C. Pellegrini, G. Stoppini: REMARKS ON NEUTRAL PION PHOTOPRODUCTION IN THE HIGH ENERGY REGION.
G. Pellegrini and G. Steppini

REMARKS ON NEUTRAL PION PHOTOPRODUCTION IN THE HIGH ENERGY REGION.

(Submitted to Nuovo Cimento for publication)

J. J. Sakurai proposed the measurement of the recoil proton polarization in the process

\[ \gamma + p \rightarrow \pi^0 + n \]

as a possible means of assigning the parity of the second resonance recently observed in the pion nucleon scattering and photo-production. In the energy region between the first and the second resonance, namely around \( E_\gamma = 600 \text{ MeV} \), the polarization predicted was as high as 80% for negative parity and zero for positive parity.

The experimental results show indeed an high polarization, though less than 80%, in the energy region 550 - 850 MeV (table 1) and this can be taken as a support to the negative parity assignment.

We already pointed out that the presence of a small s-wave amplitude can appreciably change the Sakurai's predictions, so to make the interpretation of the experimental results not completely unambiguous, if the large experimental errors are also taken into account.

As a matter of fact let us consider two models a) and b) for reaction (1) proposed respectively by R.R. Wilson.

(*) - Istituto di Fisica dell'Università di Roma.
and R. Querzoli\(^{(6)}\) according to the transition matrices

\[
M_d = i \frac{\mathcal{E}_1}{(\mathcal{E} \cdot \mathcal{E})} \mathcal{E} \cdot \mathcal{E} - \frac{1}{\mathcal{Q}^2} \left[ M_1, \left( \frac{3}{2}, \frac{1}{2} \right) + M_1, \left( -\frac{1}{2}, \frac{1}{2} \right) \right] \left( \mathcal{Q} \times \mathcal{E} \right) - i \left( \mathcal{E} \cdot \mathcal{Q} \left( \mathcal{Q} \cdot \mathcal{E} \right) - \mathcal{E} \cdot \mathcal{E} \left( \mathcal{Q} \cdot \mathcal{Q} \right) \right)
\]

**TABLE I**

<table>
<thead>
<tr>
<th>(E_x) MeV</th>
<th>Polarization of the recoil protons emitted at 90° in the cm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(550)</td>
<td>(0.30 \pm 0.12)</td>
</tr>
<tr>
<td>(560)</td>
<td>(0.40 \pm 0.14)</td>
</tr>
<tr>
<td>(585)</td>
<td>(0.50 \pm 0.16)</td>
</tr>
<tr>
<td>(610)</td>
<td>(0.60 \pm 0.27)</td>
</tr>
<tr>
<td>(650)</td>
<td>(0.59 \pm 0.24)</td>
</tr>
<tr>
<td>(658)</td>
<td>(0.50 \pm 0.18)</td>
</tr>
<tr>
<td>(700)</td>
<td>(0.56 \pm 0.11)</td>
</tr>
<tr>
<td>(720)</td>
<td>(0.57 \pm 0.06)</td>
</tr>
<tr>
<td>(720)</td>
<td>(0.82 \pm 0.29)</td>
</tr>
</tbody>
</table>

In the region 700–850 MeV preliminary results by R. Querzoli et al. seem to indicate a polarization yet equal to about 50%.

\[
M_b = i \frac{\mathcal{E}_1}{(\mathcal{E} \cdot \mathcal{E})} \mathcal{E} \cdot \mathcal{E} - \frac{1}{\mathcal{Q}^2} \left[ M_1, \left( \frac{3}{2}, \frac{1}{2} \right) \right] \left( \mathcal{Q} \times \mathcal{E} \right) - i \left( \mathcal{E} \cdot \mathcal{Q} \left( \mathcal{Q} \cdot \mathcal{E} \right) - \mathcal{E} \cdot \mathcal{E} \left( \mathcal{Q} \cdot \mathcal{Q} \right) \right)
\]
where $E_e(J, T)$ and $M_e(J, T)$ are the amplitudes for the absorption of an electric or magnetic multipole of order $\ell$ leading to a final state with total angular momentum $J$ and total isotopic spin $T$.

$\vec{e}$, $e$, $\vec{k}$, $\vec{q}$ are respectively the spin of the proton, the polarization vector of the photon and the momenta of the photon and of the pion.

In model a) it is assumed that the second resonance is excited through a magnetic dipole absorption leading to a state $T - 1/2$, $J = 3/2$, positive parity, while in model b) the multipole involved is supposed to be the electric dipole leading to the same state but with opposite parity.

In both models $E_1 (1/2)$ is the s-wave amplitude and $M_1 (3/2, 3/2)$ is the multipole giving rise to the first resonance.

Starting from (2), (3) and choosing as positive the polarization in the direction $\vec{k} \times \vec{q}$, we obtain for the polarization of protons emitted at $90^\circ$ in the center of mass system

$$P_1^+ = \frac{i}{\hbar} \frac{\langle I \mid J \rangle}{\langle J \mid J \rangle} \left[ m_e \left( \frac{3}{2}, \frac{3}{2} \right) + m_e \left( \frac{3}{2}, \frac{1}{2} \right) \right] \frac{2}{\sqrt{2}}$$

$$P_2^+ = \frac{i}{\hbar} \frac{\langle I \mid J \rangle}{\langle J \mid J \rangle} \left[ e^+ \left( \frac{3}{2}, \frac{3}{2} \right) + 4 \rho^+ \left( \frac{3}{2}, \frac{1}{2} \right) m_e \left( \frac{3}{2}, \frac{3}{2} \right) \right] \frac{2}{\sqrt{2}}$$

$A$ is the isotropic term of the angular distribution for process (1)

$$\frac{d\sigma}{d\Omega} = A + B \omega \delta + C \cos^2 \theta$$

$$\omega \delta = \frac{q \cdot k}{q \cdot k}$$

The term $E_1 (1/2)$ introduced in (2) and (3) is necessary to obtain a non-zero interference term at all energies in agreement with the experimental results (fig. I).

Models a and b are not the only possible ones. Furthermore they are not able to explain facts such as the third resonance.
W.S. McDonald, V.Z. Peterson, D.R. Corson
P.R. 107, 577 (1957)

J.I. Vette - P.R. 111, 622 (1958)

K. Berkelman, J.A. Waggoner - P.R. 117, 1364 (1960)

R.M. Worlock - P.R. 117, 537 (1960)


V.I. Goldansky et al. - Nu.Phys. 12, 327 (1959)
at about 950 MeV. However, we shall limit ourselves and consider only them.

We shall assume that they describe correctly the experimental situation around 600 MeV. Thus, roughly speaking, the polarization below the second resonance is given by (4) and (5), while above, it becomes necessary to explain the large polarization, by introducing in the models an amplitude corresponding to the third resonance.

An alternative model, which gives respectively to the resonances the parity $\pm -$, and explains all the polarization as due to the interference between the third and the first and second resonance has been proposed by Marshall and Landovitz (7). We can have a justification for not considering this model in the fact that up to 700 MeV there is no appreciable evidence for a $\cos^4 \theta$ term. This term follows from the model owing to the fact that the third resonant state is a $J = 5/2$ one.

As was pointed out by Maloy (8), $P_a$ can reach a maximum value of $\sim 31\%$. $P_b$, when $E_{1\frac{1}{2}}^1(1)$ is to be negligible, can reach a maximum of 80% per cent.

If it is assumed that the addition of $E_{1\frac{1}{2}}^1(1)$ does alter appreciably the value of $P_b$, we see that model $b$ is in better agreement with the measured values around 600 MeV.

Anyway, due to the large experimental errors, it seems preferable to have a more stringent criterion of distinction between the two models.

In fact it is difficult to obtain quantitative predictions from (4) and (5), because, in spite of the simplification assumed it is impossible to derive from the experimental $A$, $B$ and $C$ all the complex multipole amplitudes. Actually in the energy region under consideration not even the multipoles phases are known. For this reason it seems that the experimental determination of

\[ \ldots \]
the absolute polarization would serve more as additional experimental information useful for the extraction of the multipole amplitudes than for giving sharp answers of the kind required by Sakurai.

The sign of the \( P \) in (4) and (5) is a priori undetermined because of the arbitrariness in the sign of the multipole amplitudes.

We want to show that the sign of \( P_a \) and \( P_b \) can be determined if \( B \) is assumed to be known and that, within the limits of reasonable hypotheses, only model \( b \) allows to predict the sign experimentally observed. By writing

\[
\begin{align*}
\mathcal{E}_i \left( \frac{1}{2}, \frac{1}{2} \right) &= \mathcal{E}_i e^{i\alpha_i} \\
\mathcal{E}_i \left( \frac{1}{2}, \frac{3}{2} \right) &= \mathcal{E}_i e^{i\alpha_f}
\end{align*}
\]

\[
\begin{align*}
M_i \left( \frac{1}{2}, \frac{1}{2} \right) &= m_i e^{i\alpha_{i3}} \\
M_i \left( \frac{1}{2}, \frac{3}{2} \right) &= m_3 e^{i\alpha_{i33}}
\end{align*}
\]

we get

\[
\begin{align*}
a) \quad P_a &= \frac{1}{4} \mathcal{E}_i \left\{ m_3 \sin (\alpha_{i3} - \alpha') + m_i \sin (\alpha_{i3} - \alpha) \right\} \\
b) \quad P_b &= -\frac{1}{2} \mathcal{E}_i \left\{ m_3 \sin (\alpha_{i3} - \alpha') + m_i \sin (\alpha_{i3} - \alpha) \right\}
\end{align*}
\]

At \( E_y \approx 450 \text{ MeV} \), \( M_1 \) and \( E_1 \) are negligible, respectively and it results from the experiment \( B > 0 \). On the other hand \( \cos (\alpha_{i3} - \alpha') \) is less than zero. Then it is necessary to be \( e_i m_3 > 0 \). By choosing \( m_3 > 0 \) (all the results do not change if \( m_3 < 0 \) is assumed) we get \( e_i > 0 \).

At \( E_y \approx 650 \text{ MeV}, \) i.e. below the second resonance, where \( m_3 \) and \( e_i \) are negligible respectively, \( B < 0 \) and \( \cos (\alpha_{i2} - \alpha') = 90^\circ, \alpha_{i2} \leq 90^\circ \) so that \( \cos (\alpha_{i3} - \alpha') < 0, \cos (\alpha_{i3} - \alpha') > 0 \). It follows \( E_1 < 0, m_1 > 0 \) and

\[
\begin{align*}
P_a &> 0 \\
P_b &< 0.
\end{align*}
\]
The polarization measurements give a negative value so excluding model a) on the ground of the above consideration.

To obtain this result we made the hypotheses that \( e_I^+ \) does not change sign going from low to high energy. Furthermore the s-wave term should be written \( \xi_j (z_j) = e_j e_j^* \alpha_j e^*_j e^*(\alpha_j) \).

As experimentally we know that \( \alpha_j > \alpha_3 \), we neglected the T = - 3/2 state.

In the argument only for the s-wave part has it been assumed that the multipole phase coincide with the corresponding scattering phase shift. The validity of this assumption has been justified by Pellegrini and Tani (10) on the ground of a quite reasonable model.

The choice \( E_I^+ 0 \) seems to be consistent with the observed \( \gamma \) angular distribution. In fact in model b)

\[
\rho + \frac{\xi}{J} \alpha = |\xi_j (z_j)|^2 + N \xi_j \alpha_j (J - \alpha_j)
\]

and at \( E_\gamma \sim 700 \text{ MeV} \) \( C \simeq -A^{(II)} \) so \( A + \{5/3\} \) \( C < 0 \). Because \( \cos \{ J - \alpha_j \} > 0 \) this means that \( e_I^+ \) and \( E_I^+ \) have opposite sign. In the model a \( A + \{5/3\} \) \( C \simeq -A \). It is worth being noted that the addition of the \( e_3^+ e^*(\alpha_3) \) term to the s wave amplitude might change the sign of \( P_a \) but not the sign of \( A + 5/3 \) \( C \), in the model a. Instead the same addition could have the effect of changing the sign of \( A + 5/3 \) \( C \) leaving the sign of \( P_b \) unaltered, in model b.

Furthermore it has to be noted that if \( \xi_j < 0, \xi_j > 0 \) the two terms in \( P_b \) tend to cancel each other so qualitatively explaining why the measured polarization is less than the value predicted by Sakurai.

We wish to thank professor M. Cini for very helpful discussion.
BIBLIOGRAPHY


(2) - J.W. De Withe, H.E. Jackson, R. Littauer Phys. Rev. 110, 1208, (1958)

(3) - R. Quercioli, G. Salvini, A. Silverman - Private communication, in publication. The errors given by these authors includes all the corrections.
L. Bertanza, F. Franzini, I. Mannelli, V.Z. Peterson, G.V. Silvestrini - private communication.

(4) - C. Pellegrini, G. Stoppini, Reported by G. Bernardini at the kiev Conference on High Energy Physics (1959)


(8) - Maloy private communication

(9) - B. Pontecoreo - Proceedings of the Kiev Conference on High Energy Physics (1959)

(10) - C. Pellegrini, L. Tau to be published in Nuovo Cimento.

J.I. Vette, Phys. Rev. 111, 622, (1958)