energy-shell, leads to a potential with a term \(1/2 \nabla^2 \varrho\), deduced the same expression (1) of Mach\(^9\).

Durand and Gillespie\(^14\) shown that to different values of the coefficient of the \(\nabla^2 \varrho\) term in the optical potential, correspond different choices of the kinematics and of the off-shell continuation of the \(\pi\)-nucleon amplitude.

Schmit et al.\(^15\) pointed out that the kinematic approximation of Kisslinger and Tabakin were unnecessarily restrictive and that an improved potential could be obtained such that the kinematics have the correct non-relativistic limit, are properly symmetrized in the initial and final momenta and provide a unique determination of the pion-nucleon center-of-mass energy. Their coefficient for the Laplacian term \(\nabla^2 \varrho\) was one-half, in agreement with the works of Mach\(^9\) and of Cannata et al.\(^10\).

The same result was obtained by Miller\(^16\) through a coordinate space treatment of the p-wave threshold kinematics for the necessary angle transformation from the \(\pi\)-nucleon center-of-mass system to the \(\pi\)-nucleus center-of-mass system.
Fig. 1 shows the comparison between the $\pm \pi^-$, $^{12}$C elastic scattering differential cross sections values deduced at 69.5 MeV by Edelstein et al. (17) (up to 125°) and by Barbini et al. (7) (at 165° and 175°) and the optical model predictions, using the potentials of Mach (9) and of Cannata et al. (10). The agreement between experimental and theoretical results is very good, although the potentials do not include any free parameter.

**FIG. 1 - (\(\pi^-, ^{12}\text{C}\)) elastic scattering at 69.5 MeV. The experimental values up to 125° are those of Edelstein et al. (17). Large angle data are from Barbini et al., experiment(7). The theoretical curves have been calculated by Mach (9) (solid curve) and by Cannata et al. (10) (dashed curve).**

In Fig. 2, is plotted the backward elastic scattering differential cross section versus pion energy, calculated by the potentials of Mach (9) and of Cannata et al. (10). From the comparison between these two different models calculations, which both fit equally well the experimentals values, we can see that strong dependence on energy and sensitivity to particular optical potential for \(d\sigma/d\Omega\big|_{180°} (\pi^-, ^{12}\text{C})\) is obtained in the energy range 60 ± 120 MeV. The experimental backward data of Frascati (7) exhibit the expected energy behaviour; they seem to show the maximum value at about 75 MeV, while the Cannata et al. prediction (10) is at about 30 MeV and the Mach potential (9) gives the maximum at about 85 MeV.

Fig. 3 shows the "average slope" S of backward scattering differential cross section
FIG. 2 - Backward elastic scattering differential cross section versus pion energy. The experimental values are from Barbini et al. experiment. The theoretical curves have been calculated using the potentials of Mach (solid curve) and of Cannata et al. (dashed curve).

FIG. 3 - Average slope of backward scattering elastic differential cross section $S = \frac{\sigma_1 - \sigma_2}{1/2 \left( \sigma_1 + \sigma_2 \right)}$ versus pion energy. The experimental values are from ref. (17). The theoretical curves have been calculated by the potentials of Mach (solid curve) and of Cannata et al. (dashed curve).
\[
S = \frac{\frac{d\sigma}{d\Omega} \bigg|_{165^\circ} - \frac{d\sigma}{d\Omega} \bigg|_{175^\circ}}{\frac{1}{2} \left[ \frac{d\sigma}{d\Omega} \bigg|_{165^\circ} + \frac{d\sigma}{d\Omega} \bigg|_{175^\circ} \right]}
\]

This quantity goes from positive to negative values quickly in the energy region considered. The experimental backward data\(^{(7)}\) show a qualitative agreement with the theoretical curves.

REFERENCES:


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