Experimental Limits on Antigravity in Extended Supergravity

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Abstract

The available tests of the equivalence principle constrain the mass of the Higgs-like boson appearing in extended supergravity theories. We determine the constraints imposed by the present and future high precision experiments on the antigravity fields arising from \( N = 2, 8 \) supergravity.

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1 Introduction

The discovery that $N > 1$ supergravity theories lead to antigravity is due to the work of the late J. Scherk [1, 2]. In a recent paper we have revived the interest for the implications of extended supergravity theories for antigravity [3]. This interest is connected to the high precision experiment at LEAR (CERN) measuring the difference in the gravitational acceleration of the proton and the antiproton [4]. For a review of earlier ideas about antigravity the reader is referred to the extensive article by Nieto and Goldman [5] and the references therein.

The supergravity multiplet in the $N = 2, 8$ cases contains, in addition to the graviton ($J = 2$), a vector field $A_{i}^{\mu}$ ($J = 1$). ¹ The Eötvös experiment forces upon us the assumption that the field $A_{i}^{\mu}$ have a nonvanishing mass, which may have a dynamical origin [1, 2]. In any case, the vector receives a mass through the Higgs mechanism

$$m_i = \frac{1}{R_i} = k m_\phi \langle \phi \rangle,$$

where the mass of the Higgs field equals its (nonvanishing) vacuum expectation value (v.e.v.)

$$m_\phi = \langle \phi \rangle.$$

Thus, Scherk's model of antigravity leads to the possibility of violating the equivalence principle on a range of distances of order $R_i$, where $R_i$ is the $A_{i}^{\mu}$ Compton wavelength. The available limits set by the experimental tests of the equivalence principle allow us to constrain the v.e.v. of the scalar field $\phi$, and therefore its mass. It must be noted that the possibility of a massless field $A_{i}^{\mu}$ was already ruled out by Scherk using the Eötvös experiments available at that time [1].

In the present paper we review the limits provided by the present day experiments, and those obtainable from experiments currently under planning for the near future. The Compton wavelength of the gravivector thus obtained is of order $10$ m, or less [3]. Therefore, the concept of antigravity in the context of extended supergravity cannot play any role in astrophysics, except possibly for processes involving the strong gravity regime, i.e. near black holes or in the early universe.

It is worth to remind the reader that there are interesting connections between antigravity in $N = 2, 8$ supergravities and $CP$ violation experiments, via the consideration

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¹There are also two Majorana gravitini ($J = \frac{3}{2}$) for $N = 2$ [6] and a scalar field $\sigma$ for $N = 8$ [1, 2]. These fields are immaterial for our purposes and will be ignored in the following.
of the $K^0 - \bar{K}^0$ system in the terrestrial gravitational field [1]. However, the present experiments on $CP$ violations yield bounds on the range of the gravivector field which are less stringent than those obtained from the tests of the equivalence principle [3].

The present paper has the following structure: in Sec. 2 we recall the basic features of Scherk's antigravity. The limits on the Compton wavelength of the gravivector and the mass of the scalar field deriving from the current experimental verifications of the equivalence principle are discussed in Sec. 3. Improvements coming from experiments under planning are also considered. The conclusions are presented in Sec. 4.

## 2 Antigravity effects in $N > 1$ supergravity

In $N = 2, 8$ supergravity theories, the gravivector field $A^I_\mu$ couples to the fields of the matter scalar multiplet with strengths

$$g_i = \pm k m_i$$  \hspace{1cm} (3)

[6] for $N = 2$ and

$$g_i = \pm 2k m_i$$  \hspace{1cm} (4)

[7, 8] for $N = 8$. Here $k = (4\pi G)^{1/2}$, $m_i$ are the quark and lepton masses, the positive and negative signs hold for particles and antiparticles, respectively, and $g = 0$ for self-conjugated particles. As a consequence, in the interaction of an atom with the gravitational field, the vector field $A^I_\mu$ "sees" only the particles constituting the nucleon which are not self-conjugated, while the graviton couples to the real mass of the nucleon. The potential for an atom of atomic and mass numbers $(Z, A)$ in the static field of the Earth is [1]

$$V = -\frac{G}{r} \left[ MM_\oplus - \eta M^0 M_\oplus f \left( \frac{R_\oplus}{R_i} \right) \exp(-r/R_i) \right],$$  \hspace{1cm} (5)

where

$$\eta = \begin{cases} 1 & N = 2 \\ 4 & N = 8 \end{cases}$$  \hspace{1cm} (6)

$R_i$ is the Compton wavelength of the gravivector, and $R_\oplus = 6.38 \cdot 10^6$ m, $M_\oplus = 5.98 \cdot 10^{24}$ kg are the earth radius and mass, respectively. The presence of the function

$$f(x) = 3 \frac{\cosh x - \sinh x}{x^3}$$  \hspace{1cm} (7)

expresses the fact that a spherical mass distribution cannot be described by a point mass located at the center of the sphere, as in the case of a coulombic potential. The masses
in (3) are given by

\[ M = Z(M_p + m_e) + (A - Z)M_n, \]

\[ M^0 = Z(2m_u + m_d + m_e) + (A - Z)(m_u + 2m_d), \]

where \( M_p, M_n \) and \( m_e \) are the proton, neutron and electron masses, respectively. We describe the Earth by means of the average atomic composition \((Z_\oplus, 2Z_\oplus)\) which gives, from (8), (9)

\[ M^0_{\oplus} \simeq \frac{3m_u + 3m_d + m_e}{M_p + M_n} M_\oplus. \]

In \( N = 2, 8 \) supergravities, one of the scalar fields has a nonzero v.e.v. and, as a consequence, the vector field \( A^I_\mu \) acquires a mass, as described by (2) (the impossibility of a massless \( A^I_\mu \) being proved in ref. [1]). This leads to a violation of the equivalence principle, expressed by the difference between the accelerations of two atoms with numbers \((Z, A)\) and \((Z', A')\) in the field of the Earth

\[ \frac{\delta \gamma}{\gamma} = \eta \frac{(3m_u + 3m_d + m_e)(m_e + m_u - m_d)}{M_n (M_p + M_n)} \left( \frac{Z'}{A'} - \frac{Z}{A} \right) f \left( \frac{R_\oplus}{R_i} \right) \left( 1 + \frac{R_\oplus}{R_i} \right) \exp(-R_\oplus/R_i). \]

### 3 Experimental constraints on antigravity

In the Eötvös–like experiment performed at the University of Washington [9] (hereafter “Eöt–Wash”) the equivalence principle was tested using beryllium and copper and aluminum and copper. The equivalence principle was verified with a precision

\[ \left| \frac{\delta \gamma}{\gamma} \right| < 10^{-11}. \]

This number was used in ref. [3] to set a lower limit on the mass of the Higgs–like particle:

\[ m_\phi > 5 \eta^{1/2} \text{ GeV}. \]

The Moscow experiment [10], in spite of its higher precision, provides a less stringent limit on \( m_\phi \), due to the fact that it verified the equivalence principle in the gravitational field of the Sun, and (3) has to be modified accordingly [3]. Here we make use also of the experiments aimed to detect deviations from Newton’s inverse square law, which can be seen as precise tests of the equivalence principle. In these experiments it is customary to parametrize the deviations from the Newtonian form with a Yukawa–like correction
to the Newtonian potential

$$V(r) = -\frac{GM}{r} \left( 1 + \alpha e^{-r/R_i} \right).$$

(14)

In the following, we assume that, in the context of antigravity, the parameter $\alpha$ is given by the value computed for the Eötvös-Wash experiment performed using copper ($Z = 29$, $A = 63.5$) and berillium ($Z' = 4$, $A' = 9.0$), i.e.

$$\alpha = \begin{cases} 6.36 \cdot 10^{-4} & (N = 2) \\ 2.54 \cdot 10^{-3} & (N = 8). \end{cases}$$

(15)

For the materials that are likely to be used in these experiments, the values of $\alpha$ differ from those of (15) only for a factor of order unity. Moreover, our final limits on $m_\phi$ depend on the square root of $\alpha$. For these reasons, it is safe to use the values (15) of $\alpha$ in the following computations.

Equations (1) and (2) provide us with the relation

$$\frac{m_\phi(\text{new})}{m_\phi^*} = \left( \frac{R_i^*}{R_i(\text{new})} \right)^{1/2},$$

(16)

where $m_\phi^* = 5\eta^{1/2}$ GeV and $R_i^* = 34\eta^{-1}$ m are, respectively, the limits on the scalar field mass and the Compton wavelength of the vector $A_i^\mu$ derived in ref. [3], and $m_\phi(\text{new})$, $R_i(\text{new})$ are the new limits on the same quantities coming from the references considered in the following.

The $2\sigma$ limits of ref. [11] (see their fig. 3) allow the range of values of $R_i$:

$$R_i \leq 1 \text{ cm} \quad , \quad R_i \geq 5 \text{ cm}$$

(17)

for $N = 2$ and

$$R_i \leq 0.5 \text{ cm} \quad , \quad R_i \geq 16 \text{ cm}$$

(18)

for $N = 8$. This corresponds to the allowed range for the mass of the Higgs-like scalar field:

$$m_\phi \leq 130 \text{ GeV} \quad , \quad m_\phi \geq 292 \text{ GeV} \quad (N = 2)$$

(19)

$$m_\phi \leq 73 \text{ GeV} \quad , \quad m_\phi \geq 412 \text{ GeV} \quad (N = 8).$$

(20)

The curve A of fig. 13 in ref. [12] gives

$$R_i \leq 0.6 \text{ cm} \quad , \quad R_i \geq 10 \text{ cm}$$

(21)
for $N = 2$ and
\[ R_i \leq 0.4 \text{ cm} , \quad R_i \geq 32 \text{ cm} \tag{22} \]
for $N = 8$. Equivalently,
\[ m_\phi \leq 92 \text{ GeV} , \quad m_\phi \geq 376 \text{ GeV} \quad (N = 2) \tag{23} \]
\[ m_\phi \leq 52 \text{ GeV} , \quad m_\phi \geq 461 \text{ GeV} \quad (N = 8) . \tag{24} \]

The null result of the Shternberg \[13\] experiment reviewed by Milyukov \[14\] in the light of Scherk's work provides us with the limits:
\[ R_i \leq 4 \text{ cm} , \quad R_i \geq 13 \text{ cm} \tag{25} \]
for $N = 2$ and
\[ R_i \leq 2.2 \text{ cm} , \quad R_i \geq 40 \text{ cm} \tag{26} \]
for $N = 8$. These are equivalent to:
\[ m_\phi \leq 82 \text{ GeV} , \quad m_\phi \geq 146 \text{ GeV} \quad (N = 2) \tag{27} \]
\[ m_\phi \leq 46 \text{ GeV} , \quad m_\phi \geq 197 \text{ GeV} \quad (N = 8) . \tag{28} \]

Therefore, the best available limits on the mass of the scalar field are given by
\[ m_\phi \leq 82 \text{ GeV} , \quad m_\phi \geq 376 \text{ GeV} \quad (N = 2) \tag{29} \]
\[ m_\phi \leq 46 \text{ GeV} , \quad m_\phi \geq 461 \text{ GeV} \quad (N = 8) . \tag{30} \]

A high precision test of the equivalence principle in the field of the Earth is currently under planning in Moscow \[15\]. The precision expected to be achieved in this experiment is
\[ \left| \frac{\delta \gamma}{\gamma} \right| < 10^{-15} . \tag{31} \]

Adopting the upper bound $R_i \leq 34 \eta^{-1} \text{ m}$ found in ref. \[3\] and approximating the function $f(x)$ for $x = R_\oplus / R_i \gg 1$ as
\[ f(x) \simeq \frac{3}{2x^2} e^x , \tag{32} \]
we obtain
\[ \frac{|\delta \gamma/\gamma|_{\text{future}}}{|\delta \gamma/\gamma|_{\text{Eöt-Wash}}} = 10^{-4} . \tag{33} \]

In the case that the new experiment verifies the equivalence principle with the expected accuracy, the limits on $m_\phi$ would be pushed to
\[ m_\phi \geq 500 \eta^{1/2} \text{ GeV} . \tag{34} \]
4 Conclusions

The tests of the equivalence principle and the null results on deviations from Newton’s inverse square law provide constraints on the mass of the Higgs–like boson appearing in extended supergravity theories. We have reviewed the limits obtainable from the available experiments in the context of $N = 2, 8$ supergravity, and we have discussed also the impact on the field of the future high precision experiment being planned in Moscow.

There have been many papers on the effects of non–Newtonian gravity in astrophysics, in particular those due to a fifth force like the one obtainable from $N = 2, 8$ supergravity in the weak field limit (see references in [5]). However, the upper bound of $34\eta^{-1}$ m on the Compton wavelength $R_I$ of the gravivector field found in ref. [3] implies that antigravity does not play any role in nonrelativistic astrophysics since the length scales involved in stellar\(^2\), galactic and supergalactic structures dominated by gravity are much larger than $R_I$. Antigravity could affect, in principle, processes that take place in the strong gravity regime, where smaller distance scales are involved. Examples of these situations are processes occurring near black hole horizons or in the early universe, when the size of the universe is smaller than, or of the order of, $R_I$. The relevance of antigravity in such situations will be studied in future publications.

Our final remark concerns a point that apparently went unnoticed in the literature on supergravity: the detection of gravitational waves expected in a not too far future will shed light on the correctness of supergravity theories. In fact, after the dimensional reduction is performed, the action of the theory contains scalar and vector fields as well as the usual metric tensor associated to spin 2 gravitons [2]. These fields are responsible for the presence of polarization modes in gravitational waves, the effect of which differs from that of the spin 2 modes familiar from general relativity. Therefore, extended supergravities and general relativity occupy different classes in the $E(2)$ classification of Eardley et al. [17] of gravity theories. The extra polarization states are detectable, in principle, in a gravitational wave experiment employing a suitable array of detectors [17]. However, it must be noted that a detailed study of gravitational wave generation taking into account the antigravity phenomenon is not available at present. Such a work would undoubtedly have to face the remarkable difficulties well known from the studies of gravitational wave generation in the context of general relativity.

\(^2\)The conclusion that stellar structure is unaffected by antigravity might change if the non–Newtonian force alters the equation of state of the matter composing the star [16].
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References