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MEASUREMENT OF THE $\mathcal{R}e'/\epsilon$ AND $\Delta\phi$ PARAMETERS AT A $\phi$-FACTORY BY MEANS OF THE $K_L K_S \rightarrow \pi^+\pi^-\pi^0\pi^0$ DECAY TIME DIFFERENCE

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MEASUREMENT OF THE $\mathcal{R}_e^{ee}$ AND $\Delta\phi$ PARAMETERS AT A $\Phi$-FACTORY
BY MEANS OF THE $K_L K_S \to \pi^+ \pi^- \pi^0 \pi^0$ DECAY TIME DIFFERENCE

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Abstract
An analysis of the CP violation parameter $\mathcal{R}_e^{ee}$ has been made by means of the $K_L K_S \to \pi^+ \pi^- \pi^0 \pi^0$ decay time difference distribution. Non-gaussian experimental resolution on the $K_L K_S$ decay length has been considered. Background from the $\phi \to f_0(975)\gamma \to K_S K_L \gamma$ events is discussed.

Introduction
$K$ decays has been the only example of CP violation since its observation in 1964 [1]. Nevertheless several questions concerning the origin of this violation have not yet been given firm answers. The planned construction of high luminosity $\phi$-factory [2] opens new opportunities to explore the $K^0\bar{K}^0$ CP violating system with improved statistic and accuracy. A $\phi$-factory allows different analysis method to address the CP violation parameter $\mathcal{R}_e^{ee}$, but we choose to exploit the possibility of extracting information on both the real and the imaginary part of $\mathcal{R}_e^{ee}$ parameter, studying the distribution of the decay time differences in the $K_L K_S \to \pi^+ \pi^- \pi^0 \pi^0$ events.

The $\Delta t$ distribution
We considered the CP violating $\phi$ decay channel $\phi \to K_L K_S \to \pi^+ \pi^- \pi^0 \pi^0$, regardless to which kaon decays into the neutral or the charged channel. The branching ratio is $B_{\phi \to K_L K_S} = B_{K_L \to \pi^+ \pi^-} + B_{K_S \to \pi^+ \pi^-}$, $B_{K_L \to \pi^+ \pi^-} + B_{K_S \to \pi^+ \pi^-} = 4.4 \times 10^{-4}$. If CP were not violated, owing to the Bose statistic, the two kaons could not decay at the same time. More generally, taking into account the time development of the $K_L K_S$ system wave function, we obtain the expression of the decay rate as function of the difference $\Delta t = t^\pi - t^{\pi^0}$ between the decay time of the $K^0$ in the charged and in the neutral channel:

$$f(\Delta t) = e^{-\frac{|\Delta t|}{2}(1+\eta)} [C_1^2 e^{\phi_1(1-\eta)} + C_2^2 e^{\phi_2(1-\eta)} - 2C_1C_2 \cos(\chi \Delta t + \Delta \phi)]$$

(1)

$$C_1^2 = 1 + 2R_{ee}^{e^2} + (R_{ee}^{e^4})^2$$

$$C_2^2 = 1 - 4R_{ee}^{e^2} + 4(R_{ee}^{e^4})^2$$

$$\Delta \phi = \phi_+ - \phi_0 \approx 3Im \mathcal{R}_e^{ee}$$

$$\eta = \tau_s / \tau_1$$

$$\chi = \tau_s (m_{K_L} - m_{K_S})$$
\( \Delta t \) is expressed in \( \tau_e \) unit. The shape of the distribution is shown in fig.1. \( \mathcal{R}^\varepsilon \) determines the asymmetry of this function; in fact it can for \( |\Delta t| > 10 \tau_e \) :

\[
f(\Delta t) - f(-\Delta t) = 6\mathcal{R}^\varepsilon \exp(-\eta \Delta t)
\]

(2)

Figure 1: Distribution of \( \Delta t \) for the \( K_i K_e \rightarrow \pi^+ \pi^- \pi^0 \pi^0 \) events

On the other hand a shift of the interference zone of the distribution (the "hole") due to \( \Delta \phi \neq 0 \) would be a signature of CPT violation.

Since the \( \phi \) meson decays at rest the kaons are emitted back to back with \( \beta = 0.219 \), so \( \Delta t \) is measured as the difference of the two decay lengths. The \( K_i, K_e \) lifetimes correspond to decay paths \( \beta c \tau_{K_i} = 0.58 \text{ cm}, \beta c \tau_{K_e} = 338 \text{ cm} \). The resolution on \( \Delta t \) is given by the resolution on the \( K^0 \) decay vertex. Assuming a very good experimental resolution on the decay time (better than \( \tau_e \)) a fit of the \( f(\Delta t) \) to data can determine both the \( \mathcal{R}^\varepsilon \) and \( \Delta \phi \) parameters [3].

Any uncertainties on \( \Delta t \) affects mainly the reconstructed shape of the "hole" of the distribution, where the fast variation due to the interference tends to be swept out. On the contrary for \( |\Delta t| > 10 \tau_e \) the distribution is flat on typical scale of experimental resolution and is then less sensitive to the experimental \( \Delta t \) spread.

We assumed that the entire uncertainty on the \( \Delta t \) determination comes from the neutral channel and that the corresponding error distribution \( g_\sigma(y) \) is known. An "experimental" \( \Delta t \) distribution can then be defined :

\[
f_\sigma^{\text{exp}}(y) = \int_{-\infty}^{\infty} f(\Delta t)g_\sigma(y - \Delta t)d\Delta t
\]

(3)
where $y$ is the measured decay time difference. The value of the physical parameter $R_z$ and $\Delta \phi$ can be extracted fitting data with this corrected distribution. To estimate the accuracy achievable simulated events has been fitted with $f_{exp}(y)$. The results have been compared with analytical computation: the statistical accuracy on a parameter

$$
\sigma_p = \frac{1}{\sqrt{N}} \left( \int_1^f \frac{\delta f(x;p)}{\delta p} \right) \frac{1}{2} dx
$$

(4)

Figure 2: Accuracy on $R_z$ for different detector sizes

$p$ extracted from a $N$ events sample with distribution $f(x;p)$ is given by [4]:

**Results**

A luminosity of $L = 2 \times 10^{32} \text{cm}^{-2} \text{s}^{-1}$ and live time of $10^7 \text{s}$ per year were used as benchmark, with a production at the $\phi$ peak of $4 \times 10^8$ events/year. The accuracy of the $R_z$ and $\Delta \phi$ were calculated with respect either to different radius $R$ of a spherical apparatus, either to different vertex resolution. To take into account non-gaussian tails of the vertex reconstruction we considered an experimental distribution made of a sum of 1,2,3 gaussians, with $\sigma_k = k \sigma$ and weight = $1/k$.

$$
g^\sigma_n(x) = \frac{1}{\sigma \sqrt{2\pi}} \left( \sum_{k=1}^n \frac{1}{k} \right)^{-1} \sum_{k=1}^n \frac{1}{k^2} \exp \left( \frac{-x^2}{2\sigma^2 k^2} \right)
$$

(5)

Functions $g^\sigma_n(x)$ with same $\sigma$ and different index $n$ have nearly the same FWHM, approximately equal to $2.36$ times the $\sigma$ of the first term of the sum. The RMS instead grows with the index for the same "nominal" $\sigma$. From now on we will characterize the experimental resolution function by the index $n$ of the sum and by the "nominal" $\sigma$ equal to FWHM/ $2.36$. 
The accuracy on \( R^e_{\tau} \) shown in fig.2 refers to different apparatus sizes and were obtained using a \( \sigma = 0 \) (cross) and a \( \sigma = 15 \text{mm} \) , \( n = 2 \) (star) experimental resolution. The parameter is mainly affected by the number of events that can be collected. For example, because of the long decay path of the \( K_\tau (\tau = 3.4 \text{ m}) \), only 35% of the events decay in a detector with radius=150 cm. On the other hand since the \( R^e_{\tau} \) parameter is determined by the difference in height of the two flat wings of the \( \Delta t \) distribution, as expected the vertex resolution does not affect very much this measurement. On the contrary, the information on \( \Delta \phi \) lies entirely in the narrow, strong varying interference zone (\( |\Delta t| < 10 \tau_\tau \)) and the accuracy on the measurement depends critically on the vertex resolution distribution, as shown in fig.3. The accuracy on both parameters obtained in the \( \sigma = 0 \) limit is in agreement with earlier results [3].

These values have been compared with realistic simulation of a \( K^0 \rightarrow \pi^0\pi^0 \) vertex reconstruction [5]. For a fiducial volume of 100 cm, the simulated vertex resolution function is well approximated with a \( n=2 \) distribution with "nominal" \( \sigma = .7 \text{ cm} \). Therefore we define a realistic accuracy achievable by this analysis method:

\[
\sigma(R^e_{\tau}) = 2.8 \times 10^{-4} \quad \sigma(\Delta \phi) = 0.9^\circ
\]

(6)

The \( \phi \rightarrow f_0(975)\gamma \rightarrow K_\tau K_\tau \gamma \) background

The \( \phi \rightarrow f_0(975)\gamma \rightarrow K_\tau K_\tau \gamma \) events can contaminate the selected sample if the apparatus fails to detect the low energy (26 MeV) \( \gamma \). We chose to consider this specific background among the possible others because, due to the short lifetime of the two \( K_\tau \), the \( \Delta t \) distribution of this background overlap the (1) just in the interference zone, worsening the resolution on \( \Delta \phi \). Discussions of other possible backgrounds and
sistematic can be found in ref [6]. Presently the branching ratio for the $\phi \rightarrow f_0(975)\gamma$ process has only an experimental upper limit = $2 \times 10^{-3}$, and the theoretical previsions

<table>
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<th>$\sigma (mm)$</th>
<th>0</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{\phi \rightarrow f_0(975)\gamma}$ = 5 $10^{-8}$</td>
<td>0.61°</td>
<td>0.67°</td>
<td>0.80°</td>
<td>1.27°</td>
<td>1.92°</td>
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<td>$B_{\phi \rightarrow f_0(975)\gamma}$ = 0</td>
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<td>0.60°</td>
<td>0.75°</td>
<td>1.22°</td>
<td>1.88°</td>
</tr>
</tbody>
</table>

Table 1: $\sigma(\Delta \phi)$ for different values of $B_{\phi \rightarrow f_0(975)\gamma}$

range from $10^{-6}$ to $10^{-8}$ ref [7]; for our calculation we assumed a branching ratio = 5 $10^{-8}$ and 80% efficiency for the detection of the 26 MeV $\gamma$. Experimental resolution functions with n=2 and different values of the nominal $\sigma$ were used. The results are shown in the table 1 and compared with those obtained without the background. As can be seen for realistic vertex resolution the worsening is of the order of 5%.

Conclusion

The possibility to measure $R^E$ and $\Delta \phi$ at the $\phi$-factory by means of the decay times differences distribution in the $\phi \rightarrow K_i K_s \rightarrow \pi^+ \pi^- \pi^0 \pi^0$ events has been explored. The values $\sigma(R^E) = 2.8 \times 10^{-4}$ and $\sigma(\Delta \phi) = 0.9^0$ have been obtained in the case of a 100 cm fiducial volume detector and non gaussian, $\sigma = 0.7$ cm vertex resolution on the $K^0 \rightarrow \pi^0 \pi^0$ decay channel. It has also been shown that the background from the $\phi \rightarrow f_0(975)\gamma \rightarrow K_s K_s \gamma$ decay decreases the resolution on $\Delta \phi$ less than 5 per cent for branching ratio of the $\phi \rightarrow f_0(975)\gamma$ decay equal to $5 \times 10^{-8}$.

We want to thank prof. P.Franzini for calling my attention to this problem and for many useful suggestions.

References

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