S. Bellucci and D. Babusci:

**PION POLARIZABILITIES AND TESTS OF CHIRAL SYMMETRY IN TWO-PHOTON COLLISIONS AT DAPHNE**

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PION POLARIZABILITIES AND TESTS OF CHIRAL SYMMETRY IN TWO-PHOTON COLLISIONS AT DAPHNE\textsuperscript{1}

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ABSTRACT

We review the $\gamma\gamma \rightarrow \pi\pi$ data and conclude that the experiments should be redone and that an increase of $10^2$ in the rates is expected at DAPHNE. The status of the data on the pion polarizability $\alpha_\pi$ is also considered briefly. We compare the data with the prediction of chiral perturbation theory. In the case of neutral pions, by including the exchange of vector resonances in the t-channel, we prove that the $O(p^6)$ contribution cannot be neglected and gives $(20 \pm 30)\%$ of the cross-section at $(460 \pm 490)$ MeV.

Experiments on $\gamma\gamma \rightarrow \pi^0\pi^0$ have been carried out by the Crystal Ball (CB)\textsuperscript{(1)} and the JADE\textsuperscript{(2)} collaborations. The former published cross-section data, whereas the latter gave just rates. One can do the exercise of arbitrarily choosing to normalize the JADE data so as to agree with the CB data exactly at the $f_2$ resonance (1270 MeV) (see Fig. 1). Then, the JADE data normalized according to the former prescription are about one or two standard deviations above the CB experimental points, for energies $M_{\pi\pi}$ between threshold and 700 MeV (see Fig. 2). The result of this exercise cannot be taken seriously, owing to the arbitrariness of the normalization procedure, which may introduce a systematical error of order $2\sigma$ in the JADE points of Fig. 2. Hence, we concentrate on the CB data, when comparing with the prediction based on chiral symmetry. The lowest order chiral prediction, which is supposed to be valid for $M_{\pi\pi} \lesssim 0.45$ GeV, \textsuperscript{(3,4)} is shown in Fig. 2.

The expected production rates for both neutral and charged pion pairs at DAPHNE are summarized in a Table, using a 1-year integrated luminosity $\int_{\text{1-year}} L \, dt = 2.5 \times 10^{39}$ cm$^{-2}$ and an electron-beam energy $E = 500$ MeV. Both total ($2m_\pi \leq M_{\pi\pi} \leq 1$ GeV) and partial ($2m_\pi \leq M_{\pi\pi} \leq 0.45$ GeV) rates are shown. The yield production rates are obtained disregarding all efficiency factors, whereas in estimating a geometrical efficiency for the detector we introduced a blind angle $x = 8.5^\circ$ such that the observable pions in the final state would correspond to angles in the lab-system within the range $x \leq \theta_{\pi}^{\text{LAB}} \leq 180^\circ - x$.

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FIG. 1 - Data from Crystal Ball\textsuperscript{(1)} (●) and JADE\textsuperscript{(2)} (∗). The latter are normalized so as to agree with the former data at $M_{\pi\pi} = 1270$ MeV.

$\gamma\gamma \rightarrow \pi^0\pi^0$

FIG. 2 - Same data as in Fig. 1 for $M_{\pi\pi} \leq 0.7$ GeV. The theoretical predictions are: dispersive approach (full line; see Pennington in these Proceedings); chiral perturbation theory\textsuperscript{(3,4)} (dashed line).
<table>
<thead>
<tr>
<th>e⁺e⁻ → e⁺e⁻π⁺π⁻</th>
<th>Yield Production</th>
<th>8.5 ≤ θ⁺⁻LAB ≤ 171.5°</th>
</tr>
</thead>
<tbody>
<tr>
<td>M_{ππ} ≤ 1 GeV</td>
<td>1.9·10⁴ ev/y</td>
<td>1.75·10⁴ ev/y</td>
</tr>
<tr>
<td>M_{ππ} ≤ 0.45 GeV</td>
<td>0.84·10⁴ ev/y</td>
<td>0.74·10⁴ ev/y</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>e⁺e⁻ → e⁺e⁻π⁺π⁻</th>
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</tr>
</thead>
<tbody>
<tr>
<td>M_{ππ} ≤ 1 GeV</td>
<td>1.4·10⁶ ev/y</td>
<td>1.15·10⁶ ev/y</td>
</tr>
<tr>
<td>M_{ππ} ≤ 0.45 GeV</td>
<td>1.1·10⁶ ev/y</td>
<td>0.9·10⁶ ev/y</td>
</tr>
</tbody>
</table>

In the hypothesis of a smaller integrated luminosity factor \( \int_{1\text{ year}} L \ dt = 10^{39} \text{ cm}^{-2} \), in correspondence to a larger energy of the electron beam \( E = 750 \text{ MeV} \), we obtain, for neutral pion pairs, \( N_{ππ} = 0.6·10^4 \) events/year in the range \( 2m_{π} \leq M_{ππ} \leq 0.45 \text{ GeV} \). Our estimated rates are pessimistic, since we used the cross-sections obtained from the \( O(p^4) \) prediction of chiral perturbation theory (CPT), which are smaller than the measured cross-sections at low energy \( M_{ππ} \leq 0.45 \text{ GeV} \) (see Fig. 2; also see below). With the same conservative attitude, in order to evaluate the total number of events up to \( M_{ππ} = 2E \), we chose an extrapolation for \( M_{ππ} > 0.45 \text{ GeV} \) of the lowest-order chiral cross-section, corresponding to a unitarity bound \( \sigma = 10.0 \text{ nb} \), for \( 0.6 \text{ GeV} \leq M_{ππ} \leq 0.7 \text{ GeV} \), and matching the CB data about \( M_{ππ} = 0.8 \text{ GeV} \). These rates can be compared with the number of events \( N_{ππ} = (0.8 + 1.1) \times 10^2 \text{ ev} \) and \( N_{ππ} = (3.4 + 3.9) \times 10^2 \text{ ev} \), obtained by the CB experiment in the range \( 2m_{π} \leq M_{ππ} \leq 0.45 \text{ GeV} \) and \( 2m_{π} \leq M_{ππ} \leq 1.0 \text{ GeV} \) respectively. (The total number of events at CB is \( N_{ππ} = 1.0 \times 10^3 \text{ ev} \), for \( 2m_{π} \leq M_{ππ} \leq 2.0 \text{ GeV} \).) Hence, DAPHNE may increase the statistics by a factor 10 with respect to the CB data, in the energy region of interest for testing the prediction of CPT.

In ref. [5] we discuss the data on \( σ_{TOT}(γγ → π⁺π⁻) \) from PLUTO, DM1, DM2 and MARK-II. We show that they all exceed \( σ_{Born} \) at low energy \( M_{ππ} \leq 0.45 \text{ GeV} \). The only data that include systematical errors, i.e. the data from MARK-II(6) (which have also the best statistics), are in good agreement with the chiral prediction.(3) The data from MARK-II are shown in Fig. 3.

The \( γγ → ππ \) processes give information on the electric \( (α_π) \) and magnetic \( (β_π) \) \( π \)-polarizabilities, which are sensitive to the hadron internal structure. Lorentz invariance restricts the chiral lagrangian relevant for the polarizabilities to lowest order, i.e. to \( O(p^4) \), to be of the form(7)

\[
F_{μν} F^{μν} = 2( E^2 - B^2 ) .
\]

Hence, chiral and Lorentz invariances together imply

\[
α_π = - β_π + O(p^6) ,
\]

in good agreement with the indication obtained using the dispersion sum rules.(5) The measurements in the experiments with the Primakoff effect and the radiative \( π \)-photoproduction
discussed in ref. [5] show results that do not depart dramatically from the dispersive predictions. There appears to be a 2σ disagreement with the $O(p^4)$ chiral prediction for $\alpha_\pi$.

\[
\gamma \gamma \rightarrow \pi^+ \pi^-
\]

**FIG. 3** - MARK-II data\(^6\) for $M_{\pi\pi} \leq 0.5$ GeV. We show the prediction of CPTth both in the Born approximation (dashed line) and to the 1-loop order\(^3\) (full line).

The latter can be expressed, for charged pions, in terms of the ratio between the axial and the vector coupling constants of the radiative $\pi$-decay\(^8\)

\[
\alpha_\pi = \frac{8\pi^2 m_\pi F_\pi^2}{\hbar_v} \frac{\hbar_A}{\hbar_v},
\]

where $F_\pi = 93.15$ MeV is the $\pi$-decay constant. From the measured value of this ratio\(^9\)

\[
\frac{\hbar_A}{\hbar_v} = 0.46 \pm 0.02
\]

one gets

\[
\alpha_\pi = (2.7 \pm 0.1) \times 10^{-4} \text{ fm}^3
\]

The uncertainty on the chiral prediction comes from $O(p^6)$ terms in the effective action. This could amount to less than 25%, according to an estimate of the correction in eq. (2) given by Donoghue and Holstein.
The reaction $\gamma\gamma \rightarrow \pi\pi$ can be studied with $e^+e^-$ machines. One can use the equivalent photon approximation, where the photons are emitted in the forward direction by the lepton beams and only a fraction $\sqrt{s}$ of the total beam energy $2E$ is available in the $\gamma\gamma$ system. Then, neglecting single powers of $\ln \frac{E}{m_e}$, one has\(^{(10)}\)

$$\sigma(e^+e^- \rightarrow e^+e^-\pi\pi) = 2\left[ \frac{\alpha^2}{\pi} \right] \left[ \ln \frac{E}{m_e} \right]^2 \frac{4e^2}{4m^2_\pi} \int \frac{ds}{s} f\left[ \frac{\sqrt{s}}{2E} \right] \sigma_{\gamma\gamma \rightarrow \pi\pi}(s),$$

(6)

where

$$f(x) = (2 + x^2)^2 \ln \frac{1}{x} - (1 - x^2) (3 + x^2),$$

(7)
in the case with no tagging, when the scattered leptons are undetected in the final state.

The introduction of the effective action at low energy in CPTTh is described in a beautiful review by Leutwyler.\(^{(11)}\) The method of CPTTh is discussed with full generality in Gasser's contribution to these Proceedings. In the chiral limit $m_{u,d,s} = 0$, the QCD lagrangian has a SU(3)$_L \otimes$ SU(3)$_R$ symmetry. This is spontaneously broken to SU(3)$_{L+R}$. The nonvanishing quark masses provide an explicit breaking of chiral symmetry. As a consequence, the 8 pseudoscalar Goldstone bosons $(\pi, K, \eta)$ are not massless. Owing to an anomaly in the 9th axial current, $\partial_\mu J_{\mu}^A \neq 0$, $M_{\eta} \neq 0$ even if $m_q = 0$. The effective lagrangian $\mathcal{L}_{\text{eff}}(U)$ depends on a unitary $3 \times 3$ matrix U. The constraint det $U = 1$ eliminates the U(1) field associated with the $\eta'$. Expanding $\mathcal{L}_{\text{eff}}(U)$ in powers of $\partial_{\mu} U$ and $m_q$ yields

$$\mathcal{L}_{\text{eff}} = L^{(2)} + L^{(4)} + L^{(6)} + \ldots,$$

(8)

where the $O(p^{2n})$ terms $L^{(2n)}$ depend on constants determined by experiments. $L^{(2)}$ depends on the $\pi$-decay constant. $L^{(4)}$ is determined in terms of 12 renormalized parameters $L_i$. $L^{(6)}$ depends on about thirty parameters $E_a$. The new coupling constants $L_i$ and $E_a$ determine the S-matrix to $O(p^4)$ and $O(p^6)$, respectively. This is not an expansion in loops. To $O(p^2)$ the transition amplitude is obtained from the tree graphs of $L^{(2)}$. To $O(p^4)$ we have to add the 1-loop graphs of $L^{(2)}$ to the tree graphs with one vertex from $L^{(4)}$. To $O(p^6)$ we must calculate the 2-loop graphs of $L^{(2)}$, plus the 1-loop graphs with one vertex from $L^{(4)}$, plus the tree graphs with one vertex from $L^{(6)}$.

As an example, we consider the process $\gamma\gamma \rightarrow \pi^+\pi^-$. Following ref. [3], we choose a reference frame where $\varepsilon_{1,2}$ obey

$$\varepsilon_1 \cdot k_1 = \varepsilon_1 \cdot k_2 = \varepsilon_2 \cdot k_1 = \varepsilon_2 \cdot k_2 = 0.$$

(9)

To $O(p^2)$ the amplitude does not vanish

$$A^{(2)}(\gamma\gamma \rightarrow \pi^+\pi^-) = 2ie^2 \left[ \varepsilon_1 \cdot \varepsilon_2 \cdot \frac{p_+ \cdot \varepsilon_1}{p_+ \cdot k_1} \cdot \frac{p_+ \cdot \varepsilon_2}{p_+ \cdot k_2} - \varepsilon_1 \cdot \varepsilon_2 \cdot \frac{p_+ \cdot \varepsilon_1}{p_+ \cdot k_1} \cdot \frac{p_+ \cdot \varepsilon_2}{p_+ \cdot k_2} \right].$$

(10)
For charged pions the $O(p^4)$ represents merely a correction to the leading order contribution. To this order the cross-section reads

$$\frac{d\sigma}{d(\cos \theta)} = \frac{\pi}{2s} \beta \alpha^2 \left[ 2|\alpha|^2 - \text{Re}(\alpha) \frac{4 (\beta \sin \theta)^2}{1 - (\beta \cos \theta)^2} + \frac{4 (\beta \sin \theta)^4}{[1 - (\beta \cos \theta)^2]^2} \right],$$

(11)

where the parameter $\alpha$ contains the sum of three terms

$$\alpha = 1 + \frac{2s}{F_\pi^2} (L_9 + L_{10}) + L(\tau).$$

Here the first two terms represent the tree-level contributions of $L^{(2)}$ and $L^{(4)}$ and the last term originates from the 1-loop graphs of $L^{(2)}$. This loop-function has been evaluated including $\pi$ and $K$ loops.\(^{(3)}\)

$$L(s) = -\frac{1}{32\pi^2 F_\pi^2} \left[ \frac{3}{2} s + (m_\pi \ln Q_\pi)^2 + \frac{1}{2} (m_K \ln Q_K)^2 \right],$$

(13)

where

$$Q_i = \sqrt{\frac{1 - \frac{4 m_i^2}{s}}{1 - \frac{4 m_i^2}{s} - 1}}, \quad i = \pi, K.$$ 

(14)

The cross-section (11) is plotted in Fig. 3, where also the Born cross-section obtained from the amplitude (10) is shown.

The chiral Compton cross-section is related to (11) by crossing symmetry. This yields an effective $\pi^+$ polarization.\(^{(5)}\)

$$\alpha_{\pi^+}(x) = \alpha_{\pi} \left[ 1 + \frac{F_\pi^2}{L_9 + L_{10}} \frac{L(x)}{2x} \right],$$

(15)

where

$$\alpha_{\pi} = \frac{4\alpha}{m_\pi^2 F_\pi^2} (L_9 + L_{10}).$$

(16)

Note that the contribution of loops to $\alpha_{\pi}$ is negative. For a discussion of (15) and its relation to the measurements of $\alpha_{\pi}$ we refer the reader to ref. [5].

The case $\gamma \gamma \to \pi^0 \pi^0$ is more interesting. The particles involved are all neutral. Hence the Born term vanishes for $L^{(2)}$ and $L^{(4)}$. The first non-zero tree-level contribution occurs to $O(p^6)$. The fact that no counterterms are available to cancel the divergences imply that the 1-loop graphs must yield a finite result. This process tests the loop structure of chiral SU(2). There are more such cases, e.g. the weak decay $K_S \to \gamma \gamma$, where there are no $O(p^4)$ counterterms. The
latter process tests the loop structure of chiral SU(3). Another analogous case is $\pi^0 \rightarrow \gamma\gamma$. The relevant chiral symmetry for $\gamma\gamma \rightarrow \pi^0\pi^0$ is given by SU(2)$_L \otimes$ SU(2)$_R$. This is a better symmetry than SU(3)$_L \otimes$ SU(3)$_R$, as $m_K$ and $m_\eta$ are not small with respect to the chiral scale $\Lambda = 4\pi F_\pi = 1$ GeV. This is the scale where the low-energy expansion of the effective lagrangian breaks down. Hence the validity of the low-energy prediction based on this expansion requires $s \ll 1$ GeV$^2$.

The lowest order $O(p^4)$ amplitude reads$^{(3)}$

$$A^{(\pi\text{-loops})} = -ie^2 \varepsilon_1 \cdot \varepsilon_2 \frac{s}{8\pi^2 F_\pi^2} \left[ 1 - \frac{m_\pi^2}{s} \right] \left[ 1 + \frac{1}{s} (m_\pi \ln Q_\pi)^2 \right],$$

(17)

$$A^{(K\text{-loops})} = -ie^2 \varepsilon_1 \cdot \varepsilon_2 \frac{s}{32\pi^2 F_\pi^2} \left[ 1 + \frac{1}{s} (m_K \ln Q_K)^2 \right].$$

(18)

The K-loop correction is small and represents about 2% of the result at $\sqrt{s} = 500$ MeV (see Fig. 4). Far below the KK threshold we can expand the last factor in the r.h.s. of (18)

$$1 + \frac{1}{s} (m_K \ln Q_K)^2 = -\frac{s}{12m_K^2} \left[ 1 + \frac{2s}{15m_K^2} \right] + \ldots.$$  

(19)

This shows that $A^{(K\text{-loops})}$ contains one more power of $s$ than $A^{(\pi\text{-loops})}$. This corresponds to the fact that, in the limit SU(2)$_L \otimes$ SU(2)$_R$, $\pi$-loops give the full $O(p^4)$ amplitude.

**FIG. 4** - The 1-loop contribution$^{(3)}$ to $\sigma(\gamma\gamma \rightarrow \pi^0\pi^0)$ including: i) both $\pi$ and $K$ loops (full line); ii) $\pi$ loops only (dot-dashed line). The data are by CB$^{(1)}$. 
It is necessary to determine what is the range of validity of the \( O(p^4) \) result. In order to do so, we estimate the next order contribution, \( O(p^6) \). First of all, we note that the \( O(p^4) \) chiral prediction for \( \sigma(\gamma\gamma \to \pi^0\pi^0) \), plotted in Figs. 2 and 4, **underestimates** the CB data in the threshold region. Inspecting Fig. 2 shows that the \( O(p^4) \) value of \( \sigma \) is also **smaller** than a cross-section obtained from the phase-shifts for \( \pi^+\pi^- \) elastic scattering, using unitarity conditions and dispersion relations. The latter is discussed in detail by Pennington in his contribution to these Proceedings. A caveat concerning the smallness of the effect for \( (\pi^0)^2 \)'s is in order (after all, it is a 1-loop effect!). This means one has to control the systematical effects rather carefully, when measuring \( \sigma(\gamma\gamma \to \pi^0\pi^0) \).

The contribution of the exchange of vector resonances in the \( t \)-channel is of \( O(p^6) \). We consider the contribution of the low-lying vector-meson resonances in the spectrum, \( V=\rho,\omega,\phi \). The \( \gamma\pi^-V \) coupling reads

\[
L^{(\gamma\pi^-V)} = g \epsilon_{\mu\nu\lambda\rho} F^{\mu\rho} V^{\nu} D^\lambda U .
\]

(20)

The tree-level amplitude obtained from \( L^{(\gamma\pi^-V)} \) corresponds to \( O(p^6) \) terms in the lagrangian, such as \( D_\lambda U D^\lambda U^\dagger F^{\mu\rho} F_{\mu\rho} \), \( D_\mu U D_\nu U^\dagger F^{\mu\alpha} F_{\nu\alpha} \), and so on. We evaluate the coupling constant \( g \) from the \( V \)-decay width, \( \Gamma(V \to \gamma\pi^0) \). The correspondence with CPTTh is readily understood. By means of this calculation one estimates the contribution of the tree graphs with one vertex from \( L^{(6)} \). The result, displayed in Fig. 5, shows that the \( O(p^6) \) contribution cannot be ignored, for energies as low as \( \sqrt{s} = 460 \text{ MeV} \). Note that, in this energy range, the \( V \)-resonance contribution to \( \sigma \) is negative and mostly coming from the interference with the 1-loop amplitude. What is important in this result is that it shows the need for calculating the full \( O(p^6) \) contribution. This, in turn, requires that we calculate the 2-loop diagrams in CPTTh.

**FIG. 5** - The cross section for \( \gamma\gamma \to \pi^0\pi^0 \) including: i) both the 1-loop graphs and the \( O(p^6) \) vector-meson exchange contribution (full line); the 1-loop contribution only (dot-dashed line).
In the range up to $\sqrt{s} = 700$ MeV a dispersive prediction is available in the literature.\(^{(12)}\) The right-hand cut contribution to the amplitude is determined, below the K$\bar{K}$ threshold, using the I=0 and I=2 S-wave phase-shifts for $\pi-\pi$ scattering. We remark that the left-hand cut amplitude has been neglected (see however ref. [13] for a recent evaluation of this amplitude). The contribution neglected in ref. [12] corresponds to the resonance exchange in the t-channel. Our calculation shows (see Fig. 6) that this contribution is not negligible, for energies in the range $\sqrt{s} = (550 \pm 600)$ MeV (and above). Note that, at $\sqrt{s} = 600$ MeV, the contribution to the cross-section coming from the modulus squared of the V-resonance amplitude adds 30% to the prediction of ref. [12] based on the right-hand cut amplitude alone. We stress that an estimate of the $O(p^8)$ contribution in CPTh (obtained using the vector meson dominance, for instance) should be given, in order to extrapolate the cross-section to such energy range. We recall that it is possible to recover the Mandelstam structure of the amplitude at 3-loops.

\begin{center}
**R–H CUT + L–H CUT (VECTOR MESONS)**
\end{center}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{dispersive_prediction.png}
\caption{The dispersive prediction for $\sigma(\gamma\gamma \rightarrow \pi^0\pi^0)$ including: i) the sum of the left-hand (L-H) cut and the right-hand (R-H) cut amplitudes (full line; this is obtained neglecting the interference contribution); ii) the R-H cut contribution alone\(^{(12)}\) (dashed line).}
\end{figure}

Our conclusion is twofold. On the experimental side, clearly the measurements of $\sigma(\gamma\gamma \rightarrow \pi\pi)$ must be redone. In the most interesting case of $\gamma\gamma \rightarrow \pi^0\pi^0$ DAPHNE may increase the statistics by one order of magnitude with respect to CB. We stressed that a careful absolute normalization of the cross-section is crucial. This entails a very good control of the backgrounds. To this aim, the e-tagging in the process $e^+e^- \rightarrow e^+e^-\pi\pi$ is necessary. On the theoretical side, we must calculate the full $O(p^8)$ amplitude for the $\pi^0\pi^0$ case.
Acknowledgements

It is a pleasure to thank Marina Candusso for valuable help in preparing some of the material illustrated here. We had very interesting discussions with Luciano Maiani. We enjoyed informative conversations with Rinaldo Baldini, Jürg Gasser and Victor Ogievetsky.

Note added

After delivering the talk summarized in these Proceedings, we became aware of ref. [14] where the contribution of the exchange of vector mesons to $\sigma(\gamma\gamma \rightarrow \pi^0\pi^0)$ is calculated, with numerical results that agree (within 10%) with ours. We disagree, however with the unitarizing role of the vector resonances emphasized by Ko. The extrapolation carried out by Ko of the resulting cross-section, in order to fit the CB data up to energies in the range $\sqrt{s} = 800$ MeV and above, is certainly not correct. Also, near the completion of this paper, we received a preprint by Bijnens et al.\(^{(15)}\) where the corrections to the process $\gamma\gamma \rightarrow \pi^0\pi^0$ found in vector meson dominance models are briefly mentioned, in comparison with the corrections obtained within the chiral quark model. The results agree with those presented by us.

REFERENCES