R. Gatto: ELECTRON-POSITRON COLLIDING BEAM EXPERIMENTS.

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ELECTRON-POSITRON COLLIDING BEAM EXPERIMENTS

R. Gatto,
Laboratori Nazionali del CNEN, Frascati, Rome.

Electron-electron colliding beams were proposed by Barker, Gittelmann, O'Neill, Panofsky and Richter at Stanford\(^1\). Electron-positron colliding beams are under development at Stanford\(^1\) and at Frascati\(^2,3\). The Frascati project, called Adone, is intended to produce high-energy (2-3 GeV c.m. energy) colliding beams of electrons and positrons.

I shall report on work done in Frascati in collaboration with Dr. Cabibbo in continuation of some previously published work\(^4\). We shall be concerned with:

I. The "one-photon" channel: general considerations, radiative corrections.
II. \(e^+e^-\) \(\rightarrow\) pions; \(e^+e^-\) \(\rightarrow\) 2\(\pi\), 3\(\pi\); K mesons.
III. \(e^+e^-\) \(\rightarrow\) \(\pi^0+\gamma\): the 2\(\pi\) state; the 3\(\pi\) state.
IV. Resonances: general discussion.
V. Baryon pairs: general considerations; cross-sections and polarization; hopes; \(\Sigma\Lambda\).
VII. Unitarity in the "one-photon" channel.
VIII. Electrodynamical vacuum polarization and cross-sections.
IX. Weak interactions: neutral \(B^0\); pair production of \(B\); local weak interaction.

As a matter of introduction let me recall a few relevant points concerning the electron-positron colliding beam experiments.

1) Tests of validity of quantum electrodynamics.
2) Most of the annihilation processes go through the "one-photon" channel. The reactions proceed through a state of well-defined quantum numbers.
3) Systematic exploration of form factors of strong interacting particles for time-like values of their argument.
4) Panofsky programme: systematic exploration of the spectrum of elementary particles through their interaction with photons.

5) Some unstable particles would be directly observed through resonances in the cross-sections.

6) At a given energy the angular distributions of the simplest reactions are uniquely predicted by theory or they depend on a few parameters.

7) Other typical features will appear from the following.

High-energy electron-positron colliding beam experiments may become a field of spectacular development in high-energy physics.

I. THE "ONE-PHOTON" CHANNEL

1. \textit{\textbf{The annihilation}}

\[ e^+ e^- \rightarrow a + b + \ldots + c \]

is assumed to go through the "one-photon" channel, as described by graph A (Fig. 1). \( q_+ \) and \( q_- \) are the momenta of \( e^+ \) and \( e^- \), \( K \) is the virtual photon momentum.

The matrix element is

\[ \langle a, b, \ldots, c | S| e^+ e^- \rangle = \frac{2\pi e}{K^2} (v \gamma \cdot u) \langle a, b, \ldots, c | j_Y(0) | 0 \rangle \delta(K - a - b - c) \]  

\[ (1) \]

\( u \) and \( v \) are Dirac spinors, \( j_Y(x) \) is the e.m. current operator.

We define

\[ j_Y = (2\pi)^{2n} \frac{\gamma^0}{2} \langle a, b, \ldots, c | j_Y(0) | 0 \rangle \]

\[ (2) \]

where \( n \) is the number of final particles. From gauge invariance

\[ K_y J_y = 0 \]

\[ (3) \]

In the c.m. system

\[ K^2 = (q_+ + q_-)^2 = -4E^2 \]

\[ (4) \]

where \( E \) is the energy of \( e^+ \) (or \( e^- \)). Note that \( K \) is time-like. In the c.m. system \( K = (0, i2E) \). From Eq. (3)

\[ K_a J_4 = 0 \]
or \( J_4 = 0 \). Thus \( J = (\mathbf{3}, 0) \). The cross-section is (we always neglect the electron mass)

\[
\sigma = \frac{(2\pi)^{5-3n}}{16 \ E^4} \int d^3 \mathbf{a} \ d^3 \mathbf{b} \ldots d^3 \mathbf{c} \ \delta(E_a + E_b + \ldots E_c - 2E) \delta(\mathbf{a} + \mathbf{b} + \ldots \mathbf{c}) \sum_{a,b,\ldots c} R_{mn}
\]

where: \( a = \frac{1}{\gamma} \); \( \mathbf{a}, \mathbf{a}_\pm \) are the momentum and energy of \( a \);

\[
T_{mn} = \frac{1}{2} (i_{mn} - \delta_{mn}) \quad \mathbf{t} = \frac{\mathbf{q}_+}{|\mathbf{q}_+|} = - \frac{\mathbf{q}_-}{|\mathbf{q}_-|}
\]

\[
R_{mn} = - J_m J_n^* \quad \sum_{a,b,\ldots c}
\]

refers to the final polarizations. For annihilation into two particles

\[
d\sigma = \frac{\alpha}{32} \left( \frac{p}{E} \right) \frac{1}{E^2} \left( \frac{E_a E_b}{E^2} \sum_{a,b} R_{mn} \right) d(\cos \theta)
\]

where \( p \) is the final momentum of \( a \) or \( b \) in c.m. and \( \theta \) the c.m. angle.

2. Remark about radiative corrections

Inclusion of radiative corrections brings about graph B (Fig. 2) with two photons exchanged. Their interference with \( (A) \) vanishes, however, in experiments which treat the charges symmetrically (such as total cross-section or, say, \( e^+ e^- \rightarrow \pi^+ \pi^- \) at definite angles but without distinguishing the charges, etc.). In fact the interference is essentially

\[
\text{Re} \langle \mathbf{i} | S_A P_f S_B | \mathbf{i} >
\]

where \( S_A \) and \( S_B \) are the contributions to the \( S \) matrix from the one-photon channels and two-photon channels and \( P_f \) projects into the final states selected in the experiment. But \( S_B |\mathbf{i}> = S_B |i+> \), \( S_A |\mathbf{i}> = S_A |i-> \), where \( |i+> \) and \( |i-> \) are even and odd under charge conjugation, and \( C S_A P_f S_B C^{-1} = S_A P_f S_B \) for the symmetrical experiments considered. Thus

\[
(9) = \text{Re} \langle \mathbf{i} C^{-1} | S_A P_f S_B | \mathbf{C} i+ > = -(9) = 0 .
\]
Therefore the "one-channel" picture is valid including terms \( O(e^6) \). Of course radiative corrections in \( S_A \) must be considered. The electron-photon part of them is a known universal energy dependent term.

II. \( e^+ + e^- \rightarrow n \) PIONS\(^4,6,7\)

The final \( n \)-pion state, for the "one-photon" channel, has:

\[
P = -1 \\
C = -1 \\
J = 1 \\
T = 1 \text{ for } n \text{ even} \\
= 0 \text{ for } n \text{ odd.}
\]

\( P, C, J, T \) are parity, charge conjugation, angular momentum, and isotopic spin. From Eqs. (6) and (7)

\[
R_{m n} T_{m n} = \frac{1}{2} |\vec{J}|^2 \sin^2 \Theta
\]

where \( \Theta \) is the angle between \( \vec{J} \) and \( \vec{t} \). \( \vec{J} \) is a vector (pseudovector) formed out of the independent final momenta for \( n \) even (odd). For \( n = 2 \) \( \vec{J} = (\vec{p}_1 - \vec{p}_2) \); for \( n = 3 \) \( \vec{J} = (\vec{p}_1 \times \vec{p}_2) \), normal to the production plane.

1. \( e^+ + e^- \rightarrow \pi^+ + \pi^- \)

The two final pions must be produced in a p-state. The cross-section is

\[
\frac{d\sigma}{d(\cos \Theta)} = \frac{\pi}{16} \alpha^2 \frac{1}{E^2} \beta^3 |F(K^2)|^2 \sin^2 \Theta
\]

The total cross-section

\[
\sigma = \frac{1}{m^2} (0.55 \times 10^{-32} \text{ cm}^2) \frac{1}{x^2} \left(1 - \frac{1}{x^2}\right)^{3/2} |F(-4E^2)|^2
\]

where \( m \) is the boson mass in GeV, \( x = E/m \), and \( F(K^2) \) is a form factor. On the \( K^2 \) axis
\( K^2 < 0 \), time-like \( K \)

<table>
<thead>
<tr>
<th>absorptive region</th>
<th>( K^2 &gt; 0 ), space-like ( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>physical region for</td>
<td>physical region for</td>
</tr>
<tr>
<td>( e^+ + e^- \rightarrow \pi^+ + \pi^- ) ( K^2 = -(2m_\pi)^2 )</td>
<td>( K^2 = 0 ) ( e + \pi \rightarrow e + \pi )</td>
</tr>
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An interpretation of \( e^+ + e^- \rightarrow \pi^+ + \pi^- \) near threshold should be possible in terms of two-pion intermediate states only. Frazer and Fulco\(^6\) proposed \( |F(K^2)|^2 \) approximately of the form

\[
|F(K^2)|^2 = \frac{\beta^2 + (k_0^2)^2}{\beta^2 + (k^2 - k_0^2)^2} \tag{14}
\]

with \( \beta \approx 2.65 \ m_\pi^2 \) and \( k_0^2 = 10.4 \ m_\pi^2 \). According to Bowcock, Cottingham and Lurie\(^9\)

\[
F_\pi(K^2) = \frac{t_r + \gamma}{t_r + K^2 - i\gamma \left( \frac{k^2}{4} - 1 \right)^{3/2}} \tag{15}
\]

with \( t_r = 22.4 \ m_\pi^2 \) and \( \gamma = 0.4 \ m_\pi^{-1} \). With Eq. (14) at \( E = 230 \ \text{MeV} \), \( \sigma = 8.35 \times 10^{-31} \ \text{cm}^2 \) (17 times bigger than with \( F = 1 \)); with Eq. (15) at \( E \approx 350 \ \text{MeV} \), \( \sigma = 6.6 \times 10^{-31} \ \text{cm}^2 \) (33 times bigger than with \( F = 1 \)), \( e^+ + e^- \rightarrow \pi^0 + \pi^0 \) requires the exchange of two photons.

2. \( e^+ + e^- \rightarrow \pi^+ + \pi^- + \pi^0 \)

Call \( \tilde{l} \) the relative \( \pi^+ - \pi^- \) angular momentum and \( L \) the \( \pi^0 \) angular momentum relative to \( \pi^+ - \pi^- \). Then from Eq. (10) \( \tilde{l} = L = 1,2,3,5, \) etc.

\[
\frac{d^2\sigma}{d\omega_+ d\omega_- d(\cos \Theta)} = \frac{\alpha}{(2\pi)^2} \frac{1}{64E^2} |H|^2 \sin^2 \Theta \left( \frac{p^+}{p^+} \times \frac{p^-}{p^-} \right)^2 \tag{16}
\]

where \( \omega_+, \omega_- \) are the energies of \( \pi^+, \pi^- \) in c.m. and \( \frac{p^+}{p^+}, \frac{p^-}{p^-} \) their momenta; \( H \) is a form factor depending on \( E, \omega_+ \) and \( \omega_- \); \( \Theta \) is the angle between \( \tilde{l} \) and the normal to the production plane. If there is a \( T = 0, J = 1 \) bound state near the threshold

\[
H \approx (\text{constant}) \times \frac{1}{K^2 + M^2}
\]
should be a good approximation (M mass of the bound state): \( e^+e^- \rightarrow 3\pi^0 \) and, in general, \( \rightarrow n\pi^0 \) is simply forbidden by \( C \). Production of many pions would perhaps exhibit patterns reflecting the \( T = 1, J = 1 \) \( 2\pi \) resonance [also a \( 3\pi \) resonance or bound state can be thought of as occurring through the action of the \( T = 1, J = 1 \) force\(^{10}\)].

3. K mesons

Eq. (12) applies to
\[
e^+e^- \rightarrow K^+K^- \\
\rightarrow K^0\overline{K}^0
\]

with \( F \) interpreted as the relevant form factor. The final \( K^0\overline{K}^0 \) pair must have \( C = -1 \) and therefore the final amplitude is
\[
K^0\overline{K}^0 - \overline{K}^0K^0.
\]

In terms of \( K_1^0, K_2^0 \)
\[
(M) = K_1^0K_2^0 - K_2^0K_1^0.
\]

Therefore only \( K_1^0K_2^0 \) pairs are produced. Furthermore if \( K_1^0, K_2^0 \) propagate through vacuum
\[
K_1^0 \rightarrow K_1^0 e^{-(\lambda_1 + i m_1)t} \\
K_2^0 \rightarrow K_2^0 e^{-(\lambda_2 + i m_2)t}
\]

and thus Eq. (18), or equivalently Eq. (17), conserves its form at any time before an interaction occurs.

III. \( e^+e^- \rightarrow \pi^0 + \gamma \)

This reaction measures the vertex of Fig. 3 where \( p \) is the \( \pi^0 \) momentum, \( K \) the virtual photon momentum, and \( q \) the momentum of the final photon. On the real \( K^2 \) axis
The decay $\pi^0 \rightarrow 2\gamma$ occurs at $K^2 = 0$. The absorptive cut starts at $K^2 = -4m_{\pi}^2$ with two pion states. The $T=1$, $J=1$ resonance will contribute. Next, at $K^2 = -9m_{\pi}^2$, starts the $3\pi$ cut. If there is a $T=0$, $J=1$ $3\pi$ bound state there would be a pole at $K^2 = -M^2$ where $M$ is the mass ($M < 3m_{\pi}$). Decay of the bound state (into $\pi^0 + \gamma$, $2\pi + \gamma$) is slow ($\tau \sim 10^{-26} - 10^{-21}$ sec) and the associated width correspondingly very small ($\Gamma \sim 0.06 - 0.6$ MeV). Therefore it is safe to approximate its contribution with a pole at a real $K^2$, except near the pole itself. The cross-section would have a sharp narrow peak at $E = \frac{1}{2}M$. The measured cross-section is approximately

$$\sigma \approx \frac{2}{3\Delta E} \int \sigma dE \approx \frac{1}{3\Delta E} \int \sigma dE$$

where $\Delta E$ is the experimental energy resolution. Let us use a Breit-Wigner formula

$$\sigma = \frac{3}{4} \pi \frac{\lambda^2 \Gamma_i \Gamma_f}{(2E - M)^2 + \frac{\Gamma^2}{4}}$$

with: $\lambda = 1/E$, $\Gamma_i, \Gamma_f$ partial rates for decay into the initial ($e^+ e^-$) channel, final ($\pi^0 + \gamma$) channel; $\Gamma$ total disintegration rate. Since $\Gamma \ll \Delta E$

$$\sigma \approx \frac{3}{2} \pi^2 \lambda^2 B_i B_f \frac{\Gamma}{2\Delta E}$$

with $B_i = \Gamma_i / \Gamma$, $B_f = \Gamma_f / \Gamma$. For $M \approx 3m_{\pi}$, $B_i \approx 10^{-3}$.
For $\Gamma \sim 10^{20} \text{ sec}^{-1}$, \[ \bar{\sigma} = 0.8 \times 10^{-29} \text{ (2}\Delta E \text{ in MeV)}^{-1} \text{ cm}^2. \] (22)

We shall discuss $e^+ + e^- \rightarrow \pi^0 + \gamma$ on the hypothesis of the dominance of the $2\pi$ resonance and of the $3\pi$ bound state. Experiments on internal conversion of gammas in $\pi^0$ decay indicate a negative value for the derivative of the $\pi^0$ form factor at the origin. Berman and Geffen\textsuperscript{11}) explain the negative coefficient by the coherent contribution of the $2\pi$ resonant state and of nucleon-antinucleon states. Our point of view is closer to that proposed by Wong\textsuperscript{12}). The $\pi^0 \gamma \gamma$ vertex can in general be written as

\[ \frac{1}{4} G(-K^2, -q^2, -p^2) \varepsilon_{\mu \gamma \lambda \rho} F_{\mu \gamma}(q) F_{\lambda \rho}(K) \] (23)

where $G$ is a form factor, $\varepsilon$ the isotropic antisymmetric tensor, and $F_{\mu \gamma}(q)$, $F_{\lambda \rho}(K)$ are Fourier transform of the e.m. tensor. We write $G(-K^2) = G(-K^2, 0, m_\pi^2)$. The $\pi^0$ lifetime is given by

\[ \frac{1}{\tau} = \frac{m_\pi^3}{64\pi} |G(0)|^2 \] (24)

The cross-section for $e^+ + e^- \rightarrow \pi^0 + \gamma$ is given by

\[ d\sigma = \frac{m_\pi}{m_\pi^3} \frac{1}{r} \beta^3 (1 + \cos^2 \Theta) \left| \frac{G(-K^2)}{G(0)} \right|^2 d(\cos \Theta). \] (25)

The total cross-section is

\[ \sigma = \frac{8\pi}{3} \frac{a}{m_\pi^3} \frac{1}{r} \beta^3 \left| \frac{G(-K^2)}{G(0)} \right|^2. \] (26)

For $\tau = 2.2 \times 10^{-16} \text{ sec}$

\[ \sigma = 2.75 \times 10^{-35} \text{ cm}^2 \left| \frac{G(-K^2)}{G(0)} \right|^2 (1 - x^{-2})^3 \] (27)

with $x = 2E/m_\pi$. We assume a dispersion relation ($m_\pi = 1$)
\[ G(-K^2) = \frac{1}{\pi} \int \frac{\text{Im} \ G(t) dt}{t + K^2 - i\epsilon} \]  \hspace{1cm} (28)

The contribution from the $2\pi$ state can be shown to be given by

\[ G_2(-K^2) \propto \frac{1}{\pi} \int_4^\infty \frac{(t - 4)^{3/2} |F_\pi(t)|^2 dt}{t^{1/2}(t + a)(t + K^2 - i\epsilon)^{3/2}} \propto \frac{1}{\pi} \int_4^\infty \frac{(t - 4)^{3/2} |F_\pi(t)|^2 dt}{t + K^2 - i\epsilon} \]  \hspace{1cm} (29)

where we have neglected the $t$ dependence of slowly varying factors. The Bowcock form factor

\[ F_\pi(t) = \frac{t_2 + \gamma}{t_2 - t - i\Gamma(t - 4)} \]  \hspace{1cm} (30)

satisfies Eq. (28) with

\[ \text{Im} \ F_\pi(t) = \frac{\gamma}{4(t_2 + \gamma)} (t - 4)^{3/2} |F_\pi(t)|^2 \Theta(t - 4). \]  \hspace{1cm} (31)

Therefore

\[ F_\pi(t) \propto \frac{1}{\pi} \int_4^\infty \frac{(t - 4)^{3/2} |F_\pi(t)|^2 dt}{t^2 + K^2 - i\epsilon} \propto G_2(-K^2). \]  \hspace{1cm} (32)

Thus we write

\[ G(-K^2) = G_2(-K^2) + G_3(-K^2) = \frac{c_2}{t_2 + K^2 - i\Gamma(t - 4)} + \frac{c_3}{t_2 + K^2 - i\Gamma(t - 4)} \]  \hspace{1cm} (33)

where we have used Eq. (32) and added a pole term at $K^2 = -t_3 + i\Gamma$ for the $3\pi$ bound states. The constants $c_2$ and $c_3$ are determined from Eq. (24) and the value $\alpha$ measured from $\pi^0 \rightarrow \gamma + e^+ e^-$. $\alpha$ is given by

\[ -\frac{1}{G(0)} \left( \frac{\partial G(-K^2)}{\partial K^2} \right)_0 = \alpha m^2_{\pi^0}. \]  \hspace{1cm} (34)

The experimental value for $\alpha$ is $-0.24 \pm 0.16$ \cite{i3}. One finds

\[ |G(-K^2)|^2 = \frac{64\pi}{m^3_{\pi^0}} \frac{1}{(t_2 - t_3)^2} \left| \frac{t_2^2 (1 - \alpha t_3)}{t_2 + K^2 - i\Gamma(t - 4)} \right|^2 \]  \hspace{1cm} (35)
We take \( t_2 = 22.4 \, m_\pi \) and \( t_3 = 5 \, m_\pi \). The last assumption comes from identifying the bound state with the resonance observed by Abashian, Booth and Crowe\(^1\). Furthermore \( \gamma \approx 0.4 \, m_\pi^{-1} \) and \( \Gamma = 10^{20} - 10^{21} \, \text{sec}^{-1} \). According to Eq. (35) there is a peak near \( k^2 = -t_2 \), with an enhancement factor (for \( \alpha = 0.24 \))

\[
\left| \frac{G(t_2)}{G(0)} \right|^2 \approx 250.
\]

(36)

The cross-section near \( t_2 \) is \( \sim 6 \times 10^{-33} \, \text{cm}^2 \) in the above enhancement factor. There is a peak also at \( k^2 = -t_3 \) and the average (19) around the peak is

\[
\sigma \approx 3.5 \times 10^{-29} \left( \frac{t_3 - \alpha t_2 t_1}{t_3 - t_2} \right)^2 \frac{1}{2\Delta E} \, \text{cm}^2
\]

(37)

with \( \Delta E \) in MeV. With the above values for the parameters, \( \sigma \approx 10^{-28} (2\Delta E)^{-1} \, \text{cm}^2 \), not very far from the previous independent estimate (22). The energy resolution \( \Delta E \) could reasonably be assumed \( \sim 5 \, \text{MeV} \).

One of the concurrent reactions to \( e^+ + e^- \rightarrow n^0 + \gamma \) against which one has to discriminate is \( e^+ + e^- \rightarrow \gamma + \gamma + \gamma \). We also should mention that \( e^+ + e^- \rightarrow e^+ + e^- + n^0 \)\(^{15,16}\) has a large cross-section \( \sim 2.2 \times 10^{-35} \, \text{cm}^2 (E/m_\pi) \) for \( \tau = 2.2 \times 10^{-16} \, \text{sec} \).

**IV. GENERAL DISCUSSION OF THE POSSIBLE RESONANCES**

Suppose the annihilation goes through a resonant channel

\[
e^+ + e^- \rightarrow B_J \rightarrow \text{(final state)}.
\]

\( B_J \) is a boson state with spin \( J \) and mass \( M \). Assume a Breit-Wigner description for the resonance cross-section near the maximum

\[
\sigma (E) = \pi \lambda^2 \frac{2J+1}{4} \frac{\Gamma_i \Gamma_f}{(2E - M)^2 + \frac{\Gamma}{4}}.
\]

(38)

An experiment with energy resolution (in c.m.) \( 2\Delta E \) measures
\[ \bar{\sigma}_R = \frac{1}{\Delta E} \int_{\frac{1}{2} (M - \Delta E)}^{\frac{1}{2} (M + \Delta E)} \sigma(E) dE. \]  

(39)

\( \Delta E \) is not expected to be smaller than \( \sim 1 \) MeV.

We distinguish among three cases:

i) total width \( \Gamma \ll 2\Delta E; \)
ii) \( \Gamma \gg 2\Delta E; \)
iii) \( \Gamma \sim 2\Delta E. \)

In case i)
\[ \bar{\sigma}_R \sim 2\pi \kappa^2 \frac{2}{\gamma} (2J + 1) B_i B_f \frac{\Gamma}{2\Delta E}. \]  

(40)

In case ii)
\[ \bar{\sigma}_R \sim \sigma(2M) = \pi \kappa^2 (2J + 1) B_i B_f. \]  

(41)

In case iii), one can use either Eq. (40) or Eq. (41) since they differ by a factor \( \pi/2 \) for \( \Gamma \gtrsim 2\Delta E. \)

Here \( B_i = \Gamma_i/\Gamma, \) \( B_f = \Gamma_f/\Gamma \) are the branching ratios for \( B_J \to e^+e^-, B_J \to \) (final state). We can limit the discussion to final states such that \( B_f \equiv 1. \) Then the relevant quantities are

in case i)
\[ \frac{\Gamma_i}{\Delta E}, \]

in case ii)
\[ \frac{\Gamma_i}{\Gamma}, \]

and any of these two quantities in case iii).

Now there are some factors in \( \Gamma_i \) determined from charge conjugation, gauge invariance and angular momentum. If \( B_J \) has

\begin{align*}
J = 0, \ C = 1 & \text{ then } \Gamma_i \propto \alpha^4 m_e^2 \\
J = 0, \ C = -1 & \text{ then } \Gamma_i \propto \alpha^6 m_e^2 \\
J = 1, \ C = 1 & \text{ then } \Gamma_i \propto \alpha^4 \\
J = 1, \ C = -1 & \text{ then } \Gamma_i \propto \alpha^2 \\
J = 2, \ C = 1 & \text{ then } \Gamma_i \propto \alpha^4 \\
J = 2, \ C = -1 & \text{ then } \Gamma_i \propto \alpha^6, \text{ etc.}
\end{align*}
Therefore in case i) only \((J=1, C=-1)\) will produce large effects. The same applies to ii) since \(\Gamma > \Delta E, \) and also to iii). Case i), \(\Gamma << 2\Delta E \approx \Delta E \) (a few MeV), implies \(\Gamma \ll 10^{21} \text{ sec}^{-1}, \) or \(\tau = \Gamma^{-1} \gg 10^{-21} \text{ sec,} \) and it occurs if decay through strong interaction is inhibited in some way. If \(\frac{\Gamma}{\Delta E} \gg \alpha^2, \) the resonance effect is indeed very big (compare with \(e^+ + e^- \rightarrow \mu^+ + \mu^- \), for instance, whose \(\sigma \approx \frac{\pi}{3} a^2 k^2 \) for \(E > m \mu \)). The same applies to case iii). Case ii) occurs whenever \(B \rightarrow \) decays by strong interactions, \(\Gamma \approx 10^{25} \text{ - 10}^{22} \text{ sec}^{-1}. \) For instance, the \(2\pi T = 1, J = 1 \) resonance belongs to case ii); the \(3\pi \) bound state, decaying into \(\pi^0 + \gamma, \) to i) or iii).

V. BARYON PAIRS

1. In \(e^+ + e^- \rightarrow p + \bar{p}, n + \bar{n} \)

\(\rightarrow \Lambda + \bar{\Lambda} \)

\(\rightarrow \Sigma + \bar{\Sigma} \)

\(\rightarrow \Xi + \bar{\Xi} \)

the final pair is produced in \(^3S_1 \) and \(^3D_1 \)

(always limiting to one-photon channel). Therefore near threshold the cross-section is isotropic and \(\propto \beta. \) The differential cross-section is in general

\[
\frac{d\sigma}{d(\cos \Theta)} = \frac{\pi}{8} a^2 k^2 \beta \left[ |F_1(K^2) + \mu F_2(K^2)|^2 (1 + \cos^2 \Theta) + \left( \frac{m}{E} F_1(K^2) + \frac{E}{m} \mu F_2(K^2) \right)^2 \sin^2 \Theta \right]
\]

(42)

where \(F_1(K^2), F_2(K^2)\) are the analytic continuations of the electric and magnetic form factors of the fermion for large negative \(K^2 < -4m^2, \) where \(m\) is the mass of the fermion. \(\mu\) is the static anomalous magnetic moment of the fermion (the normalization is \(F_1(0) = 1, \) \(F_2(0) = 1\) for a charged fermion; \(F_1(0) = 0, \) \(F_2(0) = 1\) for a neutral fermion).

2. The form factors are in general complex along the absorptive cut, in particular for \(K^2 < -4m^2. \) Thus in \(e^+ + e^- \rightarrow \) (fermion) + (antifermion), there can be a polarization of the fermion normal to the production plane,
already at the lowest electromagnetic order. This is opposite to scattering, \( e^+ (\text{fermion}) \to e^+ (\text{fermion}) \), where there is no polarization at lowest order. The polarization \( P(\Theta) \) of the fermion is given by

\[
\frac{d\sigma}{d(\cos \Theta)} P(\Theta) = -\frac{\pi}{8} \alpha^2 \beta^3 \frac{E}{m} \text{Im} \left[ F_2(K^2)F_2(K^2) \right] \sin(2\Theta)
\]

(43)

and it is along \( \vec{p} \times \vec{q}_+ \), where \( \vec{p} \) is the momentum of the final fermion and \( \vec{q}_+ \) that of the positron. For the antifermion the polarization is \( -P(\Theta) \), by use of TCP.

3. With \( F_1 = 1, F_2 = 0 \)

\[
\sigma = m^2 (2.1 \cdot 10^{-32} \text{ cm}^2) u (1-u)^{1/2} (1+\frac{1}{2} u)
\]

(44)

with \( m \) in GeV and \( u = \frac{m}{E} \). The actual cross-section, for instance, for \( e^+ e^- \to p + \bar{p} \), or \( n + \bar{n} \), may be well above or below Eq. (44). To show a typical argument: Suppose that the core terms\(^{17}\) in the form factors given by Hofstadter and Herman\(^{17}\) comes from an absorptive contribution around, say, \( K^2 = -(3m)^2 \). For \( K^2 \approx -(3m)^2 \) the nucleon form factors could then be approximated, using Hofstadter's result, as \( F_1 \approx 1.2/D, F_2 \approx -3.4/D, F_{1n} \approx 3.2/D, \) and \( F_{2n} = 0 \), with \( D = 20 - 2\xi - i\Gamma/2 \) (all in units of \( m_\pi \)). Inserting into Eq. (42) we find for \( e^+ e^- \to n + \bar{n} \), for instance, a value 50 times bigger than Eq. (44), near \( K^2 = -(3m)^2 \), by taking \( \Gamma \), say, \( \sim m_\pi \). All this is meant only to illustrate that the cross-sections can be bigger than Eq. (44).

4. The reactions

\[
e^+ e^- \to \Sigma^0 + \Lambda^0, \Lambda^0 + \Sigma^0
\]

proceed to \( ^3 S_1, ^3 D_1 \), for relative \( \Sigma - \Lambda \) parity, \( P_{\Sigma \Lambda} = +1 \); to \( ^1 P_1, ^3 P_1 \) for \( P_{\Sigma \Lambda} = -1 \). Therefore, near threshold \( \sigma \) increases \( \propto p \) (the final c.m. momentum) and is isotropic for \( P_{\Sigma \Lambda} = +1 \), is \( \propto p^3 \), and contains also a \( \cos^2 \Theta \) term for \( P_{\Sigma \Lambda} = -1 \). The \( \gamma \Sigma \Lambda \) vertex is

\[
J_{\gamma} = \bar{u}_{\Lambda} \left[ f_1(K^2)\gamma_\gamma + f_2(K^2)\gamma_\mu \gamma_\mu + f_2(K^2)K_\gamma \right] \Gamma V_{\Sigma}
\]

(45)
where $\Gamma = 1$ ($\gamma_3$) for $P_{\Sigma^\Lambda} = +1 (-1)$.

\[
\frac{d\sigma}{d(cos \Theta)} = \frac{\alpha^2}{8} \beta^2 \cos^2 \Theta \left[ \left| f_1(K^2) \right|^2 + K^2 \left| f_2(K^2) \right|^2 \right] + \left| f_1(K^2) \right|^2 - K^2 \left| f_2(K^2) \right|^2 \frac{E_{\Sigma^\Lambda} m_{\Sigma^\Lambda}}{E^2} + 4 \frac{m_{\Sigma^\Lambda} E}{E} \frac{m_{\Sigma^\Lambda}}{E} \text{Re} \left[ f_1(K^2) f_2^*(K^2) \right]
\]

(46)

where $\beta = p/E; E_{\Lambda}, m_{\Lambda}, E_{\Sigma}, m_{\Sigma}$ are the energies and masses of $\Lambda$ and $\Sigma$. Relative $\Lambda^\Sigma$ parity could be measured in this way.

VI. VECTOR MESONS

1. In the reaction

\[ e^+ + e^- \rightarrow B + \bar{B} \]

where $B$ is a spin one meson, the possible final states are $^1P_1$, $^3P_1$, and $^3F_1$ from angular momentum, parity, and charge conjugation. The reaction measures the vertex shown in Fig. 4. $p_1, \epsilon_1, p_2, \epsilon_2$ are the four-momenta and polarization vectors of $B, \bar{B}$. They satisfy, for physical particles, $p_1^2 = p_2^2 = -m_B^2, (p_1, \epsilon_1) = (p_2, \epsilon_2)$. To form $J_\mu$ take as independent vectors: $K = p_1 + p_2$, $p = p_1 - p_2$, $\epsilon_1, \epsilon_2$. The independent scalars are $K^2$, $(\epsilon_1 K)$, $(\epsilon_2 K)$ and $(\epsilon_1 \epsilon_2)$. $J_\mu$ must be linear in each $\epsilon$ and transform like a vector. We write thus in general

\[
J_\mu = K \mu \left[ (\epsilon_1 \epsilon_2) a + (\epsilon_1 k)(\epsilon_2 k) b \right] + p_\mu \left[ (\epsilon_1 \epsilon_2) c + (\epsilon_1 k)(\epsilon_2 k) d \right] + \epsilon_1 \mu (\epsilon_2 k) e + \epsilon_2 \mu (\epsilon_1 k) f
\]

(47)

where $a, b, ..., f$ are functions of $K^2$. From $KJ = 0$

\[
a = 0 \quad \text{and} \quad -K^2 b = e + f.
\]

(48)

Because of the odd behaviour of the e.m. current under charge conjugation $J_\mu \rightarrow -J_\mu$ when $K \rightarrow K, p \rightarrow -p, \epsilon_1 \leftrightarrow \epsilon_2$: it follows $e = -f$. By convenient redefinition of the form factors we then write
\[ J_\mu = \frac{e}{\sqrt{4\omega_1\omega_2}} \left[ G_1(\epsilon_1,\epsilon_2) p_\mu + \left[ G_1 + \mu G_2 + \epsilon G_3 \right] \left[ (\epsilon_1 K) \epsilon_2 \mu - (\epsilon_2 K) \epsilon_1 \mu \right] + \epsilon G_3 m_B^{-2} \left[ (K \epsilon_1)(K \epsilon_2) - \frac{1}{2} K^2 (\epsilon_1 \epsilon_2) \right] p_\mu \right] \] (49)

where \( \omega_1 \) and \( \omega_2 \) are the c.m. energies of \( B, \bar{B} \); \( e G_1, \mu G_2, \epsilon G_3 \) describe (for small space-like \( K^2 \)) the charge distribution, the magnetic moment distribution, and the electrical dipole moment distribution of \( B \). \( G_1, G_2, G_3 \) are functions of \( K^2 \).

2. The cross-section is

\[
\frac{d\sigma}{d(\cos \Theta)} = \frac{\pi}{16} \alpha^2 \lambda^2 \beta^3 \left\{ 2 \left( \frac{E}{m_B} \right)^2 \left| G_1(K^2) + \mu G_2(K^2) + \epsilon G_3(K^2) \right|^2 (1 + \cos^2 \Theta) + \right.
\]

\[
+ \sin \Theta \left[ 2 \left| G_1(K^2) \right|^2 + 2 \left( \frac{E}{m_B} \right)^2 \left| \epsilon G_3(K^2) \right|^2 + \left| G_1(K^2) \right|^2 + 2 \left( \frac{E}{m_B} \right)^2 \mu G_2(K^2) \right] \right\} .
\]

(50)

The \( \beta^3 \) dependence (\( \beta = \text{final velocity} \)) near threshold is typical of P-state production.

With \( G_1 = 1, G_2 = G_3 = 0 \) the total cross-section is

\[
\sigma = m_B^{-2} (2.1 \times 10^{-32} \text{ cm}^2) \frac{1}{2} (1 - u)^{3/2} (\frac{u}{2} + u)
\]

(51)

where \( m_B \) is expressed in GeV and \( u = (m/E)^2 \). Therefore \( e^+ e^- \) collisions may turn out to be very efficient for detecting possible unstable vector mesons.

3. Specific angular correlations will be evident after \( B \) decays. For instance, if \( B \) is produced near threshold in \( e^+ e^- \to B + \bar{B} \), and then decays according to \( B \to \pi + \pi \) the angular correlation is

\[ 2 - \cos^2 \Theta - \cos^2 \varphi + 2 \cos \Theta \cos \varphi \cos \chi \]

where \( \Theta \) is the production angle, \( \varphi \) the angle between the incoming momentum at production and the final relative momentum at the decay, and \( \chi \) the angle between the outgoing production momentum and the decay relative momentum.
If it decays according to $B \to \mu + \nu$ or $B \to e + \nu$ the correlation is

$$3 + \cos^2 \varphi - 2 \cos \Theta \cos \varphi \cos \chi.$$  \hfill (53)

We have assumed $\mu = \epsilon = 0$ and neglected the lepton masses.

4. The cross-section (51) violates unitarity at high energies. In fact it goes to a constant at high energy, whereas it can be shown, on the basis of unitarity arguments, that the total reaction cross-section must decrease proportional to $\lambda^2$. The unitarity upper limit to the reaction cross-section is derived in the next section.

VII. LIMITATIONS FROM UNITARITY
FOR THE "ONE-PHOTON" CHANNEL

Consider

$$a + b \to \text{(final state)}.$$  

We use Jacob–Wick notation. The initial state is defined for a given c.m. momentum of $a,b$ by the helicities $\lambda_a, \lambda_b$

$$|i\rangle = |\lambda_a, \lambda_b\rangle.$$  \hfill (54)

The total cross-section from Eq. (54) into a set of final states $F$ is

$$\sigma(\lambda_a, \lambda_b; F) = (2\pi)^2 \lambda^2 <\lambda_a, \lambda_b| T(E)\, \mathbf{p}_F(E) T(E)|\lambda_a, \lambda_b>.$$  \hfill (55)

Assume $F$ rotational-invariant. Then we can define

$$\sigma_j(\lambda_a, \lambda_b; F) = \pi \lambda^2 (2J + 1) <J; \lambda_a, \lambda_b| T_j\, \mathbf{p}_F^{(j)} T_j|J; \lambda_a, \lambda_b>.$$  \hfill (56)

For reactions (as opposed to scattering) we substitute $T \to S$, with $SS^\dagger = 1$. Then, from Eq. (56)

$$\sigma_j(\lambda_a, \lambda_b; F) \leq \pi \lambda^2 (2J + 1).$$  \hfill (57)

Now consider

$$e^+ + e^- \to (\gamma) \to \text{final state}.$$
In the limit $m_e = 0$ only the superposition
\[ \frac{1}{\sqrt{2}} \left( |1, -1 > + | -1, +1 > \right) \] (58)

in the notation (54) takes part to the reaction. In fact
\[ \bar{\nu}_\gamma \mu \, u = \bar{\nu}(\bar{a} \gamma_\mu \, a + a \gamma_\mu \bar{a})u \] (59)
where $a = \frac{i}{2} (1 + \gamma_5)$, $\bar{a} = \frac{i}{2} (1 - \gamma_5)$. Thus averaging Eq. (57), for $J = 1$
\[ \sigma < \frac{3}{4} \pi \lambda^2 . \] (60)

Using Eq. (60) we see that Eq. (51) violates unitarity at high energies ($\approx 10^2 \, m_B$). At these energies, however, other neglected effects are important.

VIII. RELATION BETWEEN THE ANNIHILATION CROSS-SECTION INTO STRONG INTERACTING PARTICLES AND MODIFICATIONS OF THE PHOTON PROPAGATOR

The quantity
\[ \Pi(K^2) = -\frac{(2\pi)^3}{3K^2} \sum_{p(z)=K} <0|j_r(0)|z> <z|j_r(0)|0> \] (61)
is known to be of fundamental importance in quantum electrodynamics$^{18)}$. For instance the Fourier transform of the photon propagator can be written as
\[ D_{\mu\nu}(K) = \frac{\delta_{\mu\nu}}{K^2 - i\epsilon} + \frac{K^2 \delta_{\mu\nu} - K_{\mu} K_{\nu}}{K^2} \frac{\Pi(0) - \Pi(K^2) - i\pi}{K^2 - i\epsilon} \] (62)
\[ \Pi(K^2) = P \int_0^\infty \frac{\Pi(-a)}{K^2 + a} \, da . \] (63)

We call $\sigma_F(E)$ the cross-section (5) summed over a set $F$ of final states
\[ \sigma_F(E) = -\frac{(2\pi)^5}{16 \, E^2} \sum_{T_{mn}} \sum_{p_z=K} <0|j_m(0)|z><z|j_n(0)|0> . \] (64)
In general, from gauge invariance

\[ (2\pi)^3 \sum_{\mathcal{F}} \langle 0 | j_{\mu}(0) | z \rangle < z | j_{\gamma}(0) | 0 \rangle = \Pi_{\mathcal{F}}(k^2)(k^\mu k^\gamma - k^2 \delta_{\mu\gamma}). \]  

(65)

Substituting Eq. (65) into Eq. (64) and using Eq. (6)

\[ \sigma_{\mathcal{F}}(E) = \frac{n^2 a}{E^2} \Pi_{\mathcal{F}}(-4E^2) \]  

(66)

which is the desired relation \(^{10}\). Note that in order \(\Pi(k^2) - \Pi(0)\) to be convergent (it must be convergent from Eq. 62)

\[ \int_{-\infty}^{\infty} \frac{\sigma_{\mathcal{F}}(E)}{E} \, dE \]  

(67)

must converge (which we know to happen from unitarity). However, \(\Pi(0)\), connected to charge renormalization, is convergent only if

\[ \int_{-\infty}^{\infty} E \sigma_{\mathcal{F}}(E) dE \]  

(68)

is finite. This does not happen if \(\sigma_{\mathcal{F}}\) decreases as \(\lambda^2\). Note that \(\sigma_{\mathcal{F}}(F)\) is the cross-section in the approximation where only one photon is exchanged.

**IX. REMARKS ABOUT WEAK INTERACTIONS**

Intermediate charged vector mesons, as suggested in theories of \(^{20}\) weak interactions, can be produced according to

\[ e^+ + e^- \rightarrow B + \overline{B} \]

and the formulae and the considerations of Section VII apply to this process.

A neutral \(B^0\) cannot be coupled to the leptons if it is also coupled to the weak neutral strangeness non-conserving current. If \(B^0\) exists and is coupled to the leptons in

\[ e^+ + e^- \rightarrow B^0 \rightarrow \mu^+ + \mu^- \]
it would lead to a resonant contribution. Assuming a width $\Gamma \approx 5 \cdot 10^{17} \text{ sec}^{-1}$, a branching ratio for $B^0 \rightarrow e^+ e^-$ of the order of $\frac{\Gamma}{s}$, a mass $m_B$ of the order of the $K$ mass, we find for the resonant contribution to the above reaction a value $\sigma_R$ about three times bigger than the value for the electromagnetic cross-section. Local weak interaction, with the known strength, would be felt only at higher energies. For instance at $E \approx 30$ GeV the muons from $e^+ e^- \rightarrow \mu^+ \mu^-$ would have a strong longitudinal polarization if there is a parity non-conserving coupling $(\mu^+ \mu^-)(e^+ e^-)$ of typical weak interaction strength.

* * *

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* * *
Fig. 1. The one photon channel.

(A) \[ k = q_+ q_- \]

q_+ \hspace{1cm} q_- \hspace{1cm} k = q_+ q_-

Fig. 2. Radiative correction.

(B)

Fig. 3. \( \gamma \rightarrow \pi^0 + \gamma \).

Fig. 4. Vector meson production.

\[ P_1 \epsilon_1 \quad P_2 \epsilon_2 \]

\[ k = p_1 + p_2 \]