A. Bramon and M. Greco: THE REACTION $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ AND THE $\rho^+$ MESON.
A. Bramon and M. Greco: THE REACTION $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ AND THE $\rho'$ MESON.

In the preceding letter Barbarino et al.\(^{(1)}\) have reported on the observation of the process

\[(1) \quad e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\]

at total C.M. energies of the colliding beams $2E = \sqrt{s}$ ranging from 1.2 to 2.4 GeV. The quantum numbers of the $4\pi$ final state are well defined: $J^{PC} = 1^{--}$, $G = +$ and consequently $I = 1$, the same ones of the $\rho$ meson. The actual results however can be hardly understood under the assumption that reaction (1) proceeds only through a $\rho$ intermediate state, because of the smallness of the cross section at the total energies of 0.99 GeV (the ACO results\(^{(2)}\)), 1.2 and $\sim 2.0$ GeV. The threshold in fact for $4\pi$ production is 0.6 GeV roughly, while the $A_1\pi(3,4)$ mode has a smoother energy behaviour than the experiment actually shows.

In the other hand an attractive and, at the same time natural explanation of the data can be given assuming the existence of a new vector meson, the $\rho'$. Such a meson has been suggested by several theoretical models\(^{(5)}\) and introduced by the authors\(^{(6)}\) some time ago as a possible explanation of the rather high cross sections observed in the multi-hadron production at ADONE.

In the present note we reexamine the previous attempt in light of new experimental informations and analyse the implications concerning process (1).

The starting point of our model is to get an estimate of the coupling constants of the $\rho'$ with the photon, $g_{\rho'\gamma} = e m_{\rho'}/f_{\rho'}$, and with the mesons involved in its different decay modes. This was achieved

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(x) - On leave of absence from Departamento de Fisica Tèorica, Barcelona.
(see ref. (6) for more details) by fitting the available experimental results concerning the decay rates of some pseudoscalar and vector mesons in the spirit of a vector dominance model generalized with the inclusion of these new states. In particular, for our actual purposes, it suffices to consider the chain of processes

\[ \omega \rightarrow \pi^+ \pi^- \pi^0, \quad \omega \rightarrow \pi^0 \gamma, \quad \pi^0 \rightarrow \gamma \gamma \]

which are simply related by VMD and whose experimental values are firmly established \(^{(8)}\). By using the old ORSAY results \(^{(9)}\) for the photon-vector meson couplings, Augustin et al. \(^{(10)}\) showed that a disagreement between the VMD predictions and the experimental values, of roughly a factor of two, was obtained at each step of the chain (2). We showed \(^{(6)}\) that the existence of a new vector nonet could restore the agreement if the coupling constants of the new mesons satisfy the relation:

\[ \lambda = \frac{g^0 \langle \omega \pi \rangle}{f^0} \frac{f}{f^\rho} \propto -0.2. \]

If however the new ORSAY data \(^{(11)}\), now in agreement with those of Novosibirsk \(^{(12)}\), were used, the disagreement in ref. (10) would be reduced to a factor 1.4 roughly at each step. Correspondingly in our extended vector dominance model the parameter \(\lambda\) introduced in (3) should be given by

\[ \lambda \propto -0.12. \]

From this result and by assuming \(m_\rho' \simeq 1.6\ GeV\) we predict the partial width of the \(\rho' \rightarrow \omega \pi \rightarrow \pi^+ \pi^- \pi^0\) decay mode to be

\[ \Gamma(\rho' \rightarrow \omega \pi) = 24 \left( \frac{f}{f^\rho} \right)^2 \Gamma(\omega \rightarrow \pi^+ \pi^- \pi^0) \]

and therefore a factor of 3 smaller than our previous result \(^{(6)}\). By means of usual SU(3) and phase space evaluation techniques and introducing the experimental value \(\Gamma(\omega \rightarrow \pi^+ \pi^- \pi^0) \simeq 10\ MeV\) we obtain for the main \(\rho' \rightarrow \text{VP}\) decay modes:

\[ \Gamma(\rho' \rightarrow \omega \pi) = 24 \left( \frac{f}{f^\rho} \right)^2, \quad \Gamma(\rho' \rightarrow \rho \eta) = 7 \left( \frac{f}{f^\rho} \right)^2, \]

\[ \Gamma(\rho' \rightarrow K^+ K^-) = 4 \left( \frac{f}{f^\rho} \right)^2 \]

where all the widths are expressed in MeV. We have also estimated for the pseudoscalar-pseudoscalar decay modes \(\Gamma(\rho' \rightarrow PP) \simeq\)
\[ \simeq 8\left(\frac{f_{\rho^1}}{f}\right)^2 \text{ MeV}, \] which is compatible with the experimental information available\(^{(13)}\). By adding all these rates for the decays \( \rho^1 \to VP, PP \), we obtain \( \Gamma_{VP, PP} \simeq 43\left(\frac{f_{\rho^1}}{f}\right)^2 \text{ MeV} \simeq 215 \text{ MeV} \) where the value \( \left(\frac{f_{\rho^1}}{f}\right)^2 \approx 5 \) estimated in ref.\(^{(6)}\), has been tentatively used. Further decay modes are therefore required if the total \( \rho^1 \) width is about 0.35 GeV. This value is roughly confirmed by the new results\(^{(14)}\) on the reaction \( \gamma p \to p\pi^+\pi^-\pi^+\pi^- \), besides those of the preceding letter, both experiments suggesting a sizeable \( \rho^1 \to \pi^+\pi^-\pi^+\pi^- \) decay mode.

More in detail Davier et al.\(^{(14)}\) have studied the just mentioned reaction at photon energies up to 18 GeV, showing evidence for a \( 4 \pi^+ \) enhancement centered at \( (1,6 \pm 0,1) \text{ GeV} \), with a width of \( (0,5 \pm 0,1) \text{ GeV} \) and the following properties: a) the corresponding cross section of \( (0,9 \pm 0,2) \mu \text{b} \) is energy independent above 6 GeV; the slope parameter of \( d\sigma/dt \) is \( (5,8 \pm 0,4) \text{ GeV}^{-2} \), very near to the value \( (6,6 \pm 0,4) \text{ GeV}^{-2} \) for the analogous slope in the reaction \( \gamma p \to \rho^0 p \) at the same energies.

This effect can be understood in terms of diffractive production of a \( \rho^1 \) which subsequently decays into four pions. By assuming the equality of the elastic \( \rho \)-nucleon and \( \rho^1 \)-nucleon cross-sections, as can be expected from the quark model, and neglecting non-diagonal terms \( \rho p \to \rho^1 p \) with respect to the diagonal ones \( \rho p \to \rho p \) and \( \rho^1 p \to \rho^1 p \), we easily obtain:

\[ \frac{\Gamma(\rho^1 \to 4\pi^+)}{\Gamma_{\rho^1}} \simeq (0,08 \pm 0,02) \left(\frac{f_{\rho^1}}{f}\right)^2, \]

where we have used for the \( \rho \) photoproduction cross section the value \( \sigma_\rho = 12 \mu \text{b}\(^{(15)}\)\).

If is easy to evaluate the implications of this result in the \( 4 \pi^+ \) production in \( e^+e^- \) annihilation, by using the expression for the \( e^+e^- \to \rho^1 \to f \) cross section

\[ \sigma_f(s) = 16\pi^2\alpha^2 \frac{\rho^4}{f^2} \frac{1}{s^{3/2}} \frac{\Gamma(\rho^1 \to f)}{(s-m_{\rho^1}^2)^2 + m_{\rho^1}^2 I_{\rho^1}^2}, \]

We find:

\[ \sigma_{e^+e^- \to 4\pi^+}(m_{\rho^1}^2) = (1,6 \pm 0,5) \cdot 10^{-32} \text{ cm}^2. \]

The energy dependence is shown in Fig.\(^{(1)}\), and it is compared with the experiments. Looking at this figure it should be realized also the

\( \text{(x)} \) The experimental points are taken from ref.\(^{(1)}\).
presence of a background essentially coming from the $A_1 \pi$ contribution of the $\rho$ tail previously discussed, which can be roughly estimated to be about 5 nb in the average.

In order to have more precise informations on the four pions $\rho'$ decay modes, we have studied the reaction $\rho' \rightarrow \rho \pi \pi$ with techniques of finite energy sum rules, similar to the approach used in $\rho \pi$ scattering. We omit the details, which will be discussed elsewhere. The preliminary results of this analysis are: $\Gamma(\rho' \rightarrow \rho \pi^+ \pi^-) \approx 30 (f_{\rho'}/f_\rho)^2$ MeV and $\Gamma(\rho' \rightarrow \rho \pi^0 \pi^0) \approx 12 (f_{\rho'}/f_\rho)^2$ MeV in rough agreement with equation (7).

We are now able to estimate the strength of the $\rho'$-photon coupling by adding all the different decay rates to a total width of $\approx 0.35$ GeV. We obtain $(f_{\rho'}/f_\rho)^2 \approx 4$. This value is consistent with the typical limits given in the literature, using however realistic assumption on the $\rho'$ width and on the branching ratio to two pions[13].

We discuss now briefly the implications of our last result on proton Compton scattering and vector meson photoproduction, where discrepancies of the order of factors of two are observed for the $\rho$-photon coupling constants when compared to the values found in $e^+e^-$ annihilation.

From the VMD model follows the relation:
\[ \sigma_{\text{tot}}(\gamma p) = \sqrt{4\pi} \alpha \sum_{V=\rho, \omega, \phi} \sqrt{\frac{4}{f_V^2/4\pi}} \frac{1}{1 + \eta_V^2} \frac{d\sigma^0}{dt} (\gamma p \rightarrow Vp) \]

where \(d\sigma^0/dt\) denotes the forward cross section and \(\eta_V\) the ratio of the real to the imaginary part. It is well known that the value of \(f_{\rho}^2/4\pi\) obtained through eq. (10), from the total \(\gamma p\) cross section and from photoproduction data is smaller by a factor of about two than the results from \(e^+e^-\) colliding beams. More explicitly one gets \(f_{\rho}^2/4\pi = 1.4 \pm 0.2\) to be compared with the value \(2.56 \pm 0.22\) (11). Since equation (10) is based on VMD alone the disagreement has to be attributed to a failure of the model. The situation substantially changes in the presence of higher vector mesons coupled to the photon. Let's in fact extend the sum over the vector mesons in eq. (10), by including as next step the \(\rho^1\) and its SU(3) partners. By using the above value \((f_{\rho}/f_{\rho})^2 \approx 4\) and assuming \(d\sigma^0/dt(\gamma p \rightarrow \rho^1p) = (f_{\rho}/f_{\rho})^2 d\sigma^0/dt(\gamma p \rightarrow \rho p)\) the r.h.s. of eq. (10) rises by a factor 1.25, to be compared with 1.35 approximately.

This improvement of the VMD results suggests that moderate but significant corrections can arise from the continuum of higher mass states, and underlines the fact that the hadronic properties of the photon cannot only be described by the nearby vector mesons, namely \(\rho\), \(\omega\) and \(\phi\). Similar conclusions have been obtained by Altarelli and Preparata (16) who have estimated the contribution from the continuum by using the operator expansions near the light cone.

It is very tempting to speculate on what can be the overall contribution of the vector mesons coupled to the photon. Should the naive relation \(f_{\rho}(n)/f_{\rho} = 2^n\) be true for every state \(n\), the equation (10) has to be replaced by:

\[ \sigma_{\text{tot}}(\gamma p) = \sqrt{4\pi} \alpha \sum_{V=\rho, \omega, \phi} \sqrt{\frac{4}{f_V^2/4\pi}} \frac{1}{1 + \eta_V^2} \frac{d\sigma^0}{dt} (\gamma p \rightarrow Vp) \cdot \left\{1 + \frac{1}{4} + \frac{1}{4^2} + \cdots \right\} = \frac{4}{3} \sqrt{4\pi} \alpha \sum_{V=\rho, \omega, \phi} \frac{d\sigma^0}{dt} (\gamma p \rightarrow Vp) \cdot \sqrt{\frac{4}{f_V^2/4\pi}} \frac{1}{1 + \eta_V^2} \frac{d\sigma^0}{dt} (\gamma p \rightarrow Vp) \]

in very good agreement with experiment. This leads to an effective \(\rho\)-photon coupling measured in photon nucleon reactions:
where the value $\frac{f_\rho^2}{4\pi} = 2.56$ has been used. The value (12) is in surprising agreement with different high energy results\(^{17}\) on photoabsorption and Compton scattering experiments.

Finally we observe that experimental evidence in favor of the $1^- (\sim 1600)$ but against the $1^- (\sim 1250)$ is predicted in some Veneziano type form factors\(^{18}\), which allow the photon to couple only to the even daughters of the $\rho$ meson.

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(8) - Particle Data Group, Revs. Modern Phys. 43, April (1971).
(9) - See for instance Ref. (2).
(11) - See Ref. (2).