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HADRONIC WIDTH OF THE $\chi_b$ STATES

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Abstract

The CUSB-II Collaboration has inferred the hadronic widths of the $\chi_b(2P_J)$ states from the measured branching fractions and the potential model rates of their $E1$ decays. These widths agree qualitatively with perturbative QCD calculations. The disagreement in absolute values of the widths between the experimental and QCD predictions confirm the same situation as in the charmonium system, which means more precision measurements are sorely needed.

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1. ELECTRIC DIPOLE TRANSITION RATES

Using the CUSB-II detector\(^1\) at the Cornell Electron Storage Ring (CESR), the CUSB Collaboration collected during 1985–1990 a data sample of 288 pb\(^{-1}\) on the \(\Upsilon(3S)\), with which precision measurements of the \(\Upsilon\) system were performed.\(^2\) CUSB studied electric dipole (E1) transitions in the \(\Upsilon\) system, both in the “inclusive” photon spectrum from hadronic \(\Upsilon(3S)\) decays,\(^3\) and by reconstructing “exclusive” decay modes.\(^4\) The photon energies from \(\Upsilon(3S) \rightarrow \chi_b(2P)_\gamma, (E_\gamma)\), from both spectra agree very well and the combined results are listed in table 1. The fitted number of events give the branching ratios for the observed processes, listed in table 1. The first error is statistical, the second systematic. The branching ratios for the E1 decays of the \(\chi_b(2P)\) states have been obtained by dividing the branching ratios for the cascades by those for \(\Upsilon(3S) \rightarrow \chi_b(2P)_\gamma\) from the inclusive photon spectrum.

<table>
<thead>
<tr>
<th>(J)</th>
<th>(E_\gamma^*) MeV</th>
<th>(B(\Upsilon(3S) \rightarrow \chi_b(2P)_\gamma)%)</th>
<th>(B(\chi_b(2P)<em>\gamma \rightarrow \Upsilon(2S)</em>\gamma)%)</th>
<th>(B(\chi_b(2P)<em>\gamma \rightarrow \Upsilon(2S)</em>\gamma)%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(86.7±0.4)</td>
<td>(11.1±0.5±0.4)</td>
<td>(17.3±2.1±1.9)</td>
<td>(7.0±1.0±0.6)</td>
</tr>
<tr>
<td>1</td>
<td>(100.1±0.4)</td>
<td>(11.5±0.5±0.5)</td>
<td>(19.9±2.0±2.2)</td>
<td>(8.0±0.9±0.7)</td>
</tr>
<tr>
<td>0</td>
<td>(123.0±0.8)</td>
<td>(6.0±0.4±0.6)</td>
<td>(4.6±2.0±0.7)</td>
<td>(0.9±0.6±0.1)</td>
</tr>
</tbody>
</table>

* energy scale errors (0.9%) are not included

2. HADRONIC WIDTHS FROM QCD

The annihilation widths of \(P\) wave quarkonium states have been calculated in perturbative QCD, assuming that the annihilation amplitude factorizes into a part describing the bound state and another part describing the annihilation of two free quarks into the smallest allowed number of gluons. The states with \(J = 0, 2\) are allowed to annihilate into two gluons. Including first order QCD corrections the decay rates are given by\(^5\)

\[
\Gamma_{gg}(2P_2) = \frac{8}{5} \frac{\alpha_s^2}{m_b^4} |\Psi'(0)|^2 \left(1 + \frac{\alpha_s}{\pi}\right),
\]

(2.1)

\[
\Gamma_{gg}(2P_0) = \frac{6\alpha_s^2}{m_b^4} |\Psi'(0)|^2 \left(1 + 10.2 \frac{\alpha_s}{\pi}\right).
\]

(2.2)

To lowest order their ratio is \(\frac{15}{4}\). For the state with \(J=1\) the leading contribution to the annihilation width comes from annihilation into a quark pair and one gluon:\(^6\)

\[
\Gamma_{q\bar{q}g}(2P_1) = \frac{32}{9\pi} \frac{\alpha_s^3}{m_b^4} |\Psi'(0)|^2 \ln(m_b(\tau)),
\]

(2.3)

where \((\tau)\) is the mean radius of the \(\chi_b(2P_1)\) state. If we assume that the gluons and quarks in the final states hadronize with unit probability the hadronic widths are identical to the annihilation widths. They have been evaluated, e.g. by Kwong and Rosner,\(^7\) using their values of the derivative of the wave function at zero quark separation \(\Psi'(0)\) and
\( \alpha_s = 0.184 \pm 0.006 \) obtained from the ratio \( \Gamma_{gg}(\Upsilon)/\Gamma_{ggg}(\Upsilon) \).\(^8\) They find

\[
\begin{align*}
\Gamma_{gg}(2P_2) &= (153 \pm 13) \text{ keV}, \\
\Gamma_{q\bar{q}g}(2P_1) &= (51 \pm 5) \text{ keV}, \\
\Gamma_{gg}(2P_0) &= (866 \pm 65) \text{ keV}.
\end{align*}
\]

(2.4)

The errors come from the uncertainty in \( \alpha_s \) and the first order QCD corrections.

3. CUSB INFERRED HADRONIC WIDTHS

These widths are too small to be measured directly. We can, however, derive them from the measurement of \( \text{BR}(\chi_b(2P_J) \to \Upsilon \gamma) \), given in table 1. Let \( \sum \Gamma_{E1} \) be the sum of the partial widths of all electric dipole decays of \( \chi_b(2P_J) \). The hadronic width \( \Gamma_{\text{had}} \) of the \( \chi_b(2P_J) \) state is then

\[
\Gamma_{\text{had}} = \frac{\Gamma_{E1}(\chi_b(2P) \to \Upsilon \gamma)}{\text{BR}(\chi_b(2P) \to \Upsilon \gamma)} - \sum \Gamma_{E1}.
\]

(3.1)

The widths of the electric dipole transitions can be calculated by taking the dipole matrix element from potential model calculations. Values for \( \Gamma_{E1}(\chi_b(2P_J) \to \Upsilon \gamma) \) are given in table 2, together with the values for the hadronic widths. We have used electric dipole matrix elements from three different potential models, the inverse scattering potential of KR,\(^7\) the semirelativistic model of GRR,\(^9\) and our updated Richardson potential.\(^10\)

Table 2. Electric Dipole and Hadronic Widths of \( \chi_b(2P) \) States.

<table>
<thead>
<tr>
<th>model</th>
<th>( J )</th>
<th>( \Gamma_{E1}(\chi_b(2P_J) \to \Upsilon(1S)\gamma) ) (keV)</th>
<th>( \Gamma_{E1}(\chi_b(2P_J) \to \Upsilon(2S)\gamma) ) (keV)</th>
<th>( \Gamma_{\text{had}}(2P_J) ) (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KR</td>
<td>2</td>
<td>9.71</td>
<td>18.6</td>
<td>89±11±11</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>9.25</td>
<td>15.8</td>
<td>65±7±13</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>8.49</td>
<td>11.5</td>
<td>343±118±58</td>
</tr>
<tr>
<td>GRR</td>
<td>2</td>
<td>8.16</td>
<td>19.0</td>
<td>82±10±11</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>7.77</td>
<td>16.1</td>
<td>59±6±9</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>7.13</td>
<td>11.7</td>
<td>313±108±42</td>
</tr>
<tr>
<td>PF</td>
<td>2</td>
<td>10.0</td>
<td>18.7</td>
<td>91±11±12</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>9.54</td>
<td>15.8</td>
<td>66±7±14</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>8.76</td>
<td>11.6</td>
<td>351±120±61</td>
</tr>
</tbody>
</table>

4. COMPARISON

The variation between the values for the hadronic widths obtained using different models is small. The measured values of the hadronic widths are in qualitative agreement with the QCD predictions, although the two-gluon widths of the even spin states are about a factor of two smaller than expected. Their ratio agrees with the zeroth-order QCD expectation of \( \frac{15}{4} \).
5. \( \alpha_s \) DETERMINATION

By solving the expressions for the annihilation rates of the \( \chi_b(2P) \) states in equations (2.1)–(2.3) for the strong coupling constant \( \alpha_s \), we can use the measured values of the hadronic widths to measure \( \alpha_s \). Using the value of \( \Psi'(0) \) by Kwong and Rosner\(^7\) we obtain \( \alpha_s = 0.14 \pm 0.01, 0.20 \pm 0.02, 0.12 \pm 0.02 \) for \( J=2,1,0 \). Using our calculations with the updated Richardson potential, we obtain correspondingly \( 0.16 \pm 0.01, 0.21 \pm 0.02 \), and \( 0.14 \pm 0.03 \). These values are in agreement with the results of other methods of measuring \( \alpha_s \) in the \( b\bar{b} \) system.\(^8\)

6. CONCLUSIONS

We have seen that the ratio expected from the lowest order QCD calculation agrees with the data, and that the strong coupling constant obtained here agrees with other determinations, but the absolute values of the hadronic widths do not agree with the predictions. This is a similar situation to that found in Ted Barnes's analyses in the charmonium system\(^{11}\) that the gluonic widths calculations can not be done by simply considering loop diagrams. It may also be an indication that hybrid states are produced in gluon annihilations.\(^{12}\) It is obvious therefore that the charmonium and bottomonium systems must be mapped out in great precision before these questions can be answered.

7. ACKNOWLEDGEMENTS

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REFERENCES

11. See T. Barnes's paper in these proceedings.
12. See F. Close's paper in these proceedings.