M. Greco: DEEP INELASTIC LEPTON-NUCLEON SCATTERING WITHOUT POINT-LIKE CONSTITUENTS.
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ABSTRACT.

Neutrino- and antineutrino-nucleon scattering into hadrons is analyzed and related to electron-nucleon scattering according to a scaling model which has been recently proposed. The electromagnetic and weak structure functions are determined and explicitly given in terms of the $\pi N$ total cross sections. Predictions are made in good agreement with experiments.
The observed scaling behaviour of the structure functions in deep inelastic scattering of leptons off nucleons has suggested an intuitive picture of hadrons as composite systems of point-like constituents with simple properties \(^{(1,2)}\). The further identification of partons with quarks allows one to deduce \(^{(3,4)}\) certain selection rules, inequalities and sum rules between different structure functions and gives a consistent theoretical framework whose predictions are remarkably consistent with the experimental data, with the possible exception of the results for \(\sigma(e^+e^-\rightarrow\text{hadrons})^{(4)}\). In particular the predictions of the absolute values and the relative ratio of neutrino and antineutrinos cross sections from electron scattering are considered one of the biggest successes of the quark parton model.

On the other hand parton models do not provide enough constraints on the actual magnitude and the explicit \(\omega\) dependence of the structure functions and we are left with a complete arbitrariness of the parton distribution functions inside the nucleon, which have to be suitably chosen in order to fit the data. Furthermore most of the inequalities and sum rules of the quark parton model can be reproduced \(^{(5)}\) by imposing hadron-like duality arguments on the structure functions without having to deal explicitly with quarks as partons. Besides, since in parton models we have to face the problem of why the quarks do not show up in the final states, the question arises naturally as to whether we really need point like quarks to understand deep inelastic phenomena.

Recently we have proposed \(^{(6)}\) a scaling model of electromagnetic interactions in which the photon is coupled to a continuum of hadronic states of dual type whose properties are established consistently. With the exception of recent high energy results for \(\sigma(e^+e^-\rightarrow\text{hadrons})^{(4)}\) (which is a source of embarrassment to any scaling model - we predict \(R = \sigma(e^+e^-\rightarrow\mu^+\mu^-)/\sigma(e^+e^-\rightarrow\text{hadrons}) = \ldots\))
the model proposed gives a unified description of the electromagnetic properties of hadrons as the photon mass is continuously varied from the $q^2 \simeq 0$ region, with manifest hadron-like behaviour of the photon, to the large $q^2$ deep inelastic region, where the point-like behaviour is also clearly indicated. In particular, in the framework of two component duality, we have been able to construct the structure functions explicitly in excellent agreement with experiments, starting only with a knowledge of the photoproduction data for the $\varrho, \omega$ and $\varphi$ mesons. The scaling properties of the structure functions, and the consequent point like behaviour of the cross sections, do not follow however in this framework from the existence of any elementary constituents, but rather from common properties of scaling shared by strong and electromagnetic interactions.

The aim of the present paper is to extend our picture to deep inelastic neutrino- and antineutrino- nucleon scattering. We first compare $F_2^V(\infty)$ as predicted in our model with $F_2^A(\infty)$, obtained by Langacker and Suzuki \cite{7} using the partially conserved axial vector current hypothesis (PCAC). This allows us to prove the validity of the SKFR relation \cite{8} and to show the consistency of our approach with current algebra. Then from the $t$ channel analysis of the neutrino- and antineutrino-nucleon scattering, and using the explicit form of the electron-nucleon structure functions, derived in our previous work, we are able to determine the weak structure functions, by means of the usual tools of hadron physics. In particular we derive a whole set of relations which hold also in the quark parton model and corresponding sum rules starting from the Adler sum rule as input. The absolute values and the relative ratios of neutrino and antineutrino cross sections are also predicted, in good agreement with experiments.

Let us consider first the results previously derived for pho-
toproduction total cross sections and deep inelastic eN scattering, in the framework of two-component duality. We obtained \(^{(6)}\):

\[
\sigma^D_{\gamma N} = \frac{11}{9} \frac{4\pi a}{f^2} \sigma^D_{QN} \left( \frac{1}{2} \right)^2 \xi \left( \frac{3}{2}, \frac{1}{2} \right),
\]

\[
\sigma^R_{\gamma N}(\nu) = \frac{11}{9} \frac{4\pi a}{f^2} \sigma^R_{QN}(\nu) \left( \frac{1}{2} \right)^{3/2} \xi \left( \frac{3}{2}, \frac{1}{2} \right),
\]

and

\[
F_2^D(\omega) \rightarrow \frac{11}{9} \frac{\pi}{2} \frac{m_Q^2}{2f_Q} \sigma^D_{QN},
\]

\[
F_2^R(\omega) \rightarrow \frac{11}{9} \frac{m_Q}{4f_Q^2} \sqrt{2} \sigma^R_{QN}(\nu),
\]

where the factor \(11/9 = 1+2/9\) accounts for isovector and isoscalar production (\(\sigma_{QN} = \sigma_{Q^N} - 2 \sigma_{Q^N}\)). The \(QN\) cross section \(\sigma_{Q^N} = \sigma^D_{QN} + \sigma^R_{QN}(\nu)\) is measured in \(Q\)-photoproduction and agrees pretty well both in magnitude and \(\nu\) dependence with the average of the \(\pi^\pm N\) cross sections \(^{(9)}\).

This identification, as well known, is a nice prediction of the simple quark model \(^{(10)}\). Using this fact, we have a first simple connection among quite different processes as \(\pi N\) scattering, \(\varphi\) and total photoproduction and deep inelastic eN scattering. A similar relation was derived by Langacker and Suzuki \(^{(7)}\) and Fujikawa and O'Donnell \(^{(11)}\). More in detail, the total cross section for \(\pi N\) scattering is related through PCAC to the structure function \(F_2^A\) for the axial vector current:

\[
\sigma_{\pi N}(\nu) = \frac{\pi}{4f_\pi} F_2^A(\nu, q^2 = 0),
\]
where \( f_\pi \) is the pion decay constant. Furthermore, from the Deser-Gilbert-Sudarshan representation Langacker and Suzuki also showed that the Pomeron residue of \( F_2^A \) is independent of \( q^2 \), whence

\[
\sigma^{N \rightarrow N}_{\pi N}(\infty) = \frac{\pi}{f_\pi^2} F_2^A(\infty),
\]

having assumed Bjorken scaling\(^{(12)}\) for \( F_2^A(N, q^2) \). On the other hand, taking into account only the isovector contribution in eq. (2a) we have

\[
\sigma^{N \rightarrow N}_{\pi N}(\infty) = \frac{2 f_0^2}{m_0^2} F_2^V(\infty).
\]

By comparing (4) and (5), and using the chiral relation \( F_2^A = F_2^V \), we derive

\[
2 \frac{f_0^2}{f_\pi^2} = \frac{m_0^2}{m_\pi^2},
\]

the well known SKFR relation\(^{(3)}\), which is experimentally verified with very good accuracy. This result strongly supports our picture of deep inelastic eN scattering which leads to eqs. (1) and (2). It also reinforces Pagels's conjecture\(^{(13)}\) of the fundamental role played by \( f_\pi \) as the dimensional constant in strong interactions.

It has to be noted however that eq. (2) is strictly correct only for the transverse contribution to \( F_2^V(\omega) \). A factor \((1+R)\), with \( R \equiv \frac{\sigma_L}{\sigma_T} \), should appear in the r.h.s. of (2). It follows therefore that the exact fulfillment of (6) implies \( R = 0 \) as \( \omega \rightarrow \infty \), in agreement with the quark parton model, with only spin 1/2 constituents.

Let us consider now inelastic neutrino- and antineutrino-nucleon scattering. The main idea is to relate, in the framework of a simple Regge model, the structure functions for \( \bar{\nu} N \) and \( \nu N \) scattering to those for eN scattering, explicitly given by eqs. (2), both for protons
and neutrons. Most of our results hold more generally than in a simple Regge picture, and can be obtained applying general constraints of hadrons duality to the structure functions \(^5\).

The \( t \) channel quantum numbers and the corresponding Regge trajectories for the structure functions are shown in Table I \(^{14}\).

**TABLE I**

\( t \) channel quantum numbers for Regge contributions to the structure functions \(^{14}\).

<table>
<thead>
<tr>
<th>Structure Functions</th>
<th>((-1)^J P)</th>
<th>((-1)^J C)</th>
<th>( I^G )</th>
<th>Regge Trajectories</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{1,2}^{\gamma P} + F_{1,2}^{\gamma n} )</td>
<td>+</td>
<td>+</td>
<td>0(^+)</td>
<td>( P, f, f' )</td>
</tr>
<tr>
<td>( F_{1,2}^{\gamma P} - F_{1,2}^{\gamma n} )</td>
<td>+</td>
<td>+</td>
<td>1(^-)</td>
<td>( A_2 )</td>
</tr>
<tr>
<td>( F_{1,2}^{\gamma P} - F_{1,2}^{\gamma n} )</td>
<td>+</td>
<td>+</td>
<td>0(^+)</td>
<td>( P, f, f' )</td>
</tr>
<tr>
<td>( F_{1,2}^{\gamma P} + F_{1,2}^{\gamma n} )</td>
<td>+</td>
<td>+</td>
<td>1(^+)</td>
<td>( Q )</td>
</tr>
<tr>
<td>( F_{1,2}^{\gamma P} - F_{1,2}^{\gamma n} )</td>
<td>+</td>
<td>+</td>
<td>1(^-)</td>
<td>( A_2 )</td>
</tr>
</tbody>
</table>

Assuming SU(3) symmetry for the factorized residues with the additional constraints of the quark model, and the equality of the vector and axial vector couplings we can express all the structure functions as follows (we neglect strangeness changing interactions, i.e. \( \sin^2 \theta_c \approx 0 \)):

\[
(7a) \quad F_2^{\gamma P} + F_2^{\gamma n} = 2 (a_{V_p} + a_{S_p}) + 2 (a_{V_{p'}} + a_{S_{p'}}) \omega^{-1/2},
\]

\[
(7b) \quad F_2^{\gamma P} - F_2^{\gamma n} = 2 a_{A_2} \omega^{-1/2},
\]
\[ F_2^{\nu P} + F_2^{\nu n} = 8 a_p^V + 8 a_p^{V'} \omega^{-1/2}, \]
\[ F_2^{\nu n} - F_2^{\nu P} = 12 a_{A_2} \omega^{-1/2}, \]
\[ F_3^{\nu P} + F_3^{\nu n} = -36 a_{A_2} \omega^{1/2}, \]
\[ F_3^{\nu P} - F_3^{\nu n} = 12 a_{A_2} \omega^{1/2}. \]

Here \( a_i^V \) and \( a_i^S \) refer to the couplings of isovector and isoscalar photons respectively, \( a_p^{V}(0) = 1 \) and \( a_i^S(0) = 1/2 \) for all other trajectories. We have decoupled the nucleons from the pure strange \( \phi \) and \( f' \) trajectories and assumed pure \( F \) couplings of the vector and tensor trajectories at the nucleon vertex. The \( D/F \) ratio obtained from analysis of hadronic reaction data is \( \sim 0.2 \), although it may be nearer to zero for \( V \) exchange than for \( T \) exchange. Similar arguments give \( a_{A_2}^{V'} / a_{A_2}^V = 5 \) which agrees, through eqs. (7a-b), (1) and (2), with total photoproduction data on protons and neutrons.

Finally, using the fact that \( a_i^S \leq 2/9 a_i^V \), we are left with only two unknowns \( a_p^V \) and \( a_p^{V'} \), which are determined through eqs. (2):

\[ a_p^V \equiv F_2^V(\infty) = \frac{f_{\pi}^2}{\pi} \sigma_{\pi N}(\infty), \]
\[ a_p^{V'} = \frac{5 f_{\pi}^2 \sqrt{2 M_p}}{12 m_q} \beta_{\pi P}. \]

where \( \sigma_{\pi P}(\nu) \equiv \sigma_{\pi P}(\infty) + \beta_{\pi P} / \sqrt{\nu} \). The whole set of structure functions for both weak and electromagnetic inelastic processes is therefore determined and predicted from the knowledge of \( \pi P \) total cross sec_
tions or, equivalently, from photoproduction data.

Furthermore, from eqs. (7) we can derive a large set of relations among different structure functions which, although strictly true in the Regge region, are expected to hold, by duality, also in the small $\omega$ region. They also reproduce the results derived in the framework of the quark parton model\(^{3}\). Explicitly, from eqs. (7a) and (7c) we have:

\[
F_2^{\nu N} = \frac{1}{2} (F_2^{\nu P} + F_2^{\nu n}) = \frac{1}{2} (F_2^{\nu N} + \bar{F}_2^{\nu N}) = \frac{18}{5.5} \frac{1}{2} (F_2^{\gamma P} + F_2^{\gamma n}),
\]

where the factor 5.5 in the denominator of the r.h.s. becomes 5 if we neglect the $\varphi$ contribution to $a_1^S$ in eq. (7a) ($a_1^S = \frac{1}{9} a_1^V$).

From eqs. (7b) and (7c) we have\(^{16}\)

\[
F_2^{\nu n} - F_2^{\nu P} = 6 (F_2^{\gamma P} - F_2^{\gamma n}),
\]

which allows one to derive the Gottfried sum rule\(^{17}\) from the Adler sum rule\(^{18}\).

From eqs. (7b) and (7f) we get the Llewellyn-Smith relation\(^{19}\)

\[
F_3^{\nu P} - F_3^{\nu n} = 6 \omega (F_2^{\gamma P} - F_2^{\gamma n}) = 12 (F_1^{\gamma P} - F_1^{\gamma n}),
\]

where the last equality assumes $R = 0$.

From eqs. (7b), (7d) and (7a) we have:

\[
\frac{F_3^{\nu P} + F_3^{\nu n}}{\omega} = -3 (F_2^{\nu n} - F_2^{\nu P}) = -18 (F_2^{\gamma P} - F_2^{\gamma n}),
\]

which again relates the Gross-Llewellyn-Smith\(^{20}\) sum rule to the Adler sum rule.
We can test these relations locally by comparison with the electron and neutrino data. The ratio of total neutrino and antineutrino cross sections is given by

$$\frac{\sigma_{\bar{v}N}}{\sigma_{\nu N}} = \frac{2 - B}{2 + B},$$

with

$$B \equiv -\int_0^1 dx x F_3(x) / \int_0^1 dx F_2(x).$$

Using eqs. (9) and (12) we get:

$$B = 5.5 \frac{\int dx (F_2^p - F_2^n)}{\int dx (F_2^p + F_2^n)} \approx 5.5 \frac{(0.16 - 0.12)}{(0.16 + 0.12)} = 0.85,$$

to be compared with $B_{\text{exp}} = 0.90 \pm 0.04^{(22)}$ and $B = 1$ in the quark parton model with purely fermion partons.

Similarly, from (9), we get for the total cross section summed over neutrino and antineutrino beams:

$$\sigma_{\nu N}^{(E)} + \sigma_{\bar{v} N}^{(E)} = \frac{G_N^2 M_N E}{\pi} \left[ \frac{1}{2} \int_0^1 \frac{dx (F_2^\nu N + F_2^\bar{v} N)}{F_2^\nu N + F_2^\bar{v} N} \right] =$$

$$\approx \frac{G_N^2 M_N E}{\pi} \left[ \frac{4}{3} \right] \left[ 0.46 \right],$$

to be compared with the experimental value $(0.47 \pm 0.07)^{(22)}$.

It follows therefore that, within the present accuracy, the data support the approximate validity of the above relations also in the
low $\omega$ region, where the integrals are experimentally evaluated.

To conclude, we have been able to reproduce all the results of the quark parton model without invoking point constituents but rather accommodating scaling in a fairly natural way by imposing on the current induced processes the regularities one observes in purely hadronic reactions. Furthermore, we have found consistent links among different processes as $e^+e^-$ annihilation, photoproduction of vector mesons and total photoproduction on nucleons, $\pi N$ scattering and deep inelastic electron and neutrino scattering, in agreement with experiments.

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