A. Bramon and Y. Srivastava: $\rho - \omega$ INTERFERENCE IN KAONIC REACTIONS.
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In a recent paper, Earles and Srivastava(1) have proposed a unitary model for $\rho - \omega$ interference which has some rather surprising consequences. For instance, a study of the said interference effect in $e^+e^-$ colliding beam production of $\pi^+\pi^-$ pairs leads them to conclude that, contrary to the commonly held view, a very small $B_{\omega\pi\pi}$ ($\omega \to 2\pi$ branching ratio) is indeed consistent with experiment. This result is pleasing because it restores one's faith in the old VMD calculation proceeding through the chain $\omega \to \gamma \to \rho \to 2\pi$(2). Unfortunately, the data are not accurate enough to determine $B_{\omega\pi\pi}$ with much precision. Rather conservatively, they claim that

$$B_{\omega\pi\pi} = (15^{+25}_{-11}) \times 10^{-4} \quad (2 \text{ st. dev.}).$$

Naturally, to test the model further, one would like to investigate other reactions showing the interference, e.g. $\bar{p}p \to \pi^+\pi^- (\pi^+\pi^-\pi^0)$, $\gamma p \to \pi^+\pi^- p$, etc. The first set, ie. $\bar{p}p$ annihilation into 5 pairs has the advantage of possessing excellent statistics(3), but suffers from the problem of relative production phase and parametrization of the back-

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ground. On the other hand, photoproduction experiments of the type $\gamma p \rightarrow \pi^+\pi^- p$, suffer from poorer statistics. There do exist reactions, however, which have merits of both: relatively free from background and production phase uncertainties (unlike $p\bar{p}$ annihilation) but allowing for higher statistics and an increased interference effect (relative to the photo case)! We thus turn to processes of the type

$$K^- p \rightarrow \Lambda \pi^+\pi^- \rightarrow \Sigma^0\pi^+\pi^-,$$

and discuss why these are preferable reactions.

First, as shown in Figure 1, the production cross-section for $K^- p \rightarrow \Lambda \rho^0$ and $K^- p \rightarrow \Lambda \omega$ are (almost) equal. This is easy to understand in the quark model\(^4\). These reactions proceed through $K, K^*, K^{**}, \ldots$ exchanges and the pairwise-couplings ($K \bar{K} \rho, K \bar{K} \omega$), ($K \bar{K}^* \rho, K \bar{K}^* \omega$), ($K \bar{K}^{**} \rho, K \bar{K}^{**} \omega$) are equal in the quark model, once one admits that $\omega$ has no strange quark-antiquark admixture. This is one of the "cleaner" results of the quark model and is sustained by experiment (Fig. 1). A related prediction concerns the decay of $K^{**}(1420)$ into $K\rho$ and $K\omega$. As discussed later the equality of the two couplings leads one to calculate the relative decay rates - given a model for $\rho-\omega$ interference.

The other important point to notice is that we gain (in the amplitude) a factor of $f_\omega/f_\rho \simeq 3$ with respect to $e^+e^-$ and photoproduction experiments!

It is quite easy to obtain formulae for the resonant amplitudes ($K^- p \rightarrow \Lambda \pi^+\pi^-, K^- p \rightarrow \Sigma \pi^+\pi^-$) using the "old" model (called, Model O), which is simply a sum of $\rho$ and $\omega$ Breit-Wigners with an arbitrary phase, as well as the new unitary model\(^1\) (called Model U). Using the notation of ref. (1), we have for

Model O:

$$T_0 = t_{\rho} x_{\pi} x_{\Lambda} + e^{i\varphi} t_{\omega} y_{K} y_{\pi}, \quad (1)$$

where $t_{\rho}, t_{\omega}$ are the B.W. amplitudes for $\rho, \omega$ resonances; $x_{\pi}$ and $y_{K}$ stand for the "branching ratios" of $\rho$ and $\omega$ respectively to the $K - \bar{K}$
(exchange) system and similarly $x_\pi$ and $y_\pi$ are their b.r. to the $\pi^+\pi^-$ system. The phase $\varphi$ would be the same (to first order), via the Fermi-Watson theorem, as the one observed in Orsay $e^+e^- \rightarrow \pi^+\pi^-$; i.e., the $\rho$-phase. Indeed, one finds\(^{(5)}\) that $\varphi = \varphi(\text{ORSAY}) \approx 85.7 \pm 14.4^\circ$. We thus take $\varphi = \pi/2$. The second point concerns the b.r. Clearly $x_\pi \approx 1$. Also, as discussed earlier, the equality of the $\rho$-$\omega$ couplings to the $K$-$\bar{K}$ (exchange) system, $x_K/y_K \approx \sqrt{\Gamma_\omega/\Gamma_\rho}$, where $\Gamma_\rho, \omega$ stands for the total widths of $\rho, \omega$ resonance. The quantity $y_\pi = \sqrt{B_{\omega \rightarrow 2\pi}}$ is obtained from Orsay\(^{(5)}\) to be $0.19 \pm 0.10$ (2 st. dev.). Thus, the prediction of Model O may be simply written as

$$\left| T_0 \right|^2 \propto t_\rho \left( \frac{\Gamma_\omega}{\Gamma_\rho} \right)^{1/2} + i t_\omega y_\pi.$$  \hfill (2)

This is plotted - in arbitrary units, naturally - in Fig. 2A, along with the no interference (Model N) case - which is just the $\rho$ B.W. The resonance parameters are taken from ref.\(^{(7)}\). As expected, a very sharp interference effect appears.

Now let us turn to Model U. To first order in $r = \sum_{i} x_i y_i \propto y_\pi$, which gives the "overlap" between $\rho$ and $\omega$, $T_u$ can be written as\(^{(1)}\)

$$T_u \propto t_\rho x_{\pi} x_K t_{\omega} y_{\pi} + \frac{\delta}{\delta_{\rho} - \delta_{\omega}} \left( \frac{\Gamma_\omega}{\Gamma_\rho} \right)^{1/2} + r \frac{\delta_{\rho} - \delta_{\omega}}{\delta_{\rho} - \delta_{\omega}} (x_K y_{\pi} + y_K x_{\pi}) + O(r^2),$$

so that

$$T_u \propto t_\rho \left[ \left( \frac{\Gamma_\omega}{\Gamma_\rho} \right)^{1/2} + r \frac{\delta_{\rho} - \delta_{\omega}}{\delta_{\rho} - \delta_{\omega}} \right] - t_\omega r \frac{\delta_{\rho} - \delta_{\omega}}{\delta_{\rho} - \delta_{\omega}},$$  \hfill (3)

where $\delta_{\rho, \omega}$ are the resonant phase shifts, $\tan \delta_i = \frac{m_i I_i}{m_i - s}$.  

To make comparison with Model O as strict as possible, we choose for $r$, the value found in ref.\(^{(1)}\) by fitting the same Orsay data\(^{(5)}\) used to get $B_{\omega \rightarrow 2\pi}$ as above (for Model O). To within two standard deviations $r = \sqrt{15} \times 10^{-2}$. The prediction of Model U is shown in Figure 2B. Again, a clear interference signal can be seen. Thus, our expectations
are more than fulfilled. Models N, O, U differ so substantially with one another that even moderately good data in small mass bins should have little difficulty in discriminating between them. Unfortunately, even such a modest request is denied. There do exist data on \( K^- p \rightarrow \Lambda \pi^+ \pi^- \) as well as \( K^- p \rightarrow \Sigma \pi^+ \pi^- \) (of course, with poorer statistics), procured by Aguilar-Benitez et al.\(^6\), but the mass bins (30-40 MeV) are too broad for our purposes. Thus, we have to be content with only some semi-quantitative conclusions based on this data.

(i) Both Model O and U predict that the observed \( \rho \) mass should be less than the input value (obtained from PDG\(^7\)), i.e.

\[
m_{\rho \text{ (observed)}} < 770 \pm 5 \text{ MeV}.
\]

On the other hand, Aguilar-Benitez et al.\(^6\) quote

\[
m_{\rho \text{ (observed)}} = \begin{cases} 757 \pm 5 \quad (\Lambda) \\ 756 \pm 9 \quad (\Sigma) \end{cases}.
\]

Thus, this observation is consistent with both models.

(ii) In ref.\(^6\), it is also observed that the 740 \( \pm \) 20 MeV bin contains more \( \pi^+ \pi^- \) (\( J^P = 1^- \)) events than the higher 780 \( \pm \) 20 MeV bin. Such a depletion (in fact a rather sharp one) of events on the high mass side is obtained in Model U and hence would favor this model.

Clearly, nothing conclusive but Model U has an edge over Model O.

Let us briefly discuss now \( K^{*+} \) decay into \( \rho \) and \( \omega \). From PDG guide to the particle states for 1973\(^7\), we find

\[
R_{\text{EXP}} = \frac{K^{*+} \rightarrow K\rho^0}{K^{*+} \rightarrow K\omega} = \frac{1}{3} \frac{(9.2 \pm 2.9)\%}{(4.4 \pm 1.7)\%} \approx 0.70 \pm 0.35.
\]

From our main hypothesis (equality of couplings) and without \( \rho - \omega \) interference

\[
R_N = (\frac{k_{\rho}}{k_{\omega}})^5 \approx 1.27.
\]

Thus, interference is supposed to lower \( R \). This is again distinctly indicated by Model U but works the wrong way for Model O.
Both the above analyses lead us to suggest that future experimental work on \( \pi^+\pi^- \) production from \( K^-p \) be made with mass bins appropriate for Fig. 2. Also, for the decays of \( Q, K^{*+} \) into \( K\pi \), one first subtract the large \( (K^{*+}\pi) \) events and then plot the \( \pi^+\pi^- \) mass spectrum.

We have seen that these processes in case the interference effect over the photo case by a factor of 3. Can the effect be still increased? The answer is yes. Consider the decay of a hypothetical (?) \( I^{GC} = 1^{-+} \) meson, called \( M(6,7) \), into \( \pi^+\pi^-\gamma \). VMD implies that the decay proceeds through \( M \rightarrow \rho^0\omega \rightarrow (\pi^+\pi^-)\gamma \) and \( M \rightarrow \omega\rho^0 \rightarrow (\pi^+\pi^-)\gamma \). But now there are two factors of \( f_\omega/f_\rho \) and hence one gains (in the amplitude) a factor of 9 relative to the \( e^+e^- \) or photo case! In fact, the interference is so strong as to distort almost completely the \( \rho \)-events. Thus, for the quantum nos., \( I^{GC} = 1^{-+} \), the puzzle of why \( M(954) \), in contrast to (almost degenerate) \( \eta' \), while decaying into \( \pi^+\pi^-\gamma \) gives no \( \rho \) signal may be resolved. Since \( \eta' \) has \( I=0 \), in \( \eta' \rightarrow \pi^+\pi^-\gamma \) the \( \rho-\omega \) interference is almost an order of magnitude less, i.e. the same as in the photo case.

In connection with the last paragraph, of course, the best process would be to consider \( e^+e^- \) into \( \pi^+\pi^-\pi^0 \) and study the \( \pi^+\pi^- \) mass spectrum for \( \rho-\omega \) interference. What we said before goes for this process as well but we have the additional advantage here in that one has a variable mass "photon" and hence one can survey the situation more completely.

In summary, we have studied \( \rho-\omega \) interference in \( \pi^+\pi^- \) system produced either in \( K^- \) initiated reactions or strange meson decays and find that various models give quite different spectra. The data are not appropriate to decide among them but the model proposed in ref. (1) seems to be favored. We strongly urge that the suggested experimental analysis be made to decide on this rather crucial question.

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FIG. 1 - Cross sections for $\rho^0$ and $\omega$ production in $K^-p$ interactions (6, 8).
FIG. 2 - A) $\pi^+\pi^-$ mass spectrum using Model O; B) The same using Model U.