A. Maďecki and P. Picchi : DYNAMICAL NUCLEON-NUCLEON
CORRELATIONS IN THE Li⁶, C¹² AND O¹⁶ NUCLEI FROM
ELASTIC ELECTRON SCATTERING.
A. Małecki(x) and P. Picchi: DYNAMICAL NUCLEON-NUCLEON
CORRELATIONS IN THE Li^6, C^{12} AND O^{16} NUCLEI FROM
ELASTIC ELECTRON SCATTERING.

Elastic electron scattering is usually analyzed in terms of the e-
lastic charge form factor $F_{ch}$ defined as the Fourier transform of the
ground state charge density. Using the nuclear charge operator as given
in (1) we have the form factor:

$$F_{ch} = (G_{Ep} + G_{En})(1 + \frac{q^2}{8M^2}) e^{-\frac{q^2}{4A\alpha^2}} \left\langle \psi_{SM} \right| \sum_{j=1}^{A} e^{i\vec{q} \cdot \vec{r}_j} \left| \psi_{SM} \right\rangle$$

where $G_{Ep}$ is the electric form factor$^{(2)}$ of the nucleon; $M$, $\vec{q}$, $q^2$ are the
nucleon mass, three momentum transfer and four-momentum squared, re-
spectively. The third term in (1) is the correction for the center-of-mass
motion of the target$^{(3)}$ evaluated in the shell model with oscillator poten-
tial for the nucleus. We shall use this model henceforth; $\alpha$ is the par-
meter of the oscillator well. The last term in (1) is called the shell mo-
del elastic form factor of the nucleus $F_{SM}$; $\psi_{SM}$ being the completely anti
symmetrised shell model ground state wave function. In (1) the same num-
ber of protons and neutrons $Z = 1/2$ A is assumed.

Recently the influence of the short range nucleon-nucleon corre-
lations on the elastic form factor has been discussed extensively$^{(4)-(9)}$.
In refs. (5)-(8) the correlations were introduced by using a Jastrow type
nuclear density of the form$^{(10)}$:

$$\left| \tilde{\psi}(r_1 \ldots r_A) \right|^2 = \left| \psi_{SM}(r_1 \ldots r_A) \right|^2 \prod_{j<k} \left[ 1 - h(s_{jk}) \right] =$$

$$= \left| \psi_{SM} \right|^2 \left[ 1 - \sum_{j<k} h(s_{jk}) + \sum_{j<k, l<m} h(s_{jk})h(s_{lm}) - \ldots \right]$$

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The function $h(s_{jk})$ introduces correlations of the $(j,k)$ pair; in order to simulate at small relative distances $s_{jk} = |\vec{r}_j - \vec{r}_k|$ the hard-core repulsion between nucleons the gaussian form factor $h(s_{jk}) = e^{-c(1/2)A^2S_{jk}^2}$ was assumed.

Because of the complexity of (2) the expression for the correlated form factor (with $\phi$ instead of $\psi_{SM}$) can be evaluated exactly only for simple systems like $He^4$. In fact, in refs. (5)-(7) only the contributions from the one correlated pair part of $\phi$, i.e. terms in the series (2) with powers, of $h(s_{jk})$ smaller than two were retained. The single correlated pair approximation (s.c.p.a.) has, however, been questioned in ref. (8) where it was shown that the exact calculations for $He^4$ give quite a different result from that obtained in (5) with the help of this approximation.

Does the s.c.p.a. fail also for heavier nuclei? In order to answer this question we perform the correlation calculations for $Li^6$, $C^{12}$ and $O^{16}$: As it would be almost impossible to perform the exact Jastrow type calculations for these nuclei we use a method which has been described in our previous paper(9).

In(9) the shell model form factor $F_{SM}$ was expressed in terms of the matrix elements between the two-particle states. In the case of harmonic oscillator wave functions one can go over from the motion of two particles about a common center to a description of the relative and c.m. motion of the two particles. The nucleon-nucleon correlations were introduced in (9) by modifying the radial wave function $R_{nl}(r)$ of the relative two-nucleon motion:

$$\langle n \ell m \rangle = \frac{g(r)}{\sqrt{N_{nl}}} R_{nl} Y_{\ell m}$$

where $g(r)$ is a certain function.

Employing the Moshinsky technique(11) we obtain after some algebra the following correlation correction:

$$\Delta F_{SM} = \frac{1}{Z(2Z-1)} \left\{ 6 e^{-t_s} \Delta(000,000; s) + (Z-2) e^{-t_p} \right\} \left( 3 - 4 t_p + t_p^2 \right) x \Delta(000,000; p) + \frac{1}{2} \Delta(100,100; p) - \sqrt{\frac{2}{3}} t_p \Delta(100,000; p) +$$

$$+ \frac{\Delta \sum_m (01m,01m; p)}{t_p} - 2 t_p \Delta(011,011; p) + \frac{1}{2} \Delta \sum_m (02m,02m; p) +$$

$$+ \frac{2}{\sqrt{3}} t_p \Delta(020,000; p) \right\} + 4(Z-2) e^{-t_{sp}} \left( 1 - \frac{2}{3} t_{sp} \right) \Delta(000,000, s_p) +$$

$$+ \frac{1}{3} \Delta \sum_m (01m,01m; s_p) \right\}$$
where \( t_c = q^2 / (8 \pi^2) \); \( c = s, p, sp, (n \cdot l m, n' l' m'; c) = \langle (n \cdot l m) \right| \exp \left( i / (2qz) \right) \left| (n' \cdot l' m') \right\rangle \) and \( \Delta(\ldots) \) denotes the difference between correlated and uncorrelated magnitudes\(^{(12)}\). The formula \((4)\) is valid for nuclei with two protons in the s-shell and Z-2 protons in the p-shell; the oscillator parameters for the two shells are assumed to be different. The index \( c \) denotes the source of the correlation correction: two correlated nucleons from the s/p/-shell yield the terms with \( c = s/p/ \) while the correlation between two nucleons from the different shells introduces the terms with \( c = sp \)^{(14)}.

We use the following form of \( g(s) \) function (\( s \) being the distance between two nucleons):

\[
g(s) = \sqrt{1 - e^{-1/2 \lambda^2 s^2}}
\]

Thus we are able to compare our results with those\(^{(6)}\) obtained with help of the Jastrow method and s.c.p.a. The results of our analysis for the Li\(^6\), C\(^{12}\) and O\(^{16}\) nuclei are presented in Figs. 1, 2 and 3.

Our conclusions and comments are the following:

a) In the case of Li\(^6\) the elastic electron scattering turns out to be sensitive to the nucleon-nucleon correlations only at large momentum transfers. The low momentum transfer data (up to \( q = 2 \text{ fm}^{-1} \)) for this nucleus are well explained in the oscillator shell model provided one assumes that the s- and p-shell protons move in different wells. At large momentum transfers there is, however, a deviation from the uncorrelated shell model. Introducing the correlations one improves the situation. We have obtained a good fit to the experimental data over the wide range of momentum transfer (see Fig. 1). Our calculations predict a diffraction minimum for Li\(^6\) at \( q = 3.7 \text{ fm}^{-1} \).

b) In the elastic electron scattering from C\(^{12}\) and O\(^{16}\) the short range correlations are less important. The experimental results for these nuclei are fairly well explained in the oscillator shell model without the correlations.

c) Our calculations are in a drastic disagreement with the results\(^{(6)}\) which were based on the Jastrow method with the s.c.p.a. Using the oscillator and correlation parameters as given in ref. \((6)\) we have evaluated the dotted lines in Figs. 1, 2 and 3. These curves are inconsistent with the experimental results while in ref. \((6)\) applying the same parameters, a good accord with experiment was obtained. This comparison allows us to state that the single correlated pair approximation is wrong. In our opinion, this approximation considerably overestimates the effect of the short range correlations.

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REFERENCES AND FOOTNOTES -

(3) - L. J. Tassie and F. C. Barker, Phys. Rev. 111, 940 (1958). If one assumes for the s-p nuclei the oscillator model in which the s- and p- nucleons move in different wells ($\propto_s \neq \propto_p$) the parameter in the c.m. correction is given by:

$$A \propto^2 \rightarrow 4 \propto^2_s + 2(Z-2) \propto^2_p.$$  

(12) - We use Moszinsky's notation for the radial quantum number $n$. The more usual $n'$ can be obtained by adding unity to Moszinsky's values. In order to obtain orthonormality of all the correlated states involved in calculations for the $\uparrow 100 \downarrow$ state a slightly different from (3) radial dependence should be introduced (13):

$$\widetilde{R}_{10} = 6^{1/2} \pi^{-1/4} \propto^{3/2} \frac{g(r)}{N_{10}} \left(1 + \delta - \frac{2}{3} \propto^2 r^2\right) e^{-1/2 \propto^2 r^2}$$

where

$$N_{10} = -3/2 \frac{N_{01}}{N_{00}} + \frac{5}{2} N_{02} \quad \text{and} \quad \delta = \frac{N_{01}}{N_{00}} - 1.$$

(14) - The average oscillator parameter $\propto_{sp}$ was assumed to be:

$$\propto_{sp} = \sqrt{\frac{5Z-4}{\propto_{s}^{-2} + 5(Z-2) \propto_{p}^{-2}}}^{-1/2}.$$  

FIG. 1 - Elastic form factor of Li$^6$. The experimental points are from ref. (15). The dashed line represents the uncorrelated form factor with $\alpha_s = 118$ MeV, $\alpha_p = 106$ MeV. The full line was obtained with the same $\alpha$'s and with the correlation parameter $\Lambda = 2$ fm$^{-1}$. The dotted line, calculated with $\alpha_s = 127.5$ MeV, $\alpha_p = 100.6$ MeV and $\Lambda = 1.878$ fm$^{-1}$, is given for a comparison with ref. (6).
FIG. 2 - Elastic form factor of C\textsuperscript{12}. The experimental data are those reported in ref. (6). The dashed line represents the uncorrelated form factor with \( \alpha_g = \alpha_p = \alpha = 119.5 \text{ MeV} \). The full line was obtained with the same \( \alpha \) and with the correlation parameter \( \Lambda = 2 \text{ fm}^{-1} \). The dotted line, calculated with \( \alpha_g = 168.2 \text{ MeV} \), \( \alpha_p = 126.2 \text{ MeV} \) and \( \Lambda = 2.156 \text{ fm}^{-1} \), is given for the comparison with the result of ref. (6).
FIG. 3 - Elastic form factor of $^{16}$O. The experimental data are those reported in ref. (6). The dashed line represents the uncorrelated form factor with $\alpha_s = \alpha_p = \alpha = 10.9$ MeV. The full line was obtained with the same $\alpha$ and with the correlation parameter $\Lambda = 2$ fm$^{-1}$. The dotted line, calculated with $\alpha_s = 175.1$ MeV, $\alpha_p = 122.3$ MeV and $\Lambda = 2.426$ fm$^{-1}$, is given for the comparison with the result of ref. (6).