CP VIOLATION

Juliet Lee-Franzini

INFN-Laboratori Nazionali di Frascati Via E. Fermi 40, I-00044 Frascati, Italy

Table of Contents

1. Introduction
2. Historical background
3. CP Violation in Two Pion Decay Modes
4. CP Violation at a $\phi$-Factory
5. CP Violation in Other Modes
6. Determinations of Neutral Kaon Properties
7. Three Precision CP Violation Experiments
8. The CKM Mixing Matrix
9. Unitarity triangles
10. Rare K decays
11. $B$ decays

Lectures given at
CERN Summer School 1999, August 3–6, 1999
1 Introduction

The origin of $CP$ violation, to my mind, is one of the two most important questions to be understood in particle physics (the other one being the origin of mass). In the meantime we are finally getting proof - after 51 years of hard work - that $CP$ belongs to the weak interaction with 6 quarks and a unitary mixing matrix. This June 1999, “kaon physicists” had a celebratory get together in Chicago. Many of the comments in these lectures reflect the communal reassessments and cogitations from that workshop. It is clear that a complete experimental and theoretical $albeit$ phenomenological solution of the $CP$ violation problem will affect in a most profound way the fabric of particle physics.

2. Historical background

It is of interest, at this junction, to sketch with broad strokes this evolution. With hindsight, one is impressed by how the $K$ mesons are responsible for many of the ideas which today we take for granted.

1. Strangeness which led to quarks and the flavor concept.
2. The $\tau-\theta$ puzzle led to the discovery of parity violation.
3. The $\Delta I = 1/2$ rule in non leptonic decays, approximately valid in kaon and all strange particle decays, still not quite understood.
4. The $\Delta S = \Delta Q$ rule in semileptonic decays, fundamental to quark mixing.

5. Flavor changing neutral current suppression which led to 4 quark mixing - GIM mechanism, charm.

6. $CP$ violation, which requires 6 quarks - KM, beauty and top.

2.1 $K$ Mesons and Strangeness

$K$ mesons were discovered in 1944 in cosmic radiation$^{[1]}$ and their decays were first observed in 1947.$^{[2]}$ A pair of two old cloud chamber pictures of their decay is on the website http://hepweb.rl.ac.uk/ppUKpics/pr_971217.html demonstrating that they come both in neutral and charged versions.

On December 1947 Rochester and Butler* published Wilson chamber pictures showing evidence for what we now call $K^d \rightarrow \pi^+ \pi^-$ and $K^{-} \rightarrow \pi^- \nu$.

2.1.1 The Strange Problem

Fig. 1. Production and decay of V particles.

In few triggered pictures, $\sim 1000$ nuclear interactions, a few particles which decay in few cm were observed. A typical strong interaction cross section is $(1 \text{ fm})^2 = 10^{-26} \text{ cm}^2$, corresponding to the production in a $1 \text{ g/cm}^2$ plate of:

$$N_{\text{events}} = N_{\text{in}} \times \sigma \times \frac{\text{nucleons}}{\text{cm}^2} = 10^3 \times 10^{-26} \times 1 \times 6 \times 10^{23} = 6$$

Assuming the V-particles travel a few cm with $\gamma \beta \sim 3$ their lifetime is $\mathcal{O}(10^{-10} \text{ s})$, typical of weak interactions. We conclude that the decay of V-particles is weak while the production is strong, strange indeed since pions and nucleons appear at the beginning and at the end!!

This strange property of $K$ mesons and other particles, the hyperons, led to the introduction of a new quantum number, the strangeness, $S^{[3]}$. Strangeness is conserved in strong interactions, while first order weak interaction can induce transitions in which
strangeness is changed by one unit.

Today we describe these properties in terms of quarks with different “flavors”, first suggested in 1964 independently by Gell-Mann and Zweig, reformulating the $SU(3)$ flavor, approximate, global symmetry. The “normal particles” are bound states of quarks: $q\bar{q}$, the mesons, or $qqq$, baryons, where

$$q = \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \text{up} \\ \text{down} \end{pmatrix}.$$  

$K$’s, hyperons and hypernuclei contain a strange quark, $s$:

$$K^0 = d\bar{s} \quad \bar{K}^0 = \bar{d}s$$  
$$K^+ = u\bar{s} \quad K^- = \bar{u}s$$  
$$S = +1 \quad S = -1.$$  

The assignment of negative strangeness to the $s$ quark is arbitrary but maintains today the original assignment of positive strangeness for $K^0$, $K^+$ and negative for the $\Lambda$ and $\Sigma$ hyperons and for $\bar{K}^0$ and $K^-$. Or, mysteriously, calling negative the charge of the electron.

An important consequence of the fact that $K$ mesons carry strangeness, a new additive quantum number, is that the neutral $K$ and anti neutral $K$ meson are distinct particles!!

$$C|K^0\rangle = |\bar{K}^0\rangle, \quad S|K^0\rangle = |K^0\rangle, \quad S|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

An apocryphal story says that upon hearing of this hypothesis, Fermi challenged Gell-Mann to devise an experiment which
shows an observable difference between the $K^0$ and the $\bar{K}^0$. Today we know that it is trivial to do so. For example, the process $p\bar{p} \rightarrow \pi^-K^+\bar{K}^0$, produces $\bar{K}^0$'s which in turn can produce $\Lambda$ hyperons while the $K^0$'s produced in $p\bar{p} \rightarrow \pi^+K^-K^0$ cannot.

Since the '50's $K$ mesons have been produced at accelerators, first amongst them was the Cosmotron.

2.2 Parity Violation

Parity violation, $P$, was first observed through the $\theta-\tau$ decay modes of $K$ mesons.

Incidentally, the $\tau$ there is not the heavy lepton of today, but is a charged particle which decays into three pions, $K^+ \rightarrow \pi^+\pi^+\pi^-$ in todays language.

The $\theta$ there refers to a neutral particle which decays into a pair of charged pions, $K^0 \rightarrow \pi^+\pi^-$. The studies of those days were done mostly in nuclear emulsions where JLF contributions included pulling out long strands of hair from her head to make the cross grids in between emulsion plates, to enable tracing tracks from one plate to the next...

The burning question was whether these two particle were the same particle with two decay modes, or two different ones. And if they were the same particle, how could the two different final states have opposite parity?

This puzzle was originally not so apparent until Dalitz ad-
vanced an argument which says that one could determine the spin of \( \tau \) by looking at the decay distribution of the three pions in a “Dalitz” (what he calls phase space) plot, which was in fact consistent with \( J=0 \).

The spin of the \( \theta \) was inferred to be zero because it did not like to decay into a pion and a photon (a photon cannot be emitted in a \( 0 \rightarrow 0 \) transition).

\[
\begin{array}{c}
\text{Fig. 2. Definition of } l \text{ and } L \text{ for the three pion decays of } \tau^+.
\end{array}
\]

For neutral \( K \)'s one of the principal decay modes are two or three pions.

\[
\begin{array}{c}
\text{Fig. 3. Definition of } l \text{ and } L \text{ for } K^0 \rightarrow \pi^+\pi^-\pi^0.
\end{array}
\]

The relevant properties of the neutral two and three pion systems with zero total angular momentum are given below.

1. \( \ell = L = 0, 1, 2 \ldots \)
2. \( \pi^+\pi^-, \pi^0\pi^0 \): \( P = +1, \; C = +1, \; CP = +1 \).
3. \( \pi^+\pi^-\pi^0 \): \( P = -1, \; C = (-1)^l, \; CP = \pm 1 \), where \( l \) is the
angular momentum of the charged pions in their center of mass. States with \( l > 0 \) are suppressed by the angular momentum barrier.

4. \( \pi^0 \pi^0 \pi^0 \): \( P = -1, \ C = +1, \ CP = -1 \). Bose statistics requires that \( l \) for any identical pion pair be even in this case.

Note that the two pion and three pion states have opposite parity.

2.3 Mass and \( CP \) Eigenstates

While the strong interactions conserve strangeness, the weak interactions do not. In fact, not only do they violate \( S \) with \( \Delta S = 1 \), they also violate charge conjugation, \( C \), and parity, \( P \), as we have just seen. However, at the end of the 50’s, the weak interaction does not manifestly violate the combined \( CP \) symmetry. For now let’s assume that \( CP \) is a symmetry of the world. We define an arbitrary, unmeasurable phase by:

\[
CP | K^0 \rangle = | \bar{K}^0 \rangle
\]

Then the simultaneous mass and \( CP \) eigenstates are:\[5\]

\[
| K_1 \rangle \equiv \frac{| K^0 \rangle + | \bar{K}^0 \rangle}{\sqrt{2}}
\]

\[
| K_2 \rangle \equiv \frac{| K^0 \rangle - | \bar{K}^0 \rangle}{\sqrt{2}}, \tag{2.1}
\]

where \( K^0_1 \) has \( CP=+1 \) and \( K^0_2 \) has \( CP=-1 \).

While \( K^0 \) and \( \bar{K}^0 \) are degenerate states in mass, as required
by CPT invariance, the weak interactions, which induces to second order $K^0\leftrightarrow\bar{K}^0$ transitions, induces a small mass difference between $K_1^0$ and $K_2^0$, $\Delta m$. We expect that $\Delta m\sim \Gamma$, at least as long as real and imaginary parts of the amplitudes of fig. 4 are about equal, since the decay rate is proportional to the imaginary part and the real part contributes to the mass difference. Dimensionally, $\Gamma=\Delta m=G^2m_\pi^5 = 5.3\times10^{-15}$ GeV, in good agreement with measurements. The $K_1^0$ mass is the expectation value $\langle K_1^0|H| K_1^0 \rangle$. With $K_1^0=(K^0+\bar{K}^0)/\sqrt{2}$ and analogously for $K_2^0$, we find

$$m_1 - m_2 = \langle K^0|H|\bar{K}^0 \rangle + \langle \bar{K}^0|H| K^0 \rangle,$$

that is the mass difference is due to $K^0 \leftrightarrow \bar{K}^0$ transitions induced by a $\Delta S=2$ interaction.

![Diagrams](image)

**Fig. 4.** Contributions to the $K_1^0$-$K_2^0$ mass difference.

### 2.4 $K_1^0$ and $K_2^0$ lifetimes and mass difference

If the total Hamiltonian conserves $CP$, i.e. $[H, CP] = 0$, the decays of $K_1$’s and $K_2$’s must conserve $CP$. Thus the $K_1$’s with $CP = 1$, must decay into two pions (and three pions in an $L=\ell=1$ state, surmounting an angular momentum barrier $-\sim(kr)^2(KR)^2\sim1/100$), while the $K_2$’s with $CP = -1$,
must decay into three pion final states. Since the energy available in the two pion decay mode is approximately 220 MeV, while that for the three pion decay mode is only about 90 MeV, the lifetime of the $K_1$ is much much shorter than that of the $K_2$.

The first verification of the above consideration was obtained as early as 1956 by Lederman et al.\cite{6} who observed in a cloud chamber that, in fact, neutral $K$ meson were still present at times much larger than the then accepted value of the neutral $K$ lifetime, which was in fact the $K^0$ lifetime.

From the most recent measurements, giving $\tau_1 = (0.8959 \pm 0.0006) \times 10^{-10}$ s and $\Delta m$ as below, ignoring for the moment $CP$ violation, we have:†

\[
\begin{align*}
\Gamma_1 &= (1.1162 \pm 0.0007) \times 10^{10} \text{ s}^{-1} \\
\Gamma_2 &= (1.72 \pm 0.02 \times) 10^{-3} \times \Gamma_1 \\
\Delta m &= m(K_2) - m(K_1) = (0.5296 \pm 0.0010) \times 10^{10} \text{ s}^{-1} \\
\frac{\Delta m}{\Gamma_1 + \Gamma_2} &= 0.4736 \pm 0.0009.
\end{align*}
\]

\[
\text{†We use natural units, i.e. } \hbar = c = 1. \text{ Conversion is found using } \hbar c = 197.3 \ldots \text{ MeV} \times \text{fm}.
\]
Unit Conversion

<table>
<thead>
<tr>
<th>To convert from</th>
<th>to</th>
<th>multiply by</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/MeV</td>
<td>s</td>
<td>6.58 x 10^{-22}</td>
</tr>
<tr>
<td>1/MeV</td>
<td>fm</td>
<td>197</td>
</tr>
<tr>
<td>1/GeV²</td>
<td>mb</td>
<td>0.389</td>
</tr>
</tbody>
</table>

2.5 Strangeness Oscillations

The mass eigenstates \( K_1^0 \) and \( K_2^0 \) evolve in vacuum and in their rest frame according to

\[
i \frac{d}{dt} \Psi = H \Psi = M \psi, \tag{2.3}
\]

where \( M \) is the complex mass \( M_1, _2 = m_1, _2 - i \Gamma_1, _2 / 2 \), with \( \Gamma = 1/\tau \). The state evolution is therefore given by:

\[
| K_{1,2}, t \rangle = | K_{1,2}, t = 0 \rangle e^{-im_{1,2}t - t\Gamma_{1,2}/2}
\]

If the initial state has definite strangeness, say it is a \( K^0 \) as from the production process \( \pi^- p \rightarrow K^0 \Lambda^0 \), it must first be rewritten in terms of the mass eigenstates \( K_1^0 \) and \( K_2^0 \) which then evolve in time as above. Since the \( K_1^0 \) and \( K_2^0 \) amplitudes change phase differently in time, the pure \( S=1 \) state at \( t=0 \) acquires an \( S=-1 \) component at \( t > 0 \). Using equations (2.1) the wave function at time \( t \) is:

\[
\Psi(t) = \sqrt{1/2} \left[ e^{(i m_1 - \Gamma_1/2)t} | K_1 \rangle + e^{(i m_2 - \Gamma_1/2)t} | K_2 \rangle \right] = 1/2 \left[ (e^{(i m_1 - \Gamma_1/2)t} + e^{(i m_2 - \Gamma_2/2)t}) | K^0 \rangle + (e^{(i m_1 - \Gamma_1/2)t} - e^{(i m_2 - \Gamma_2/2)t}) | \bar{K}^0 \rangle \right].
\]
The intensity of $K^0$ ($\bar{K}^0$) at time $t$ is given by:

$$I(K^0 \ (\bar{K}^0) , t) = |\langle K^0 \ (\bar{K}^0) | \Psi(t) \rangle|^2 =$$

$$\frac{1}{4}[e^{-t\Gamma_1} + e^{-t\Gamma_2} + (-) 2e^{-t(\Gamma_1+\Gamma_2)/2} \cos \Delta m \ t]$$

which exhibits oscillations whose frequency depends on the mass difference, see fig. 5.

![Figure 5](image)

**Figure 5.** Evolution in time of a pure $S = 1$ state at time $t = 0$.

The appearance of $\bar{K}^0$'s from an initially pure $K^0$ beam can detected by the production of hyperons, according to the reactions:

$$\bar{K}^0 p \to \pi^+ \Lambda^0, \quad \to \pi^+ \Sigma^+, \quad \to \pi^0 \Sigma^+, \quad \to \pi^+ \Sigma^+,$$

$$\bar{K}^0 n \to \pi^0 \Lambda^0, \quad \to \pi^0 \Sigma^0, \quad \to \pi^- \Sigma^-.$$

The $K_L-K_S$ mass difference can therefore be obtained from the oscillation frequency.
2.6 REGENERATION

Another interesting, and extremely useful phenomenon, is that it is possible to regenerate $K_1$’s by placing a piece of material in the path of a $K_2$ beam. Let’s take our standard reaction,

$$\pi^- p \rightarrow K^0 \Lambda^0,$$

the initial state wave function of the $K^0$’s is

$$\Psi(t = 0) \equiv | K^0 \rangle = \frac{| K_1 \rangle + | K_2 \rangle}{\sqrt{2}}.$$

Note that it is composed equally of $K_1$’s and $K_2$’s. The $K_1$ component decays away quickly via the two pion decay modes, leaving a virtually pure $K_2$ beam. This $K_2$ beam has equal $K^0$ and $\bar{K}^0$ components, which interact differently in matter. For example, the $K^0$’s undergo elastic scattering, charge exchange etc. whereas the $\bar{K}^0$’s also produce hyperons via strangeness conserving transitions. Thus we have an apparent rebirth of $K_1$’s emerging from a piece of material placed in the path of a $K_2$ beam! See fig. 6.

\begin{center}
\includegraphics[width=0.8\textwidth]{figure6.png}
\end{center}

**Figure 6.** $K_1^0$ regeneration.

Virtually all past and present experiments, with the exception of
a couple which will be mentioned explicitly, use this method to obtain a source of \(K_1^0\)'s (or \(K_S\)'s, as we shall see later). Denoting the amplitudes for \(K^0\) and \(\bar{K}^0\) scattering on nuclei by \(f\) and \(\bar{f}\) respectively, the scattered amplitude for an initial \(K_2^0\) state is given by:

\[
\sqrt{1/2}(f | K^0 \rangle - \bar{f} | \bar{K}^0 \rangle) = \frac{f + \bar{f}}{2\sqrt{2}}(| K^0 \rangle - | \bar{K}^0 \rangle) + \\
\frac{f - \bar{f}}{2\sqrt{2}}(| K^0 \rangle + | \bar{K}^0 \rangle) = 1/2(f + \bar{f})| K_2 \rangle + \\
1/2(f - \bar{f})| K_1 \rangle.
\]

The so called regeneration amplitude for \(K_2^0 \rightarrow K_1^0\), \(f_{21}\) is given by \(1/2(f - \bar{f})\) which of course would be 0 if \(f = \bar{f}\), which is true at infinite energy.

Another important property of regeneration is that when the \(K_1^0\) is produced at non-zero angle to the incident \(K_2^0\) beam, regeneration on different nuclei in a regenerator is incoherent, while at zero degree the amplitudes from different nuclei add up coherently.

The intensity for coherent regeneration depends on the \(K_1^0\), \(K_2^0\) mass difference. Precision mass measurements have been performed by measuring the ratio of coherent to diffraction regeneration. The interference of \(K_1^0\) waves from two or more regenerators has also allowed us to determine that the \(K_2^0\) meson is heavier than the \(K_1^0\) meson. This perhaps could be expected, but it is nice to have it measured.
Finally we note that the $K^0_1$ and $K^0_2$ amplitudes after regeneration are coherent and can interfere if $CP$ is violated.

3. CP Violation in Two Pion Decay Modes

3.1 DISCOVERY

For some years after the discovery that $C$ and $P$ are violated in the weak interactions, it was thought that $CP$ might still be conserved. $CP$ violation was discovered in '64,[7] through the observation of the unexpected decay $K^0_2 \rightarrow \pi^+\pi^-$. This beautiful experiment is conceptually very simple, see fig. 7.

![Figure 7](image)

**Figure 7.** The setup of the experiment of Christenson *et al.*

Let a $K^0_2$ beam pass through a long collimator and decay in an empty space (actually a big helium bag) in front of two spectrometers. The decay products are viewed by spark chambers and scintillator hodoscopes in the spectrometers placed on ei-
ther side of the beam. Two pion decay modes are distinguished from three pion and leptonics decay modes by the reconstructed invariant mass $M_{\pi\pi}$, and the direction $\theta$ of their resultant momentum vector relative to the beam. In the mass interval between 490 MeV and 510 MeV, 50 events were found which were exactly collinear with the beam ($\cos\theta > 0.999$), which demonstrated for the first time that $K_2^0$'s decayed into two pions, with a branching ratio of the order of $1/10^{-3}$, thus $CP$ is shown to be violated! The $CP$ violating decay $K_L \rightarrow \pi^0\pi^0$ has also been observed.

### 3.2 $K^0$ Decays with $CP$ Violation

Since $CP$ is violated in $K$ decays, the mass eigenstates are no more $CP$ eigenstate and can be written, assuming $CPT$ invariance, as:

$$K_S = p|K^0\rangle + q|\bar{K}^0\rangle$$

$$K_L = p|K^0\rangle - q|\bar{K}^0\rangle$$

with $p^2 + q^2 = 1$. Defining $\epsilon$ via

$$\frac{p}{q} = \frac{1 + \epsilon}{1 - \epsilon}$$

we have:

$$|K_S\rangle = \frac{|K_1^0\rangle + \epsilon|K_2^0\rangle}{\sqrt{1 + |\epsilon|^2}}$$

$$|K_L\rangle = \frac{|K_2^0\rangle + \epsilon|K_1^0\rangle}{\sqrt{1 + |\epsilon|^2}}$$

with $|\epsilon| = (2.259 \pm 0.018) \times 10^{-3}$ from experiment. Note that
the $K_S$ and $K_L$ states are not orthogonal states, contrary to the case of $K_1^0$ and $K_2^0$. Equation (2.3) can be rewritten, to lowest order, as:

$$\frac{d}{dt} |K_{S,L}\rangle = -i\mathcal{M}_{S,L} |K_{S,L}\rangle$$

with

$$\mathcal{M}_{S,L} = M_{S,L} - i\Gamma_{S,L}/2$$

and the values of masses and decay widths given in eq. (2.2) belong to $K_S$ and $K_L$, rather than to $K_1^0$ and $K_2^0$. We further introduce the so called superweak phase $\phi_{SW}$ as:

$$\phi_{SW} = \text{Arg}(\epsilon) = \tan^{-1} \frac{2(M_{K_L} - M_{K_S})}{\Gamma_{K_S} - \Gamma_{K_L}} = 43.63^\circ \pm 0.08^\circ.$$  

A superweak theory, is a theory with a $\Delta S=2$ interaction, whose sole effect is to induce a $CP$ impurity $\epsilon$ in the mass eigenstates.

Since 1964 we have been asking the question: is $CP$ violated directly in $K^0$ decays, *i.e.* is the $|\Delta S|=1$ amplitude $\langle \pi\pi | K_2 \rangle \neq 0$ or the only manifestation of $CP$ is to introduce a small impurity of $K_1$ in the $K_L$ state, via $K^0 \leftrightarrow \bar{K}^0$, $|\Delta S|=2$ transitions?

Wu and Yang,[8] have analyzed the two pion decays of $K_S$, $K_L$ in term of the isospin amplitudes:

$$A(K^0 \rightarrow 2\pi, I) = A_I e^{i\delta_I}$$

$$A(\bar{K}^0 \rightarrow 2\pi, I) = A^*_I e^{i\delta_I}$$

where $\delta_I$ are the $\pi\pi$ scattering phase shifts in the $I=0, 2$ states. W-Y chose an arbitrary phase, by defining $A_0$ real. They also
introduce the ratios of the amplitudes for \( K \) decay to a final state \( f_i \), \( \eta_i = A(K_L \to f_i)/A(K_S \to f_i) \):

\[
\eta_{+-} \equiv |\eta_{+-}| e^{-i\phi_{+-}} = \frac{\langle \pi^+\pi^- \mid K_L \rangle}{\langle \pi^+\pi^- \mid K_S \rangle} = \epsilon - 2\epsilon' \\
\eta_{00} \equiv |\eta_{00}| e^{-i\phi_{00}} = \frac{\langle \pi^0\pi^0 \mid K_L \rangle}{\langle \pi^0\pi^0 \mid K_S \rangle} = \epsilon + \epsilon',
\]

with

\[
\epsilon' = \frac{i}{2\sqrt{2}} e^{i(\delta_2-\delta_0)} \frac{\mathcal{R}A_2}{A_0} \frac{\Re A_2}{\mathcal{R}A_2}
\]

Since \( \delta_2 - \delta_0 \sim 45^\circ \), \( \text{Arg}(\epsilon') \sim 135^\circ \) \emph{i.e.} \( \epsilon' \) is orthogonal to \( \epsilon \). Therefore, in principle, only two real quantities need to be measured: \( \Re \epsilon \) and \( \Re(\epsilon'/\epsilon) \), \emph{with sign}.

In terms of the measurable amplitude ratios, \( \eta \) and \( \epsilon \) are given by:

\[
\epsilon = (2\eta_{+-} + \eta_{00})/3 \\
\text{Arg}(\epsilon) = \phi_{+-} + (\phi_{+-} - \phi_{00})/3.
\]

\( \epsilon' \) is a measure of direct \( CP \) violation and its magnitude is \( \mathcal{O}(A(K_2^0 \to \pi\pi)/A(K_1^0 \to \pi\pi)) \).

Our question above is then the same as: is \( \epsilon' \neq 0 \)? Since 1964, experiments searching for a difference in \( \eta_{+-} \) and \( \eta_{00} \) have been going on.

If \( \eta_{+-} \neq \eta_{00} \) the ratios of branching ratios for \( K_{L,S} \to \pi^+\pi^- \) and \( \pi^0\pi^0 \) are different. Most experiments measure the quantity \( \mathcal{R} \), the so called double ratio of the four rates for \( K_{L,S} \to \pi^0\pi^0 \), \( \pi^+\pi^- \), which is given, to lowest order in \( \epsilon \) and \( \epsilon' \) by:
\[ \mathcal{R} \equiv \frac{\Gamma(K_L \to \pi^0 \pi^0)}{\Gamma(K_L \to \pi^+ \pi^-)} / \frac{\Gamma(K_S \to \pi^0 \pi^0)}{\Gamma(K_S \to \pi^+ \pi^-)} \equiv \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 = 1 - 6\Re(\epsilon'/\epsilon). \]

Observation of \( \mathcal{R} \neq 0 \) is proof that \( \Re(\epsilon'/\epsilon) \neq 0 \) and therefore of “direct” \( CP \) violation, \textit{i.e.} that the amplitude for \(|\Delta S| = 1\), \( CP \) violating transitions \( A(K_2 \to 2\pi) \neq 0 \).

All present observations of \( CP \) violation, \( \mathcal{C}_P \), \textit{i.e.} the decays \( K_L \to 2\pi, \pi^+ \pi^- \gamma \) and the charge asymmetries in \( K_{\ell 3} \) decays are examples of so called “indirect” violation, due to \(|\Delta S| = 2\) \( K^0 \leftrightarrow \bar{K}^0 \) transitions introducing a small \( CP \) impurity in the mass eigenstates \( K_S \) and \( K_L \).

Because of the smallness of \( \epsilon \) (and \( \epsilon' \)), most results and parameter values given earlier for \( K_1^0 \) and \( K_2^0 \) remain valid after the substitution \( K_1^0 \to K_S \) and \( K_2^0 \to K_L \).

### 3.3 Experimental Status

We have been enjoying a roller coaster ride on the last round of \( CP \) violation precision experiments.

One of the two, NA31, was performed at CERN and reported a tantalizing non-zero result:\[^9\]

\[ \Re(\epsilon'/\epsilon) = (23 \pm 6.5) \times 10^{-4}. \]
NA31 alternated $K_S$ and $K_L$ data taking by the insertion of a $K_S$ regenerator in the $K_L$ beam every other run, while the detector collected both charged and neutral two pion decay modes simultaneously.

The other experiment, E731 at Fermilab, was consistent with no or very small direct $\mathcal{CP}$: \cite{10}

$$\Re(\epsilon'/\epsilon) = (7.4 \pm 5.9) \times 10^{-4}.$$  

E731 had a fixed $K_S$ regenerator in front of one of the two parallel $K_L$ beams which entered the detector which, however, collected alternately the neutral and charged two pion decay modes.

Both collaborations have completely redesigned their experiments. Both experiments can now observe both pion modes for $K_S$ and $K_L$ simultaneously. Preliminary results indicate that in fact the answer to the above question is a resounding NO!!!

In fact, the great news in HEP for 1999 is that, combining the results of few years ago and the new ones, the value of $\epsilon'$ is $(21.2 \pm 4.6) \times 10^{-4}$, which means that there definitely is direct $CP$ violation. The observed value is $4.7\sigma$ away from zero.

Attention is now turning from asking whether $\epsilon'$ is zero or not to its precise determination.
4. \( CP \) Violation at a \( \phi \)-factory

4.1 \( e^+e^-\to\phi, \phi \to K\bar{K} (\Upsilon'' \to B^0\bar{B}^0) \)

The cross section for production of a bound \( q\bar{q} \) pair of mass \( M \) and total width \( \Gamma \) with \( J^{PC}=1^{--} \), a so called vector meson \( V \), \((\phi\) in the following and the \( \Upsilon(4S) \) later) in \( e^+e^- \) annihilation, see fig. 8, is given by:

\[
\sigma_{q\bar{q},\text{res}} = \frac{12\pi}{s} \frac{\Gamma_{\text{ee}}\Gamma_V M_V^2}{(M_V^2 - s)^2 + M_V^2 \Gamma_V^2} =
\]

\[
\frac{12\pi}{s} B_{\text{ee}} B_{Vq\bar{q}} \frac{M_V^2 \Gamma_V^2}{(M^2 - s)^2 + M_V^2 \Gamma_V^2}
\]

Figure 8. Amplitude for production of a bound \( q\bar{q} \) pair.

The \( \phi \) meson is a \( s\bar{s} \) \( ^3S_1 \) bound state with \( J^{PC}=1^{--} \), just as a photon and the cross section for its production in \( e^+e^- \) annihilations at 1020 MeV is

\[
\sigma_{s\bar{s}}(s = (1.02)^2 \text{ GeV}^2) \sim \frac{12\pi}{s} B_{ee}
\]

\[
= 36.2 \times (1.37/4430) = 0.011 \text{ GeV}^{-2} \sim 4000 \text{ nb},
\]

compared to a total hadronic cross section of \( \sim 4(3) \)
×87≈100 nb.

The production cross section for the $\Upsilon(4S)$ at $W=10,400$ MeV is $\sim 1$ nb, over a background of $\sim 2.6$ nb.

The Frascati $\phi$–factory, DA$\Phi$NE, will have a luminosity $\mathcal{L} = 10^{33}$ cm$^{-2}$ s$^{-1} = 1$ nb$^{-1}$s$^{-1}$. Integrating over one year, taken as $10^7$ s or one third of a calendar year, we find

$$\int_{1\text{ y}} \mathcal{L} dt = 10^7 \text{ nb}^{-1},$$

corresponding to the production at DA$\Phi$NE of

$\sim 4000 \times 10^7 = 4 \times 10^{10}$ $\phi$ meson per year or approximately $1.3 \times 10^{10}$ $K^0$, $\bar{K}^0$ pairs, a large number indeed.

One of the advantages of studying $K$ mesons at a $\phi$–factory, is that they are produced in a well defined quantum state. Neutral $K$ mesons are produced as collinear pairs, with $J^{PC} = 1^{--}$ and a momentum of about 110 MeV/c, thus detection of one $K$ announces the presence of the other and gives its direction.

Since in the reaction:

$$e^+e^- \rightarrow \gamma \rightarrow \phi \rightarrow K^0\bar{K}^0$$

we have

$$C(K^0\bar{K}^0) = C(\phi) = C(\gamma) = -1.$$

we can immediately write the 2-$K$ state. Define $|i\rangle = |KK\rangle$, $t =$
0, \( C = -1 \). Then \(|i\rangle\) must have the form:

\[
| i \rangle = \frac{| K^0, p \rangle | \bar{K}^0, -p \rangle - | \bar{K}^0, p \rangle | K^0, -p \rangle}{\sqrt{2}}
\]

From eq. (3.1), the relations between \( K_S, K_L \) and \( K^0, \bar{K}^0 \), to lowest order in \( \epsilon \), we find:

\[
| K_S (K_L) \rangle = \frac{(1 + \epsilon)| K^0 \rangle + (-)(1 - \epsilon)| \bar{K}^0 \rangle}{\sqrt{2}}
\]

\[
| K^0 (\bar{K}^0) \rangle = \frac{| K_S \rangle + (-)| K_L \rangle}{(1 + (-)\epsilon)\sqrt{2}}
\]

from which

\[
| i \rangle = \frac{1}{\sqrt{2}} \left( | K_S, -p \rangle | K_L, p \rangle - | K_S, p \rangle | K_L, -p \rangle \right)
\]

so that the neutral kaon pair produced in \( e^+e^- \) annihilations is a pure \( K^0, \bar{K}^0 \) as well as a pure \( K_S, K_L \) for all times, in vacuum.

What this means, is that if at some time \( t \) a \( K_S (K_L, K^0, \bar{K}^0) \) is recognized, the other kaon, if still alive, is a \( K_L (K_S, \bar{K}^0, K^0) \).

The result above is correct to all orders in \( \epsilon \), apart from a normalization constant, and holds even without assuming CPT invariance.

### 4.2 Correlations in \( K_S, K_L \) Decays

To obtain the amplitude for decay of \( K(p) \) into a final state \( f_1 \) at time \( t_1 \) and of \( K(-p) \) to \( f_2 \) at time \( t_2 \), see the diagram below,
we time evolve the initial state in the usual way:

\[
|t_1, \ p; \ t_2, -p\rangle = \frac{1 + |\epsilon^2|}{(1 - \epsilon^2)^{\sqrt{2}}} \times \\
\left( |K_S(-p)\rangle |K_L(p)\rangle e^{-i(M_{st2} + M_{lt1})} - \\
|K_S(p)\rangle |K_L(-p)\rangle e^{-i(M_{st1} + M_{lt2})} \right)
\]

\[
\begin{array}{ccc}
& t_1 & \\
K_S, K_L & \phi & K_L, K_S \\
& t_2 & \end{array}
\]

\[f_1 \quad \bullet \quad \phi \quad \bullet \quad f_2\]

**Fig. 9.** $\phi \rightarrow K_L, K_S \rightarrow f_1, f_2.$

where $M_{s,l} = M_{s,l} - i\Gamma_{s,l}/2$ are the complex $K_S, K_L$ masses.

In terms of the previously mentioned ratios $\eta_i = \langle f_i | K_L \rangle / \langle f_i | K_S \rangle$ and defining $\Delta t = t_2 - t_1$, $t = t_1 + t_2$, $\Delta M = M_L - M_S$ and $\mathcal{M} = \mathcal{M}_L + \mathcal{M}_S$ we get the amplitude for decay to states 1 and 2:

\[
A(f_1, f_2, t_1, t_2) = \langle f_1 | K_S \rangle \langle f_2 | K_S \rangle e^{-i\mathcal{M}t/2} \times \\
\left( \eta_1 e^{i\Delta M \Delta t/2} - \eta_2 e^{-i\Delta M \Delta t/2} \right) / \sqrt{2}.
\] (4.1)

This implies $A(e^+e^- \rightarrow \phi \rightarrow K^0\overline{K}^0 \rightarrow f_1 f_2) = 0$ for $t_1 = t_2$ and $f_1 = f_2$ (Bose statistics).

For $t_1 = t_2$, $f_1 = \pi^+\pi^-$ and $f_2 = \pi^0\pi^0$ instead, $A \propto \eta_{++} - \eta_{00} = 3 \times \epsilon'$ which suggest a (unrealistic) way to measure $\epsilon'$.

The intensity for decay to final states $f_1$ and $f_2$ at times $t_1$ and $t_2$ obtained taking the modulus squared of eq. (4.1) depends on magnitude and argument of $\eta_1$ and $\eta_2$ as well as on $\Gamma_{L,S}$ and
$\Delta M$. The intensity is given by

\[
I(f_1, f_2, t_1, t_2) = |\langle f_1 \mid K_S \rangle|^2 |\langle f_2 \mid K_S \rangle|^2 e^{-\Gamma_s t/2} \times \\
(|\eta_1|^2 e^{\Gamma_s \Delta t/2} + |\eta_2|^2 e^{-\Gamma_s \Delta t/2} - \\
2|\eta_1||\eta_2| \cos(\Delta m t + \phi_1 - \phi_2))
\]

where we have everywhere neglected $\Gamma_L$ with respect to $\Gamma_S$.

Thus the study of the decay of $K$ pairs at a $\phi$–factory offers the unique possibility of observing interference pattern in time, or space, in the intensity observed at two different points in space.

This fact is the source of endless excitement and frustration to some people.

Rather than studying the intensity above, which is a function of two times or distances, it is more convenient to consider the once integrated distribution. In particular one can integrate the intensity over all times $t_1$ and $t_2$ for fixed time difference $\Delta t = t_1 - t_2$, to obtain the intensity as a function of $\Delta t$. Performing the integrations yields, for $\Delta t > 0$,

\[
I(f_1, f_2; \Delta t) = \frac{1}{2\Gamma} |\langle f_1 \mid K_S \rangle\langle f_2 \mid K_S \rangle|^2 \\
\quad \times \left(|\eta_1|^2 e^{-\Gamma_L \Delta t} + |\eta_2|^2 e^{-\Gamma_s \Delta t} - \\
2|\eta_1||\eta_2| e^{-\Gamma \Delta t/2} \cos(\Delta m \Delta t + \phi_1 - \phi_2)\right)
\]

and a similar expression is obtained for $\Delta t < 0$.

The interference pattern is quite different according to the
choice of \( f_1 \) and \( f_2 \) as illustrated in figs. 10-12.

![Graph](image1)

**Fig. 10.** Interference pattern for \( f_1 = \pi^+ \pi^- \), \( f_2 = \pi^0 \pi^0 \).

The strong destructive interference at zero time difference is due to the antisymmetry of the initial \( KK \) state, decay amplitude phases being identical.

![Graph](image2)

**Fig. 11.** Interference pattern for \( f_1 = \ell^- \), \( f_2 = \ell^+ \).

The destructive interference at zero time difference becomes
constructive since the amplitude for $K^0 \to \ell^-$ has opposite sign to that for $\bar{K}^0 \to \ell^+$ thus making the overall amplitude symmetric.

One can thus perform a whole spectrum of precision “kaon-interferometry” experiments at DAΦNE by measuring the above decay intensity distributions for appropriate choices of the final states $f_1$, $f_2$. Four examples are listed below.

1. With $f_1=f_2$ one measures $\Gamma_S$, $\Gamma_L$ and $\Delta m$, since all phases cancel. Rates can be measured with a $\times 10$ improvement in accuracy and $\Delta m$ to $\sim \times 2$.

2. With $f_1=\pi^+\pi^-$, $f_2=\pi^0\pi^0$, one measures $\Re(e'/e)$ at large time differences, and $\Im(e'/e)$ for $|\Delta t| \leq 5\tau_s$. Fig. 10 shows the interference pattern for this case.

3. With $f_1 = \pi^+\ell^-\nu$ and $f_2 = \pi^-\ell^+\nu$, one can measure the $CPT$–violation parameter $\delta$, see our discussion later concerning tests of $CPT$. Again the real part of $\delta$ is measured at large time differences and the imaginary part for $|\Delta t| \leq 10\tau_s$. Fig. 11 shows the interference pattern.

4. For $f_1 = 2\pi$, $f_2 = \pi^+\ell^-\nu$ or $\pi^-\ell^+\nu$ small time differences yield $\Delta m$, $|\eta_{\pi\pi}|$ and $\phi_{\pi\pi}$, while at large time differences, the asymmetry in $K_L$ semileptonic decays provides tests of $T$ and $CPT$. The vacuum regeneration interference is shown in fig. 12.
5. CP Violation in Other Modes

5.1 Semileptonic decays

$K$-mesons also decay semileptонically, into a hadron with charge $Q$ and strangeness zero, and a pair of lepton-neutrino. These decays at quark levels are due to the elementary processes

\[ s \rightarrow W^- u \rightarrow \ell^- \bar{\nu} u \]
\[ \bar{s} \rightarrow W^+ \bar{u} \rightarrow \ell^+ \nu \bar{u}. \]

Physical $K$-mesons could decay as:

\[ K^0 \rightarrow \pi^- \ell^+ \nu, \ \Delta S = -1, \ \Delta Q = -1 \]
\[ \bar{K}^0 \rightarrow \pi^+ \ell^- \bar{\nu}, \ \Delta S = +1, \ \Delta Q = +1 \]
\[ \bar{K}^0 \rightarrow \pi^- \ell^+ \nu, \ \Delta S = +1, \ \Delta Q = -1 \]
\[ K^0 \rightarrow \pi^+ \ell^- \bar{\nu}, \ \Delta S = -1, \ \Delta Q = +1. \]

In the standard model, $SM$, $K^0$ decay only to $\ell^-$ and $\bar{K}^0$ to
\( \ell^+ \). This is commonly referred to as the \( \Delta S = \Delta Q \) rule, experimentally established in the very early days of strange particle studies. Semileptonic decays enable one to know the strangeness of the decaying meson - and for the case of pair production to “tag” the strangeness of the other meson of the pair.

Assuming the validity of the \( \Delta S = \Delta Q \) rule, the leptonic asymmetry

\[
A_\ell = \frac{\ell^- - \ell^+}{\ell^- + \ell^+}
\]

in \( K_L \) or \( K_S \) decays is

\[
2\Re\epsilon \simeq \sqrt{2} |\epsilon| = (3.30 \pm 0.03) \times 10^{-3}.
\]

The measured value of \( A_\ell \) for \( K_L \) decays is \((0.327 \pm 0.012)\)%), in good agreement with the above expectation, a proof that \( CP \) violation is, mostly, in the mass term.

In strong interactions strangeness is conserved. The strangeness of neutral \( K \)-mesons can be tagged by the sign of the charge kaon (pion) in the reaction

\[
p + \bar{p} \rightarrow K^0(\bar{K}^0) + K^{-(+)} + \pi^{(+)}.
\]

5.2 \( CP \) Violation in \( K_S \) Decays

\( CP \) violation has only been seen in \( K_L \) decays (\( K_L \rightarrow \pi\pi \) and semileptonic decays). This is because, while it is easy to prepare an intense, pure \( K_L \) beam, thus far it has not been possible to prepare a pure \( K_S \) beam.
However, if the picture of CP we have developed so far is correct, we can predict quite accurately the values of some branching ratios and the leptonic asymmetry. It is quite important to check experimentally such predictions.

5.2.1 $K_S \rightarrow \pi^0\pi^0\pi^0$

At a $\phi$–factory such as DAΦNE, where $\mathcal{O}(10^{10})$ tagged $K_S/\gamma$ will be available, one can look for the CP decay $K_S \rightarrow \pi^0\pi^0\pi^0$, the counterpart to $K_L \rightarrow \pi\pi$.

The branching ratio for this process is proportional to $|\epsilon + \epsilon'_{000}|^2$ where $\epsilon'_{000}$ is a quantity similar to $\epsilon'$, signalling direct CP violation. While $\epsilon'_{000}/\epsilon$ might not be as suppressed as the $\epsilon'/\epsilon$, we can neglect it to an overall accuracy of a few %. Then $K_S \rightarrow \pi^+\pi^-\pi^0$ is due to the $K_L$ impurity in $K_S$ and the expected $BR$ is $2 \times 10^{-9}$. Assuming a hermetic detector, the signal at DAΦNE is at the 30 event level, and therefore there is here the possibility to see the CP impurity of $K_S$, never observed before, though not direct CP violation.

The current limit on $BR(K_S \rightarrow \pi^+\pi^-\pi^0)$ is $3.7 \times 10^{-5}$.

5.2.2 $BR(K_S \rightarrow \pi^\pm\ell^\mp\nu)$ and $A_\ell(K_S)$

The branching ratio for $K_S \rightarrow \pi^\pm\ell^\mp\nu$ can be predicted quite accurately from that of $K_L$ and the $K_S$-$K_L$ lifetimes ratio, since the two amplitudes are equal assuming CPT invariance. In this way we find
\[ BR(K_S \rightarrow \pi^\pm e^\mp \nu) = (6.70 \pm 0.07) \times 10^{-4} \]
\[ BR(K_S \rightarrow \pi^\pm \mu^\mp \nu) = (4.69 \pm 0.06) \times 10^{-4} \]

The leptonic asymmetry in \( K_S \) (as for \( K_L \)) decays is \( 2\Re \epsilon = (3.30 \pm 0.03) \times 10^{-3} \).

Some tens of leptonic decays of \( K_S \) have been seen recently by CMD-2 at Novosibirsk resulting in a value of \( BR \) of 30\% accuracy, not in disagreement with expectation. The leptonic asymmetry \( A_L \) in \( K_S \) decays is not known. At DAΦNE an accuracy of \( \sim 2.5 \times 10^{-4} \) can be obtained. The accuracy on \( BR \) would be vastly improved.

This is again only a measurement of \( \epsilon \), not \( \epsilon' \), but the observation for the first time of \( CP \) violation in two new channels of \( K_S \) decay would be nonetheless of considerable interest.

5.3 \( CP \) Violation in Charged \( K \) Decays

Evidence for direct \( CP \) violation can be also be obtained from the decays of charged \( K \) mesons. \( CP \) invariance requires equality of the partial rates for \( K^\pm \rightarrow \pi^\pm \pi^+\pi^- \ (\tau^\pm) \) and for \( K^\pm \rightarrow \pi^\pm \pi^0\pi^0 \ (\tau'^\pm) \).

With the luminosities obtainable at DAΦNE one can improve the present rate asymmetry measurements by two orders of magnitude, although \( alas \) the expected effects are predicted from standard calculations to be woefully small.

One can also search for differences in the Dalitz plot distri-
butions for $K^+$ and $K^-$ decays in both the $\tau$ and $\tau'$ modes and reach sensitivities of $\sim 10^{-4}$. Finally, differences in rates in the radiative two pion decays of $K^\pm$, $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$, are also proof of direct $CP$ violation. Again, except for unorthodox computations, the effects are expected to be very small.

6. Determinations of Neutral Kaon Properties

6.1 CPLEAR

The CPLEAR experiment\textsuperscript{[11]} studies neutral $K$ mesons produced in equal numbers in proton-antiproton annihilations at rest:

$$pp \rightarrow K^- \pi^+ K^0 \quad BR=2 \times 10^{-3}$$

$$\rightarrow K^+ \pi^- \bar{K}^0 \quad BR=2 \times 10^{-3}$$

The charge of $K^\pm (\pi^\pm)$ tags the strangeness $S$ of the neutral $K$ at $t=0$.

They have presented several results\textsuperscript{[12,13]} from studying $\pi^+ \pi^-$, $\pi^+ \pi^- \pi^0$ and $\pi^\pm \ell^\mp \bar{\nu} (\nu)$ final states. Of particular interest is their measurement of the $K_L-K_S$ mass difference $\Delta m$ because it is independent of the value of $\phi_{+-}$, unlike in most other experiments.

They also obtain improved limits on the possible violation
of the $\Delta S = \Delta Q$ rule, although still far from the expected SM value of about $10^{-7}$ arising at higher order.

The data require small corrections for background asymmetry $\sim 1\%$, differences in tagging efficiency, $\varepsilon(K^+\pi^-) - \varepsilon(K^-\pi^+) \sim 10^{-3}$ and in detection, $\varepsilon(\pi^+e^-) - \varepsilon(\pi^-e^+) \sim 3 \times 10^{-3}$. Corrections for some regeneration in the detector are also needed.

6.1.1 $K^0(\bar{K}^0) \to e^+(e^-)$

Of particular interest are the study of the decays $K^0(\bar{K}^0) \to e^+(e^-)$. One can define the four decay intensities:

\[
\begin{align*}
I^+(t) & \text{ for } K^0 \to e^+ \\
\overline{I}^- (t) & \text{ for } \bar{K}^0 \to e^- \\
\overline{I}^+(t) & \text{ for } \bar{K}^0 \to e^+ \\
I^-(t) & \text{ for } K^0 \to e^- 
\end{align*}
\]

where $\Delta S = 0, 2$ mean that the strangeness of the decaying $K$ is the same as it was at $t=0$ or has changed by 2, because of $K^0 \leftrightarrow \bar{K}^0$ transitions.

One can then define four asymmetries:

\[
A_1(t) = \frac{I^+(t) + \overline{I}^- (t) - (\overline{I}^+(t) + I^-(t))}{I^+(t) + \overline{I}^- (t) + \overline{I}^+(t) + I^-(t)}
\]

\[
A_2(t) = \frac{\overline{I}^- (t) + \overline{I}^+(t) - (I^+(t) + I^-(t))}{\overline{I}^- (t) + \overline{I}^+(t) + I^+(t) + I^-(t)}
\]
\[ A_T(t) = \frac{\overline{I}^+(t) - I^-(t)}{\overline{I}^+(t) + I^-(t)}, \]

\[ A_{CPT}(t) = \frac{\overline{I}^-(t) - I^+(t)}{\overline{I}^-(t) + I^+(t)} \]

From the time dependence of \( A_1 \) they obtain: \( \Delta m = (0.5274 \pm 0.0029 \pm 0.0005) \times 10^{10} \text{ s}^{-1} \), and \( \Delta S = \Delta Q \) is valid to an accuracy of

\[ (12.4 \pm 11.9 \pm 6.9) \times 10^{-3}. \]

Measurements of \( A_T \), which old timers like myself call the Kabir test, involves comparing time “conjugate” processes (which in fact are also \( CP \) conjugate) is now hailed as a direct measurement of \( T \) violation. Assuming \( CPT \), the expected value for \( A_T \) is \( 6.52 \times 10^{-3} \) (4\( \times \text{Re} \epsilon \)). The CPLEAR result is \( A_T = (6.6 \pm 1.3 \pm 1.6) \times 10^{-3} \).

6.1.2 \( \pi^+\pi^- \) Final State

From an analysis of \( 1.6 \times 10^7 \pi^+\pi^- \) decays of \( K^0 \) and \( \overline{K}^0 \) they determine \( |\eta_{+-}| = (2.312 \pm 0.043 \pm 0.03 \pm 0.011_{\tau_S}) \times 10^{-3} \) and \( \phi_{+-} = 42.6^\circ \pm 0.9^\circ \pm 0.6^\circ \pm 0.9_{\Delta m}^\circ \).
Fig. 13. Decay distributions for $K^0$ and $\bar{K}^0$.

Fig. 14. Difference of decay distributions for $K^0$ and $\bar{K}^0$.

Fig. 13 shows the decay intensities of $K^0$ and $\bar{K}^0$, while fig. 14 is a plot of the time dependent asymmetry $A_{+-} = \left( I(\bar{K}^0 \to \pi^+\pi^-) - \alpha I(K^0 \to \pi^+\pi^-) \right) / \left( I(\bar{K}^0 \to \pi^+\pi^-) + \alpha I(K^0 \to \pi^+\pi^-) \right)$. Most systematics cancel in the ratio and the residual difference in efficiencies for $K^0$ and $\bar{K}^0$ decays is determined from a fit to
the same data: $\alpha = 0.9989 \pm 0.0006$.

6.2 E773 AT FNAL

E773 is a modified E731 setup, with a downstream regenerator added. Results have been obtained on $\Delta m$, $\tau_S$, $\phi_{00} - \phi_{+-}$ and $\phi_{+-}$ from a study of $K \rightarrow \pi^+\pi^-$ and $\pi^0\pi^0$ decays.\textsuperscript{[14]}

\textit{E773 figure}

6.2.1 Two Pion Final States

This study of $K \rightarrow \pi\pi$ is a classic experiment where one beats the amplitude $A(K_L \rightarrow \pi\pi)_i = \eta_i A(K_S \rightarrow \pi\pi)$ with the coherently regenerated $K_S \rightarrow \pi\pi$ amplitude $\rho A(K_S \rightarrow \pi\pi)$, resulting in the decay intensity

$$I(t) = |\rho|^2 e^{-\Gamma_ST} + |\eta|^2 e^{-\Gamma_LT} + 2|\rho||\eta| e^{-\Gamma_T} \cos(\Delta mt + \phi_\rho - \phi_{+-})$$

Measurements of the time dependence of $I$ for the $\pi^+\pi^-$ final state yields $\Gamma_S$, $\Gamma_L$, $\Delta m$ and $\phi_{+-}$. They give: $\tau_S = (0.8941 \pm 0.0014 \pm 0.009) \times 10^{-10}$ s.

Setting $\phi_{+-} = \phi_{SW} = \tan^{-1} 2\Delta m/\Delta \Gamma$ and floating $\Delta m$; they
get:
\[ \Delta m = (0.5297 \pm 0.0030 \pm 0.0022) \times 10^{10} \text{ s}^{-1}. \]

Including the uncertainties on \( \Delta m \) and \( \tau_S \) and the correlations in their measurements they obtain: \( \phi_{+-} = 43.53^\circ \pm 0.97^\circ \)

From a simultaneous fit to the \( \pi^+\pi^- \) and \( \pi^0\pi^0 \) data they obtain \( \Delta \phi = \phi_{00} - \phi_{+-} = 0.62^\circ \pm 0.71^\circ \pm 0.75^\circ \), which combined with the E731 result gives \( \Delta \phi = -0.3^\circ \pm 0.88^\circ \).

6.2.2 \( K^0 \to \pi^+\pi^-\gamma \)

From a study of \( \pi^+\pi^-\gamma \) final states \( |\eta_{+-\gamma}| \) and \( \phi_{+-\gamma} \) are obtained. The time dependence of the this decay, like that for two pion case, allows extraction of the corresponding parameters \( |\eta_{+-\gamma}| \) and \( \phi_{+-\gamma} \). The elegant point of this measurement is that because interference is observed (which vanishes between orthogonal states) one truly measures the ratio

\[ \eta_{+-\gamma} = \frac{A(K_L \to \pi^+\pi^-\gamma, CP) \sum \rho}{A(K_S \to \pi^+\pi^-\gamma, CP \ OK)} \]

which is dominated by E1, inner bremsstrahlung transitions. Thus again one is measuring the \( CP \) impurity of \( K_L \). Direct \( CP \) could contribute via E1, direct photon emission \( K_L \) decays, but it is not observed within the sensitivity of the measurement.

The results obtained are: \(^{15}\) \( |\eta_{+-\gamma}| = (2.362 \pm 0.064 \pm 0.04) \times 10^{-3} \) and \( \phi_{+-\gamma} = 43.6^\circ \pm 3.4^\circ \pm 1.9^\circ \). Comparison with \( |\eta_{+-}| \sim |\epsilon| \sim 2.3 \), \( \phi_{+-} \sim 43^\circ \) gives excellent agreement. This implies
that the decay is dominated by radiative contribution and that all one sees is the $CP$ impurity of the $K$ states.

6.3 Combining Results for $\Delta m$ and $\phi_{+-}$ from Different Experiments

![Graph showing $\Delta m$ vs $\phi_{+-}$]

**Fig. 15.** A compilation of $\Delta m$ and $\phi_{+-}$ results, from ref. 16. The CPLEAR collaboration$^{[16]}$ has performed an analysis for obtaining the best value for $\Delta m$ and $\phi_{+-}$, taking properly into account the fact that different experiments have different correlations between the two variables. The data$^{[12,13,14,17-23]}$ with their correlations are shown in fig. 15. A maximum likelihood analysis of all data gives
\[ \Delta m = (530.6 \pm 1.3) \times 10^7 s^{-1} \]

\[ \phi_{+-} = 43.75^\circ \pm 0.6^\circ. \]

Note that \( \phi_{+-} \) is very close to the superweak phase \( \phi_{SW} = 43.44^\circ \pm 0.09^\circ. \)

### 6.4 Tests of CPT Invariance

In local field theory, CPT invariance is a consequence of quantum mechanics and Lorentz invariance. Experimental evidence that CPT invariance might be violated would therefore invalidate our belief in either or both QM and L-invariance. We might not be so ready to abandon them, although recent ideas\(^{[24]} \) such as distortions of the metric at the Planck mass scale or the loss of coherence due to the properties of black holes might make the acceptance somewhat more palatable. Very sensitive tests of CPT invariance, or lack thereof, can be carried out investigating the neutral K system at a \( \phi \)-factory.

In general, CPT requires

\[ M_{11} - M_{22} = M(K^0) - M(\bar{K}^0) = 0. \]

CPLEAR recently used a very complex analysis to obtain that this mass difference is \( 1.5 \pm 2.0 \pm 10^{-18} \).

KTEV, using combined values of \( \tau_s, \Delta m, \phi_{SW}, \) and \( \Delta \phi = (-0.01 \pm 0.40) \) obtained the bound \( (M(K^0) - M(\bar{K}^0))/\langle M \rangle \)
= (4.5 ± 3) \times 10^{-19}, with some simplifying assumptions.

If we note that \( m_K^2 / M_{\text{Planck}} \) is approximately a few times \( 10^{-20} \) it is clear that we are probing near that region, and future experiments, especially at a \( \phi \)-factory is very welcome for confirmation.

7. Three Precision \( CP \) Violation Experiments

Three new experiments: NA48\(^{[25]}\) in CERN, KTEV\(^{[26]}\) at FNAL and KLOE\(^{[27]}\) at LNF, have begun taking data, with the primary aim to reach an ultimate error in \( \Re(\epsilon'/\epsilon) \) of \( \mathcal{O}(10^{-4}) \).

The sophistication of these experiments takes advantage of our experience of two decades of fixed target and \( e^+e^- \) collider physics. Fundamental in KLOE is the possibility of continuous self-calibration while running, via processes like Bhabha scattering, three pion and charged \( K \) decays.
7.1 NA48

The layout of the NA48 experiment, with its main components is shown in fig. 16. A new feature of NA48, with respect to its predecessor NA31, is that $K_L$ and $K_S$ beams simultaneously illuminate the detector, by the very clever use of a bent crystal to deflect a portion of the incident proton beam. This deflected
beam is brought to a $K_S$ production target located close to the detector, reducing systematic errors due to different dead times when detecting $\pi^+\pi^-$ or $\pi^0\pi^0$ $K$ decays.

The superior resolution of the liquid krypton calorimeter further improves the definition of the fiducial regions and improves rejection of $3\pi^0$'s decays. A magnetic spectrometer has also been added in order to improve resolution and background rejection for the $K^0 \rightarrow \pi^+\pi^-$ decays.

This experiment has announced in June 1999, using 10% of their data, the result:

$$\mathcal{R}(\epsilon'/\epsilon) = (18.5 \pm 7.3) \times 10^{-4},$$

which is $2.5 \sigma$ away from zero.

7.2 KTEV

Fig. 17 gives a plan view of the KTEV experiment at FNAL; note the different longitudinal and transverse scales. The KTEV experiment retains the basic principle of E731, with several significant improvements, the most important being the use of CsI crystals for the electromagnetic calorimeter. This results in better energy resolution which is important for background rejection in the $\pi^0\pi^0$ channel as well as in the search for rare $K$
decays.

![Plan view of the KTEV experiment at FNAL.](image)

**Fig. 17.** Plan view of the KTEV experiment at FNAL.

This experiment has announced in Feb. 1999, based on 20% of their data, the result

$$\Re(\epsilon'/\epsilon) = 0.00280 \pm 0.00041,$$

some 7σ from zero. Interestingly, their new result is more in agreement with the old NA31 result than with their own of a few years ago.

### 7.3 KLOE

The KLOE detector[^28] designed by the KLOE collaboration and under construction by the collaboration at the Labora-
The KLOE detector looks very much like a collider detector and will be operated at the DAΦNE collider recently completed at the Laboratori Nazionali di Frascati, LNF. At DAΦNE $K$-mesons are produced in pairs at rest in the laboratory, via the reaction $e^+e^- \rightarrow \phi \rightarrow KK$. $\sim5000$ $\phi$-mesons are produced per second at a total energy of $W=1020$ MeV and full DAΦNE luminosity.

The main motivation behind the whole KLOE venture is
the observation of direct $CP$ violation from a measurement of $\mathcal{R}(e'/e)$ to a sensitivity of $10^{-4}$. The first requirement for achieving such accuracy is to be able to collect enough statistics, which in turn requires studying of the order of few $\times 10^{10}$ $K_L$ decays. The dimensions of the detector are dictated by one parameter, the mean free path for decay of $K_L$'s which is about 3.4 m.

The detectors consists of a 2 m radius drift chamber, employing helium rather than argon, to control multiple scattering at energies below 500 MeV and to minimize regeneration. The chambers has 13,000 W sense wires plus 39,000 Al field wires. The chamber is surrounded by a sampling electromagnetic calorimeter consisting of 0.5 mm Pb foils and 1 mm diameter scintillating fibers. The calorimeter resolution in energy is $\sigma(E)/E=4.7\%$ at 1 GeV and timing resolution is $\sigma(t)=55$ ps, also at 1 GeV.

At full DAΦNE luminosity, $\mathcal{L}=10^{33}$ cm$^{-2}$ s$^{-1}$, KLOE will collect almost 2000 $K_S$, $K_L$ decays per second. Measurements of the leptonic decays mentioned in section 3.1 is possible with KLOE because of the large statistics and the tagged $K_S$ beam unique to a $\phi$–factory. The two neutral $K$ mesons are produced in a pure $C$-odd quantum state. This implies that, to a very high level of accuracy, the final state is always $K_SK_L-K_LK_S$ or $K^0\bar{K}^0-\bar{K}^0K^0$.

Tagging of $K_S$, $K_L$, $K^0$, $\bar{K}^0$ is therefore possible. The pro-
duced $K$ mesons are monochromatic, with $\beta \sim 0.2$. This allows measurement of the flight path of neutral $K$’s by time of flight. A pure $K_S$ beam of about $10^{10}$ per year is a unique possibility at DAΦNE at full luminosity. A very high $K_S$ tagging efficiency, $\sim 75\%$, can be achieved in KLOE by detecting $K_L$ interactions in the calorimeter, in addition to $K_L$ decays in the tracking volume.

Finally because of the well defined quantum state, spectacular interference effects are observable, allowing a totally different way of measuring $\Re(e'/e)$, in addition to the classical method of the double ratio $R$.

KLOE has begun to take data beginning July 29, 1999.

8. The $CKM$ Mixing Matrix

The Standard Model has a natural place for $CP$ violation (Cabibbo, Kobayashi and Maskawa). In fact, it is the discovery of $CP$ violation which inspired KM to expand the original Cabbibo-GIM $2 \times 2$ mass mixing matrix, to a $3 \times 3$ one, which allows for a phase and therefore for $CP$ violation. This also implied an additional generation of quarks, now known as the $b$
and $t$, matching the $\tau$ in the SM.

$$\Gamma(s\rightarrow d) \propto \sin^2 \theta_C$$  \text{Strange part. decays}

\[ \begin{array}{ccc}
  u & c \\
  d & s \\
\end{array} \]

GIM, neutral currents,
2 by 2 unitary matrix,
calculable loops

$$V_{C/GIM} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} = \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C \cos \theta_C \end{pmatrix}$$

$$J_\mu^{+}(udcs) = \overline{u[..]\mu}(\cos \theta_C d + \sin \theta_C s) + \text{No FCNC}$$

$$\overline{e[..]\mu}(-\sin \theta_C d + \cos \theta_C s)$$

\[ \begin{array}{ccc}
  u & b \\
  d & s \\
\end{array} \]

$$\Gamma(b\rightarrow u) \propto \sin^2 \theta_1$$  \text{$B\rightarrow \pi, \rho$ ...}

$$V_{1,-1,2,3} = (\cos \theta_C \cos \theta_1 \quad \sin \theta_C \cos \theta_1 \quad \sin \theta_1)$$

$$J_\mu^{+}(u,dsb) = \overline{u[..]\mu}(\cos \theta_C \cos \theta_1 d +$$

$$\sin \theta_C \cos \theta_1 s + \sin \theta_1 b)$$
The complete form of the matrix, in the Maiani notation, is:

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & c_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

with $c_{12} = \cos \theta_{12} = \cos \theta_C$, etc.

While a phase can be introduced in the unitary matrix $V$ which mixes the quarks

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix},$$

the theory does not predict the magnitude of the effect. The constraint that the mixing matrix be unitary corresponds to the desire of having a universal weak interaction.

Our present knowledge of the magnitude of the $V_{ij}$ elements
is given below.

\[
\begin{pmatrix}
0.9745 - 0.9757 & 0.219 - 0.224 & 0.002 - 0.005 \\
0.218 - 0.224 & 0.9736 - 0.9750 & 0.036 - 0.047 \\
0.004 - 0.014 & 0.034 - 0.046 & 0.9989 - 0.9993 \\
\end{pmatrix}
\]

The diagonal elements are close but definitely not equal to unity. If such were the case there could be no $CP$ violation.

However, if the violation of $CP$ which results in $\epsilon \neq 0$ is explained in this way then, in general, we expect $\epsilon' \neq 0$. For technical reasons, it is difficult to compute the value of $\epsilon'$. Predictions are $\epsilon'/\epsilon \leq 10^{-3}$, but cancellations can occur, depending on the value of the top mass and the values of appropriate matrix elements, mostly connected with understanding the light hadron structure.

A fundamental task of experimental physics today is the determination of the four parameters of the CKM mixing matrix, including the phase which results in $\mathcal{CP}$. A knowledge of all parameters is required to confront experiments. Rather, many experiments are necessary to complete our knowledge of the parameters and prove the uniqueness of the model or maybe finally break beyond it.

8.1 Wolfenstein Parametrization

Wolfenstein found it convenient to parameterize the mixing matrix above in a way which reflects more immediately our present knowledge of the value of some of the elements and has the $CP$
violating phase appearing in only two off-diagonal elements.

The Wolfenstein\textsuperscript{[31]} approximate parameterization of the mixing matrix including up to $\lambda^3$ terms is

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$$

$$\begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$  

$\lambda=0.2215\pm0.0015$ is the Cabibbo angle, a real number describing mixing of $s$ and $d$ quarks. $A$, also real, is close to one $A\sim0.84\pm0.06$ and $|\rho - i\eta|\sim0.3$.

$CP$ violation occurs only if $\eta$ does not vanish. $\eta$ and $\rho$ are not really known.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig_19.png}
\caption{Box and “penguin” FCNC diagrams for $K^0\to\bar{K}^0$.}
\end{figure}

Several constraints on $\eta$ and $\rho$ can however be obtained from the values of measured parameters. The value of $\epsilon$ can be calculated from the $\Delta S=2$ amplitude of fig. 19, the so called box
diagram. At the quark level the calculations is straightforward, but complications arise in estimating the correct matrix element between \( K^0 \) and \( \bar{K}^0 \) states. Apart from the uncertainties in these estimates \( \epsilon \) depends on \( \eta \) and \( \rho \) as:

\[
|\epsilon| = a\eta + b\eta\rho
\]

which is a hyperbola in the \( \eta, \rho \) plane whose central value is shown in figure 22.

The calculation of \( \epsilon' \) is more complicated. There are three \( \Delta S=1 \) amplitudes that contribute to \( K\to\pi\pi \) decays:

\[
A(s \to u\bar{u}d) \propto V_{us}V_{ud}^* \sim \lambda \tag{8.1}
\]

\[
A(s \to c\bar{c}d) \propto V_{cs}V_{cd}^* \sim -\lambda + i\eta A^2\lambda^5 \tag{8.2}
\]

\[
A(s \to t\bar{t}d) \propto V_{ts}V_{td}^* \sim -A^2\lambda^5(1 - \rho + i\eta) \tag{8.3}
\]

where the amplitude (8.1) correspond to the natural way for computing \( K\to\pi\pi \) in the standard model and the amplitudes (8.2), (8.3) account for direct \( \mathbb{CP} \). If the latter amplitudes were zero there would be no direct \( \mathbb{CP} \) violation in the standard model. The flavor changing neutral current (FCNC) diagram of fig. 19 called the penguin diagram, contributes to the amplitudes (8.2), (8.3). The calculation of the hadronic matrix elements is even more difficult because there is a cancellation between the electroweak (\( \gamma, Z \)) and the gluonic penguins, for
$m_t$ around 200 GeV, close to the now known top mass. Estimates of $R(e'/\epsilon)$ range from few $\times 10^{-3}$ to $10^{-4}$.

9. “Unitarity” triangles

We have been practically inundated lately by very graphical presentations of the fact that the $CKM$ matrix is unitary, ensuring the renormalizability of the $SU(2) \otimes U(1)$ electroweak theory. The unitarity condition

$$V^\dagger V = 1$$

contains the relations

$$\sum_i V_{ij}^* V_{ik} = \sum_i V_{ji}^* V_{ki} = \delta_{jk}$$

which means that if we take the products, term by term of any one column (row) element with the complex conjugate of another (different) column (row) element their sum is equal to 0. Geometrically it means the three terms are sides of a triangle. Two examples are shown below. The second one has the term $V_{cd} V_{cb}^*$ pulled out, and many of you will recognize it as a common figure used when discussing measuring $CP$ violation in the $B$ system.
1, 2 triangle

\[
\begin{align*}
V_{ud} & \quad V_{us}^* \\
V_{cd} & \quad V_{cs}^* \\
V_{ld} & \quad V_{ls}^*
\end{align*}
\]

1, 3 triangle

\[
\rho - i \eta = \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} = \frac{V_{ld} V_{lb}^*}{V_{cd} V_{cb}^*} = 1 - \rho - i \eta
\]

\[
\frac{V_{cd} V_{cb}^*}{V_{cd} V_{cb}^*} = 1
\]

**Fig. 20:** The (1,2) and (1,3) unitarity triangles, not to scale.

**Fig. 21:** The “B” and “K” unitarity triangles, almost to scale.

Cecilia Jarlskog in 1984 observed that any direct \( CP \) violation is proportional to twice the area which she named \( J \) (for Jarlskog ?) of these unitary triangles, whose areas are of course are equal, independently of which rows/columns one used to form them.
In terms of the Wolfenstein parameters,

$$J \simeq A^2 \lambda^6 \eta$$

which according to present knowledge is \((2.7 \pm 1.1) \times 10^{-5}\), very small indeed! The smallness of this number also illustrates why the \(\epsilon'\) experiments are so hard to do, and also why \(B\) factories have to be built in order to study \(CP\) violation in the \(B\) system, despite the large value of the angles in the \(B\) unitary triangle.

Measuring the various \(J\)'s to high precision, to check for deviations amongst them, is a sensitive way to probe for new physics!

### 10. Rare \(K\) Decays

Rare \(K\) decays offer several interesting possibilities, which could ultimately open a window beyond the standard model. They allow the determination of the CKM matrix parameters, as for instance from the \(\xi R\) decay \(K_L \to \pi^0 \nu \bar{\nu}\), as well as from the \(CP\) conserving one \(K^+ \to \pi^+ \nu \bar{\nu}\). The connection between measurements of neutral \(K\) properties and branching ratios and the \(\rho\) and \(\eta\) parameters of the Wolfenstein parameterization of the CKM matrix, is shown schematically in fig. 22.

Rare decays also permit the verification of conservation laws which are not strictly required in the standard model, for instance by searching for \(K^0 \to \mu e\) decays.

In general the situation valid for the more abundant \(K\) de-
cays, *i.e.* that the $\mathcal{CP}_{\text{direct}}$ decays have much smaller rates than the $\mathcal{CP}_{\text{indirect}}$ ones, can be reversed for very rare decays.

![Diagram](image)

**Fig. 22.** Constraints on $\eta$ and $\rho$ from measurements of $\epsilon$, $\epsilon'$, rare decays and $B$ meson properties.

In addition while the evaluation of $\epsilon'$ is particularly unsatisfactory because of the uncertainties in the calculation of the hadronic matrix elements, it is not the case for some rare decays. A classification of measurable quantities according to increasing uncertainties in the calculation of the hadronic matrix elements is given by Buras$^{[32]}$ as:

1. $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$,
2. $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$,
3. $BR(K_L \rightarrow \pi^0 e^+ e^-)$, $\epsilon_K$, and
4. $\epsilon'_K$, $BR(K_L \rightarrow \mu \bar{\mu}]_{SD}$, where SD stands for *short distance* contributions.
The observation $\epsilon' \neq 0$ remains a unique proof of direct CP.
Measurements of 1 through 3, plus present knowledge, over determine the CKM matrix.

Rare $K$ decay experiments are not easy however, just like measuring $\Re(\epsilon'/\epsilon)$ has turned out to be difficult.

Typical expectations for some of the interesting decays are:
\[
BR(K_L \to \pi^0 e^+ e^- , \, \mathcal{CP}_{dir}) \sim (5 \pm 2) \times 10^{-12}
\]
\[
BR(K_L \to \pi^0 \nu \bar{\nu}) \sim (3 \pm 1.2) \times 10^{-11}
\]
\[
BR(K^+ \to \pi^+ \nu \bar{\nu}) \sim (1 \pm 0.4) \times 10^{-10}
\]

The most extensive program in this field has been ongoing for a long time at BNL and large statistics have been collected recently and are under analysis. Sensitivities of the order of $10^{-11}$ will be reached, although $10^{-(12 \text{ or } 13)}$ is really necessary. Experiments with high energy kaon beams have been making excellent progress toward observing rare decays.

10.1 Search for $K^+ \to \pi^+ \nu \bar{\nu}$

This decay, CP allowed, is best for determining $V_{td}$. At present after analyzing half of their data, E781-BNL obtains $BR$ is about $2.4 \times 10^{-10}$. This estimate is based on ONE event which surfaced in 1995 from about $2.55 \times 10^{12}$ stopped kaons. The SM expectation is about half that value.

At least 100 such decays are necessary for a first $V_{td}$ measurements.
10.2 $K \rightarrow \mu \mu$

Second order weak amplitudes give contributions which depend on $\rho$, with $BR|_{SD} \sim 10^{-9}$. However, one needs to confirm the calculations for $K \rightarrow \gamma \gamma \rightarrow \mu \mu$, which can confuse the signal. To this end a measurement of another rare decay ($K_L \rightarrow e^+e^-$) whose short-distance part is helicity suppressed helps to check the validity of calculations of the long distance contributions to both decays.

The present experimental status is:

At BNL the experiment E871 has measured that

$$B(K_L \rightarrow \mu^+\mu^-) = (7.18 \pm 0.17) \times 10^{-9}$$

from 6200 candidate events.

With four candidate events, they have measured the smallest branching ratio ever measured in particle physics,

$$B(K_L \rightarrow e^+e^-) = (8.7 + 3.7 - 4.1) \times 10^{-12}$$

which agrees well with the $SM$ expectation of $9 \times 10^{-12}$.

Unfortunately, for this experiment, background will probably prevent any further improvement in the measurement.

10.3 $K_L \rightarrow \pi^0 e^+e^-$

The direct $CP BR$ is expected to be $\sim 5 \times 10^{-12}$. There are however three contributions to the rate plus a potentially dangerous
background.

1. \( K_2 \rightarrow \pi^0 \gamma \gamma \rightarrow \pi^0 e^+ e^- \), a \( CP \) allowed transition. A new KTEV measurement found

\[
B(K_L \rightarrow \gamma \gamma) = (1.68 \pm 0.1) \times 10^{-9}
\]

which implies that the \( CP \) allowed contribution would be only 1/5th that from direct \( CP \) violation, so it is good news.

2. \( K_L \rightarrow \pi^0 e^+ e^- \), from the \( K_L \) \( CP \) impurity (\( \epsilon \mid K_1 \)).

3. Direct \( \mathcal{CP} \) from short distance, second order weak contributions, via \( s \rightarrow d + Z \), the signal of interest.

4. Background from \( K_L \rightarrow \gamma \gamma^* \rightarrow e^+ e^- \gamma \rightarrow e^+ e^- \gamma \gamma \), with a photon from final state radiation.

The background from point 4 above is not too dangerous for the new experiments (KTEV and NA48), because of the superior resolution of their new electromagnetic calorimeter.

The present experimental limit from KTEV is \( B(K_L \rightarrow \pi^0 e^+ e^-) < 5.64 \times 10^{-10} \), so there is still some way to go.

10.4 \( K_L \rightarrow \pi^0 \nu \bar{\nu} \)

This process is a pure direct \( \mathcal{CP} \) signal. At the June '99 kaon workshop this process was hailed as the holy grail of rare kaon decay physics. Measuring its branching ratio is a must for kaon physics and \( \mathcal{CP} \). It is theoretically particularly "pristine", with
only about 1-2\% uncertainty, since the hadronic matrix element need not be calculated, but is directly obtained from the measured $K_{\ell 3}$ decays. Geometrically we see it as being the altitude of the $J_{12}$ triangle.

$$J_{12} = \lambda(1 - \lambda^2/2)\text{Im}V_{td}V_{ts}^* \approx 5.6[B(K_L \to \pi^0\nu\bar{\nu})]^{1/2}$$

The relevant amplitudes are shown in the next figure.

---

**Fig. 23:** Feynman Diagrams for $K_L \to \pi^0\nu\bar{\nu}$

In Wolfenstein parameters the branching ratio is approximately $(m_t/m_W)^{2.2}A^4\eta^2 = 3 \times 10^{-11}$

The experimental signature is the appearance of a single unbalanced $\pi^0$ in a hermetic detector. The difficulty of the experiment is seeing in the present experimental limit from E799-I, that the branching ratio is less than $5.8 \times 10^{-5}$, very far away indeed. The sensitivities claimed for E799-II and at KEK are around $10^{-9}$, thus another factor of 100 improvement is necessary.

There is a great deal of activities on both sides of the ocean
to hope to make this measurement a reality sometime after the second millenium.

11. B decays

11.1 Introduction

The discovered at Fermilab in 1977 of the $\Upsilon$, with mass of $\sim 10$ GeV, was immediately taken as proof of the existense of the $b$ quark, heralded by KM and already so christened: $b$ for beauty or bottom. The $b$ quark has $B$ flavor $B=-1$. The fourth $\Upsilon$ is barely above threshold for decaying into a $B\bar{B}$ pair, where the $B$ meson are $b\bar{u}$, $b\bar{d}$ and their charge conjugate states, in complete analogy to charged and neutral kaons. $B^0$ and $\bar{B}^0$ are not self conjugate states. The fact that the $\Upsilon(4S)$ decays only to such pairs was demonstrated by CUSB searching and not finding low energy (about 50 MeV) photons from $B^*$ (the first excited state of $B$ meson) decays, nor finding high energy photons indicative of $\Upsilon(4S)$ decaying directly into $\gamma$ plus hadrons. The neutral $B$’s, $B^0\bar{B}^0$’s, are produced in the same state as $K^0\bar{K}^0$ and can be used for $CP$ studies in the $B$ system, once the short $B$ lifetime problem is overcome.

Measurements the hadronic cross section as a function of the total center of mass energy at CESR, a series of resonances with masses higher than the $\Upsilon(4S)$ ($\Upsilon(5S)$, $\Upsilon(6S)$...) were seen. A coupled channel analysis of the hadronic cross section, and
studying the photon and lepton spectra, CUSB first inferred the thresholds at which $B^*B$, $B^*B^*$, $B_s\bar{B}_s$ etc. were produced.\textsuperscript{[33]}

11.2 B semileptonic decays

Because of their massiveness, $B$’s can decay weakly into many more channels than the $K$’s. We might recall that we owe the long lifetime of the $K_L$ to the smallness of the phase space for 3 body decays. The average particle multiplicity in the decay of $B$ and $\bar{B}$ pair is about six. Its leptonic mode, which has a branching ratio of only about 12%, has a very tell tale signature, namely a lepton with energy up to half that of the parent $\Upsilon$. It was in fact through the observation of the sudden appearance of high energy electrons that the existence of the $b$ quark was unambiguously proved in 1980, since the $\Upsilon$ after all has $B=0$.

The elementary leptonic decays of beauty are: $b \to c\ell^-\bar{\nu}$ and $b \to c\ell^-\bar{\nu}$ with the selection rule $\Delta B=\Delta Q$

The endpoint of the lepton spectrum and its shape depend on the flavor of the hadronic system appearing in the final state. We define as $X_c$ a hadronic system with charm $C=\pm 1$ and $U=0$ where $U$ is the uppityness. Likewise $X_u$ has $U=\pm 1$ and $C=0$. The leptonic decays are:

\[ B \to \ell^\pm + \nu(\bar{\nu}) + X_c \]

\[ B \to \ell^\pm + \nu(\bar{\nu}) + X_u \]

where $X_c=D$, $D^*\ldots$ with $\overline{M}(X_c) \sim 2$ GeV and $X_u=$
π, ρ.. with $\overline{M}(X_u) \sim 0.7$ GeV. The expected lepton spectra are shown in figure.

![Leptonic Spectrum in B semileptonic decays](image)

**Fig. 24:** Leptonic Spectrum in $B$ semileptonic decays

Total decay rates, *i.e.* the inverse of the lifetimes, and branching ratios of $B$ mesons provide the determination of $|V_{cb}|$ and $|V_{ub}|$. A preferred way is to measure the semileptonic branching ratio by integrating over the whole spectrum. An early determination by CUSB already indicated that $|V_{ub}/V_{cb}|$ is very small, less than 0.06. However, uncertainties in the calculation of the hadronic matrix elements and the shape of the spectrum near the end point introduce errors in the extraction of $|V_{ub}/V_{cb}|$. Methods (HQET) have been developed to make use of exclusive channels, a good ten years has been spent in refining such measurements.
11.3 Mixing

11.3.1 discovery

Just as with $K$-mesons, neutral $B$ mesons are not $C$ eigenstates and can mix, i.e., transitions $B^0 \leftrightarrow \overline{B}^0$ are possible. The first observation of mixing was reported by Argus at the DESY DORIS collider running on the $\Upsilon(4S)$.

They observed mixing by comparing the $\ell^+\ell^+$ and $\ell^-\ell^-$ decay rates from $B\overline{B}$ pairs. Defining the ratio

$$r = \frac{\ell^+\ell^+ + \ell^-\ell^-}{\ell^+\ell^- + \ell^-\ell^+ + \ell^+\ell^+ + \ell^-\ell^-}$$

$r \neq 0$ is proof of mixing, not however of $\mathcal{CP}$.

Today, instead of $r$, the $\chi_d$ parameter, which is a measure of the time-integrated $B^0-\overline{B}^0$ mixing probability that a produced $B^0(\overline{B}^0)$ decays as as $\overline{B}^0(B^0)$, is used. They are related simply by $r = \chi_d/(1 - \chi_d)$. The present value of $\chi_d$ is $0.172 \pm 0.01$.

11.3.2 Formalism

We define, analogously to the $K^0\overline{K}^0$ system,

$$B_L = p |B^0\rangle + q |\overline{B}^0\rangle$$

$$B_H = p |B^0\rangle - q |\overline{B}^0\rangle$$

with $p^2 + q^2 = 1$ Here L, H stand for light and heavy, because the $B_d$'s have different masses but very similar decay widths.

Mixing is calculated in the SM by evaluating the standard “box” diagrams with intermediate $u, c, t$ and $W$ states. We de-
fine:

\[ \Delta M = M_H - M_L, \quad \Delta \Gamma = \Gamma_H - \Gamma_L \]

note that \( \Delta M \) is positive by definition. The ratio \( q/p \) is given by:

\[
\frac{q}{p} = \frac{\Delta M - i/2\Delta \Gamma}{2(M_{12} - i/2\Gamma_{12})} =\]

\[
2(M_{12}^* - i/2\Gamma_{12}^*/(\Delta M - i/2\Delta \Gamma)
\]

where

\[
\Gamma_{12} \propto [V_{ub}V_{ud}^* + V_{cb}V_{cd}^*]^2 m_b^2 = (V_{tb}V_{td}^*)^2 m_b^2
\]

and \( M_{12} \propto (V_{tb}V_{td}^*)^2 m_t^2 \), so they have almost the same phase. \( x \) and \( y \), for \( B_d \) and \( B_s \) mesons are:

\[
x_{d,s} = \Delta M_{d,s}/\Gamma_{d,s}, \quad y_{d,s} = \Delta \Gamma_{d,s}/\Gamma_{d,s}
\]

\( y_d \) is less than \( 10^{-2} \), and \( x_d \) is about 0.7, and if we ignore the width difference between the two \( B_d \) states,

\[
\frac{q}{p}\bigg|_{B_d} = \frac{(V_{tb}^*V_{td})}{(V_{tb}V_{td}^*)} = e^{-2i\phi_{\text{mixing}}}
\]

Therefore \( |q/p|_d \) is very close to 1 and since \( 2\Re\epsilon_{B_d} \approx 1 - |q/p|_d \), \( \epsilon_{B_d} \) is imaginary. \( y_s \) is about 0.2, and \( x_s \) theoretically could be as large as 20, so far only lower bounds are quoted.

\( \chi_d \) as defined before, in terms of \( q, p, x, y \) is

\[
\chi_d = \frac{1}{2} \frac{|q|^2}{p} \frac{x_d^2 - y_d^2/4}{(1 + x_d^2)(1 - y_d^2/4)}
\]

which reduces to a good approximation:
\[ \chi_d = \frac{x_d^2}{2(1 + x_d^2)}, \]

from which one obtains that \( x_d = 0.723 \pm 0.032 \).

In summary, from evaluating the box diagrams, one finds:

\[ x_l \propto m_t^2 \tau_{B_l} m_{B_l} |V_{tl}V_{tb}^*|^2. \]

where the subscript \( l \) refers to the light meson partner which makes up the \( B \) meson, i.e. \( l = s \) or \( d \).\(^{[34]}\)

An amusing historical note. The surprisingly large amount of mixing seen resulted in a prediction that the mass of the top was larger than the then considered values of about 20 GeV. Using a very generous estimate of \( |V_{ub}| \), about four times the present value for \( |V_{tb}| \) and \( r \sim 0.22 \), a lower limit on the top mass of \( \sim 40 \) GeV was obtained, clearly the early CUSB limit should have been used.

Using the top mass today known, \( \Delta M \) measured from \( 2\pi \) times the \( B^0 \overline{B^0} \) oscillation frequency in time-dependent mixing experiments at FNAL and LEP inserted in the above formula, one obtains bounds on \( |V_{td}| = (8.4 \pm 1.4) \times 10^{-3} \) and \( \Im(V_{td}V_{tb}^*) = (1.33 \pm 0.30) \times 10^{-4} \).

From a fit to available data,\(^{[35]}\) obtain that

\[ \sin 2\beta = 0.71 \pm 0.13, \quad \sin \gamma = 0.85 \pm 0.15. \]
11.4 CP Violation

Semileptonic decays of $B_s$ allow, in principle, to observe $\mathcal{CP}$ by studying the dilepton and total lepton charge asymmetries. This however has turned to be rather difficult because of the huge background and so far yielded no evidence for $\mathcal{CP}$ in $B$.

We can estimate the magnitude of the leptonic asymmetry from

$$4\Re\epsilon_B = \Im\left(\frac{\Gamma_{12}}{M_{12}}\right) = \frac{|\Gamma_{12}|}{|M_{12}|}\text{Arg}\left(\frac{\Gamma_{12}}{M_{12}}\right)$$

or approximately

$$\frac{m_b^2}{m_t^2} \times \frac{m_c^2}{m_b^2}$$

which is $\mathcal{O}(10^{-4})$. 

**Figure 25.** Fit to data in the $\eta-\rho$ plane.
11.4.1 \( \alpha, \beta \text{ and } \gamma \)

Sensitivity to \( CP \) violation in the \( B \) system is usually discussed in terms of the 3 interior angles of the \( U_{13} \) triangle.

\[
\alpha = \text{Arg} \left( -\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right)
\]

\[
\beta = \text{Arg} \left( -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right)
\]

\[
\gamma = \text{Arg} \left( -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right)
\]

As in the case in the \( K \) system, direct \( CP \) need interfere of two different phases. When one compares the amplitude \( A \) for decay to a \( CP \) eigenstate \( A = \sum_i A_i e^{i(\delta_i+\phi_i)} \) versus the amplitude \( \bar{A} \) for decay to the \( CP \) conjugate process \( \bar{A} = \sum_i A_i e^{i(\delta_i-\phi_i)} \), one notes that the strong phases \( \delta \) do not change sign while the weak phases (CKM related) do. Direct \( CP \) violation requires \( |A| \neq |\bar{A}| \), while indirect \( CP \) violation only requires \( |q/p| \neq 1 \).

Let’s consider examples of neutral \( B \) decays to \( CP \) eigenstates denoted by \( f_{CP} \), \( J/\psi(1S)K_S \) or \( \pi\pi \), since they allow particularly clean theoretical interpretations in terms of the CKM parameters.

\[
A_{f_{CP}} \equiv \langle f_{CP}|B^0 \rangle \quad \bar{A}_{f_{CP}} \equiv \langle f_{CP}|\bar{B}^0 \rangle \quad \text{and}
\]

\[
\lambda_{f_{CP}} \equiv (q/p)(\bar{A}_{f_{CP}}/A_{f_{CP}}).
\]

The sign of \( \lambda_{f_{CP}} \) is given by the \( CP \) eigenvalue of the final state \( f_{CP} \) if there were no \( CP \).
The time-dependent $\mathcal{CP}$ asymmetry

$$a_{fCP}(t) \equiv \frac{\Gamma(B^0 \rightarrow f_{CP}) - \Gamma(\overline{B}^0 \rightarrow f_{CP})}{\Gamma(B^0 \rightarrow f_{CP}) + \Gamma(\overline{B}^0 \rightarrow f_{CP})}$$

is

$$= \frac{(1 - |\lambda_{fCP}|^2) \cos(\Delta M t) - 2 \text{Re}(\lambda_{fCP}) \sin(\Delta M t)}{1 + |\lambda_{fCP}|^2}.$$

11.5 CDF AND DØ

CDF at the Tevatron is the first to profit from the idea suggested by Toni Sanda, is to study asymmetries in the decay of tagged $B^0$ and of $\overline{B}^0$ to a final state which is a $CP$ eigenstate. The most promising channel is

$$B^0, \overline{B}^0 \rightarrow J/\psi K_S.$$ 

Large asymmetries are expected, since the channel is strongly suppressed by many powers of $\lambda$, which results in a strong dependence of the $\mathcal{CP}$ phase. Hadronic complications are removed since the main contribution to the asymmetry is from $B^0 \rightarrow \overline{B}^0 \rightarrow J/\psi K_S$ interfering with $B^0 \rightarrow J/\psi K_S$.

The time integrated asymmetry $a_{fCP}$, in this case reduces to

$$x_d/(1 + x_d^2) \times \sin 2\beta \approx 0.5 \sin 2\beta.$$ 

In addition, at FNAL CDF can also measure the time dependence of $a_{fCP}$, $a_{fCP}(t) = \sin 2\beta \sin(\Delta m_d t)$. CDF obtains $\sin 2\beta = 0.79 + 0.41 - 0.44$ from measuring $a_{fCP}$ and $\sin 2\beta = 0.71 \pm 0.63$ from measuring $a_{fCP}(t)$. 
Their very lucky central value agrees with the aforementioned SM fit, but there is at least a two fold ambiguity in the determination of $\beta$ which they can not differentiate with their present errors. In the coming Tevatron runs, CDF (and DØ) not only expect to improve the determinations of $\sin 2\beta$ by a factor of four, so $\Delta \sin 2\beta \approx 0.1$, but to measure $\sin(2\alpha)$ from using the asymmetry resulting from $B^0 \to \pi^+\pi^-$ interfering with $\overline{B}^0 \to \pi^+\pi^-$ to a similar accuracy. By being very optimistic, they hope to get a first measurement of $\sin(\gamma)$ by using $B^0_s/\overline{B}^0_s \to D_s^{\pm}K^{\mp}$ from about 700 signal events.

11.6 HERAB

At DESY there is a dedicated experiment for B physics: HERAB. It is unique in that that they use a wire target in a proton beam. There physics reach is estimated to be about the same as that of CDF and DØ. This experiment in several aspects, such as triggering amongst incredible background and radiation hardness in a very hot environment, which are similar to the ones being contemplated (and approved?) at the hadron colliders, LHCb, BTeV, ATLAS/CMS, for attaining ten times better accuracy than the presently running ones (including Belle and Babar), so bear serious watching.
11.7  B FACTORIES

In order to overcome the short $B$ lifetime problem, and still profit from the coherent state property of $B$’s produced on the $\Upsilon(4S)$, two asymmetric $e^+e^-$ colliders have been built, PEP-II and KEKB. The two colliders both use a high $\approx 9$ GeV beam colliding against a $\approx 3.1$ GeV beam, so that the center of mass energy of the system is at the $\Upsilon(4S)$ energy, but the $B$’s are boosted in the laboratory, so they travel detectable distances before their demise. In order to produce the large number of $B^0\bar{B}^0$ pairs, the accelerators must have luminosities on the order of $3 \times 10^{33}$ cm$^{-2}$s$^{-1}$, about two orders of magnitude that of CESR.

Two large detectors, Babar and Belle have also been built in SLAC and KEK respectively, to perform the experiments. The collaborations are medium large in terms of today’s high energy standards: 600 physicists (about 1/3 of an LHC experiment, and 6 times that of the KLOE collaboration). Both detectors are relatively standard $e^+e^-$ collider detectors, with slight modifications to accommodate the boost of the center of mass system. These two detectors have seen collisions one month after KLOE did. It was a marvelous confluence of activities in $\mathcal{CP}$ physics!

First results will be available in 1 year at accuracies of about 0.1 on $\sin 2\beta$. Ultimately more precise results will come from the aforementioned experiments at hadron colliders, because of
the larger production cross section, the larger path lengths of high energy $b$-quark or -hadrons and also because time integrated asymmetries at $B$-factories do not retain memory of $\mathcal{CP}$, because the initial $B^0\bar{B}^0$ pair produced in $e^+e^-$ annihilations at the $\Upsilon(4S)$ peak is $C$-odd.

REFERENCES

28. “KLOE, A General Purpose Detector for DAΦNE”, the KLOE Collaboration, Internal Report LNF- 019,
April 1, 1992.


33. For a review of results on Υ and B see J. Lee-Franzini, Hidden an Open Beauty in CUSB, in Proc. of Twenty Beautiful Years of Bottom Physics, Burnstein et al. eds, p. 85, The American Institute of Physics, and references therein.
