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Coincidence Measurement of Quasifree Scattering of 9-keV Electrons on K and L Shells of Carbon

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Momentum distributions of electrons bound in the K and L shells of carbon have been separately measured. Data obtained from the angular distribution of coincidences between the two electrons occurring in a quasifree (e, 2e) process are compared with theoretical predictions.

The problem of measuring electron momentum distributions (EMD's) in atoms, molecules, and crystals is a long-standing one. The techniques used until now to obtain these distributions include Compton scattering of x rays, quasifree electron scattering at large angles, and positron annihilation. In the published experiments only one of the three reaction products has been detected. In this way it is impossible to resolve EMD's due to electrons belonging to different bound states. In spite of this limitation, interesting information has been obtained, particularly from the study of the broadening of the Compton lines. Even when the initial state of the bound electron is defined, as for instance when the scattered x ray is taken in coincidence with a fluorescence photon, the Compton profile does not give direct information on the EMD because it is determined by an integral of this function. A possible way to overcome these difficulties consists in using high-energy electrons and in measuring the angular correlation of the scattered and ejected electrons detected in coincidence. To simplify the interpretation of the results, the process has to be a "quasifree" scattering and the energies and momenta of the particles must be measured with great relative accuracy. In this way direct measurements of EMD's of well-defined initial states can be obtained. Of course the improvement of information goes together with an increase in experimental difficulties because of the smaller values of the cross sections involved and the much more stringent experimental requirements.

In the last few years theoretical papers have been published suggesting the use of electron coincidence techniques for measuring EMD's in atoms, molecules, and solids, but experimental results are still lacking. Only Amaldi et al. have demonstrated the feasibility of these (e, 2e) experiments using an apparatus whose resolving power is enough to separate (e, 2e) events produced by scattering on K and L shells of carbon. Preliminary measurements of the angular distributions on Formvar targets have also been presented.

In the framework of the impulse approximation, justified by the high kinetic energies of the electrons and by the high momentum transfer in the
collision, the cross section for the process can be written as

\[
\frac{d^5\sigma}{d\Omega_e d\Omega_s dE_e} = \frac{m_e^2 \rho_{el}}{\hbar^3} \frac{d^3\sigma}{d\Omega_e} \rho_{el}(\vec{q}),
\]

(1)

where \(\Omega_e\) and \(\Omega_s\) are the directions of the final electrons, \(E_e\) is the final energy of the incident electron, \(\frac{d^3\sigma}{d\Omega_e}\) is the elastic electron-electron cross section, \(\rho_{el}(\vec{q})\) is a suitable form factor, and \(\vec{q}\) and \(\vec{p}_e\) are the momenta of the bound and of the emitted electron, respectively. In the self-consistent-field approximation the factor

\[ \chi^2 = 40.9 \]

--- MBS

\[ \chi^2 = 28.0 \]

--- Roothan

FIG. 2. Angular distribution of the coincidence rate for the 1s state of carbon. Beam energy \(E_0 = 9.3\) keV; final energies of the electrons \(E_e = E_s = 4.5\) keV; current intensity, \(3.5 \times 10^{-8}\) A. The data belong to two different series of measurements on two 50-Å-thick carbon films. The correspondence between scattering angle and initial momentum of the atomic electron is also given. The dashed and solid curves are the predictions obtained with MBS and Roothaan wave functions, respectively.
\( \rho_{ni}(\vec{q}) \) involves only the Fourier transform of the initial wave function of a single bound electron and becomes

\[
\rho_{ni}(\vec{q}) = \frac{1}{(2\pi)^3} \sum_{m=-1}^{1} \frac{1}{(2\pi)^3} \int \exp \left( -\frac{\vec{q} \cdot \vec{F}}{\hbar} \right) \varphi_{ni}(\vec{r}) d^3r.
\]  

(2)

The atomic state \((nl)\) and the initial electron momentum \(\vec{q} = \hbar \vec{k}\) are defined by energy and momentum conservation laws. Under symmetrical coplanar scattering conditions, i.e., by keeping the final momenta of the outgoing electrons fixed and by varying symmetrically the angles, only the components of momentum \(\vec{q}\) of the bound electron parallel to the direction of the incident beam are measured.

The background on the counting rate due to unavoidable accidental coincidences puts a limit on the study of internal states of heavy elements. They are in fact characterized by a broad EMD function and consequently by a low counting rate. These and other experimental difficulties, such as energy losses and damage effects in solid targets, are fully discussed in the literature.

Coincidence measurements on evaporated carbon films of nominal 250- and 100-Å thicknesses have been carried out with the apparatus schematically shown in Fig. 1, essentially the same as that described in Ref. 6, where also typical energy spectra are reported. Typically 7 coincidences per minute are registered at the maximum of the angular distribution, with an incident beam current of \(3.5 \times 10^{-8}\) A. The angular distribution of the true coincidences rate for the 1s state in carbon is shown in Fig. 2. The solid and dashed curves are obtained using Roothaan and minimal-basis-set (MBS) wave functions, respectively, and integrating Eqs. (1) and (2) over the acceptance of the apparatus. The best fit to the data is obtained by minimizing the \(\chi^2\) function with an arbitrary normalization constraint. The \(\chi^2\) test gives a confidence level of \(\approx 7\%\) for the more realistic Roothaan wave function and only \(0.3\%\) for the MBS approximation.

Since a 7\% confidence level is acceptable, we conclude that the Roothaan model gives a good description of the true wave function, while the

![Figure 3](image-url)  
**FIG. 3.** Angular distribution of the coincidence rate for the external L shell in carbon films 250 Å (solid circles) and 100 Å (open circles and squares) thick. The data have been normalized to the same area. Experimental conditions are the same as in Fig. 2, except for the incident energy, which was \(E_0 = 9.0\) keV. The continuous curve is predicted in the basis of the Roothaan 2s2p configuration, while the dotted curve is an eye interpolation to the measured points.
MBS approximation has to be rejected. Our result adds independent and new support to previous theoretical considerations on the goodness of the various approximations which have been based up to now on the comparison of the computed and measured energy values. We believe that this result is not only important in itself but also shows the capability of this new experimental method to measure the wave functions directly and to discriminate among different models.

As far as the absolute counting rate is concerned, it contains the unknown efficiency of the apparatus in detecting the coincident outgoing electrons. It involves the effect of the energy losses suffered by the electrons in the sample and damage effects of the target under the beam, besides the efficiency of the Channeltron detectors. In general one can say that measured rates agree with computed ones. More details are presented elsewhere.\textsuperscript{10}

The angular distribution of the outer electrons, whose binding energy is \( \leq 20 \text{ eV} \), has also been measured and is reported in Fig. 3. The lack of energy resolution (\( \Delta E = \pm 45 \text{ eV} \) for each channel)\textsuperscript{11} prevents us from resolving \( 2p \) from \( 2s \) states, but a quite accurate determination of the characteristic width of the curve is still possible. It appears about 1\textsuperscript{o} wider than the one computed by integrating a \( 2s2p^3 \) Rothean configuration (solid curve of Fig. 3), commonly employed in carbon bonding calculations. The presence in the EMD of momenta higher than expected from purely atomic wave functions is in fair agreement with the EMD broadening previously observed in Compton scattering and in positron annihilation, on various allotropic forms of carbon.\textsuperscript{1} Thus our data independently confirm the already existing information on the external atomic states in carbon and, conversely, the agreement constitutes further evidence in support of the idea to use this new technique in measuring directly electron wave functions.

Further work is in progress along this line, mainly in improving the resolution of the apparatus, to provide new and direct information on the outer states in molecules and crystals.

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\textsuperscript{2}M. Cooper, Advan. Phys. 20, 453 (1971), and references cited therein.
\textsuperscript{3}A. L. Hughes and M. M. Mann, Phys. Rev. 55, 50 (1937).
\textsuperscript{4}P. E. Mijares, Phys. Rev. 160, 512 (1967), and references cited therein.
\textsuperscript{5}T. Fukamachi and S. Hosoya, Phys. Lett. 28A, 341 (1967).