Eikonalized Mini–Jet Cross–Sections in $\gamma\gamma$ Collisions$^1$

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Abstract

In this note we assess the validity and uncertainties in the predictions of the eikonalised mini-jet model for $\sigma^{\text{inel}}$. We are able to find a choice of parameters where the predictions are compatible with the current data. Even for this restricted range of parameters the predictions at the high c.m. energies, which can be reached at the TeV energy $e^+e^-$ colliders, differ by about $\pm 25\%$. LEP 2 data can help pinpoint these parameters and hence reduce the uncertainties in the predictions.

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Eikonalized mini-jet cross-sections

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In this note we wish to assess the validity and uncertainties of the eikonalized mini-jet model in predicting $\sigma^{\text{incl}}_{ab}$ and further to ascertain whether measurements at LEP-200 and HERA can constrain various parameters of the model. In its simplest formulation, the eikonalized mini-jet cross-section is given by

$$\sigma^{\text{incl}}_{ab} = P_{ab}^{\text{had}} \int d^2 \vec{b} [1 - e^{-n(b,s)}]$$

(1)

where the average number of collisions at a given impact parameter $\vec{b}$ is obtained from

$$n(b,s) = A_{ab}(b)(\sigma^{\text{soft}}_{ab} + \frac{1}{P_{ab}^{\text{had}}} \sigma^{\text{jet}}_{ab})$$

(2)

with $A_{ab}(b)$ the normalized transverse overlap of the partons in the two projectiles and $P_{ab}^{\text{had}}$ to give the probability that both colliding particles $a,b$ be in a hadronic state. $\sigma^{\text{soft}}_{ab}$ is the non-perturbative part of the cross-section from which the factor of $P_{ab}^{\text{had}}$ has already been factored out and $\sigma^{\text{jet}}_{ab}$ is the hard part of the cross-section. The rise in $\sigma^{\text{jet}}_{ab}$ drives the rise of $\sigma^{\text{incl}}_{ab}$ with energy [1]. We have also assumed the factorization property

$$P_{\gamma^{(*)}}^{\text{had}} = P_{\gamma}^{\text{had}}, \quad P_{\gamma^{(*)}}^{\text{had}} = (P_{\gamma}^{\text{had}})^2.$$

The predictions of the eikonalized mini-jet model [2] for photon induced processes [3] depend on 1) the assumption of one or more eikons, 2) the hard jet cross-section $\sigma^{\text{jet}}_{ab} = \int_{p_{\text{min}}} d^2 q F_a(q) F_b(q) e^{i\vec{q} \cdot \vec{b}}$, which in turn depends on the minimum $p_t$ above which one can expect perturbative QCD to hold, viz. $p_{\text{min}}$, and the parton densities in the colliding particles $a$ and $b$, 3) the soft cross-section $\sigma^{\text{soft}}_{ab}$, 4) the overlap function $A_{ab}(b)$, defined as

$$A_{ab}(b) = \frac{1}{(2\pi)^2} \int d^2 \vec{q} F_a(q) F_b(q) e^{i\vec{q} \cdot \vec{b}}$$

(3)

where $F$ is the Fourier transform of the $b$-distribution of partons in the colliding particles and 5) last but not the least $P_{ab}^{\text{had}}$.

In this note we shall restrict ourselves to a single eikonal. The hard jet cross-sections have been evaluated in LO perturbative QCD. The dependence of $\sigma^{\text{jet}}_{ab}$ on $p_{\text{min}}$ is strongly correlated with the parton densities used. Here we show the results using GRV densities.
[4] (see ref. [5] for the results using the DG densities [6]). For the purposes of this note, we determine $\sigma^{soft}_{\gamma\gamma}$ from $\sigma^{soft}_{\gamma p}$ which is obtained by a fit to the photoproduction data. We use the Quark Parton Model suggestion $\sigma^{soft}_{\gamma\gamma} = \frac{3}{2} \sigma^{soft}_{\gamma p}$.

In the original use of the eikonal model, the overlap function $A_{\gamma\gamma}(b)$ of eq. (3) is obtained using for $\mathcal{F}$ the electromagnetic form factors and thus, for photons, a number of authors [7, 8] have assumed for $\mathcal{F}$ the pole expression used for the pion electromagnetic form factor, on the basis of Vector Meson Domiance (VMD). We shall investigate here another possibility, i.e. that the b-space distribution of partons in the photon is the Fourier transform of their intrinsic transverse momentum distributions. This will correspond to use the functional expression expected for the perturbative part [9]

$$\frac{dN_\gamma}{dk^2_\perp} = \frac{1}{k^2 + k^2_0}$$

(4)

Recently this expression was confirmed by the ZEUS [10] Collaboration, with $k_0 = 0.66 \pm 0.22$ GeV. For $\gamma\gamma$ collisions, the overlap function is now simply given by

$$A(b) = \frac{1}{4\pi} k^2_0 b K_1(bk_0)$$

(5)

with $K_1$ the Bessel function of the third kind. It is interesting to notice that for photon-photon collisions the overlap function will have the same analytic expression for both our ansätze: the VMD inspired pion form factor or the intrinsic transverse momentum; the only difference being that the former corresponds to a fixed value of $k_0 = 0.735$ GeV whereas the latter allows us to vary the value of the parameter $k_0$. Thus both possibilities can be easily studied by simply changing $k_0$ appropriately. Notice that the region most important to this calculation is for large values of the parameter $b$, where the overlap function changes trend, and is larger for smaller $k_0$ values.

As for $\rho^{had}_{\gamma\gamma}$, this is clearly expected to be $O(\alpha_{em})$ and from VMD one would expect 1/250. From phenomenological considerations [8] and fits to HERA data, one finds a value 1/200, which indicates at these energies a non-VMD component of $\approx 20\%$. It should be noticed that the eikonalized minijet cross-sections do not depend on $A_{\gamma\gamma}$ and $\rho^{had}$ separately, but depend only on the ratio of the two [11, 12].

Having thus established the range of variability of the quantities involved in the calculation of total photonic cross sections, we now proceed to calculate and compare with existing data the eikonalized minijet cross-section for $\gamma\gamma$ collisions. We use GRV (LO) densities and values of $p_{\text{min}}$ deduced from a best fit to photoproduction. As discussed in [15], it is possible to include the high energy points in photoproduction using GRV densities and $p_{\text{min}} = 2$ GeV, but the low energy region would be better described by a smaller $p_{\text{min}}$. This is the region where the rise, according to some authors, notably within the framework of the Dual Parton Model, is attributed to the so-called soft Pomeron. For our studies here we use $p_{\text{min}} = 2$. GeV. We also use $\rho^{had}_{\gamma} = 1/204$ and $A(b)$ from eq.(5) with different values of $k_0$. One choice for $k_0$ is the pole parameter value in the photon b-distribution expression, which includes both the intrinsic transverse momentum option
\[ p_{t_{\text{min}}} = 2.0 \text{ GeV}, \ k_0 = 0.66(\text{dashes}), 0.88(\text{dots}), 1 \text{ GeV}(\text{full}) \]

\[ \sigma_{\text{soft}} = 20.8 \text{mb} + 6.7(\text{mb GeV}) / \sqrt{s} + 25.27(\text{mb GeV}^2) / s \]

\[ 1 / P_{\text{had}} = 204 \text{ and GRV densities} \]

Figure 1: Total inelastic photon-photon cross-section for \( p_{t_{\text{min}}} = 2 \text{ GeV} \) and different parton b-distribution in the photon. The solid line corresponds to \( k_0 = 1 \text{ GeV} \).

0.66 ± 0.22 GeV as well as the pion form factor value, 0.735 GeV. The other value, 1 GeV, is a possible choice which appears to fit the present data better than everything else. Our predictions are shown in Fig.(1). A comparison with existing \( \gamma \gamma \) data shows that all of our choices are compatible with the data within the present experimental errors. At high energies, however, like the ones reachable with the proposed linear photon colliders, these predictions vary by about ±25%. Reducing the error in the LEP1 region and adding new data points in the c.m. region attainable at LEP2, can help pinpoint and restrict the choices. Were the LEP1 and LEP2 data to confirm the present values, we believe that the best representation of the present data is obtained with the higher \( k_0 \) value.

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