Laboratori Nazionali di Frascati

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G. Bologna, G. Diambrini, G. P. Murtas: HIGH ENERGY BREMSSTRAHLUNG FROM A SILICON SINGLE CRYSTAL.
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HIGH ENERGY BREMSSTRAHLUNG FROM A SILICON SINGLE CRYSTAL.

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In a previous letter\(^1\) we reported the results relative to
electron pair production from a silicon single crystal. In
this letter we give the results relative to several measure-
ments on bremsstrahlung from a similar target. We also com-
pare these results with the theoretical prediction by Uberall\(^2\)
and Schiff\(^3\).

We used about the same experimental arrangement described
in ref. 1. The only differences are that the single crystal
is now mounted within the synchrotron chamber and that the
spectrometer converter is an aluminum one; moreover we added
another counting channel, for measuring simultaneously at
two different photon energies.

The silicon single crystal is in the form of a half-
circular plate 15 mm in diameter and 2.7 \( \times 10^{-3} \) radiation
length thickness; it is cut perpendicular to the axis [111]
within \( \pm 4 \) mrad, as determined by a Laue X-ray back-reflec-
tion method\(^(\circ)\). A goniometric device allows the single cry-
stral to be rotated both about a horizontal and a vertical
axis; the precision in the measurement of the angles is

\(^{(\circ)}\) We are indebted to Dr. F. Sgorlata, of the Istituto Supe-
riore di Sanità, Roma, for this analysis.
First we successively measure the numbers $N(\vartheta, k)$, $N(\vartheta, k_0)$ of symmetrical pairs per fixed numbers of monitor units (corresponding to $10^{10}$ equivalent quanta), as a function of the angle $\vartheta$ between the incident $1$ Gev electron beam and the crystal axis, and for the central values $k, k_0$ of the photon energies. We subtract for delayed coincidences and for background as in the previous work.

We choose $k_0 \approx 900$ Mev because, as it follows from Uebel's theory, the bremsstrahlung intensity at such an energy has a negligible dependence on $\vartheta$, so that $N(\vartheta, k_0)$ may be used as a normalization factor. The $\gamma$-ray beam monitor units cannot be used, because the shape of the spectrum depends on $\vartheta$. $N(\vartheta, k_0)$ ranges from $\sim 3,000$ to $\sim 12,000$ counts/monitor units.

Due to the multiple traversals of the electrons through the crystalline target, the $\gamma$-ray beam intensity after the extreme collimation ($0.8 \cdot 10^{-3}$ rad) is still rather strong: $\sim 3 \cdot 10^9$ eq. quanta/minutes.

On the other hand we have determined the number of pairs $N(k, \vartheta)$ as a function of $k$, for several fixed values of $\vartheta$.

Some of the results obtained are shown in this letter. In Figs. 1(a) and 1(b) we give the experimental ratio $R_{\text{ex}}$:

$$R_{\text{ex}}(\vartheta) = \frac{N(\vartheta, k)}{N(\vartheta, k_0)} \frac{\sigma(\vartheta, k_0)}{\sigma(\vartheta, k)},$$

where $\sigma(\vartheta, k)\,dk$ is the cross section for symmetrical pair production in aluminum at the photon energy between $k$ and $k + dk$. The solid line in both figures represents the value of the theoretical ratio $R_{\text{th}}$.
\[ R_{th}(\psi) = \frac{I(\psi, x)}{I(\psi, x_0)} \quad , \]  

where:

\[ I(\psi, x) = I_n(\psi, x) + I_e(x) \quad ; \]  

\[ I_n \] a quantity proportional to the bremsstrahlung intensity in the field of the nuclei of the single crystal, has been calculated by Überall.

\[ I_e \] is a quantity proportional to the bremsstrahlung intensity in the electron field of a non crystalline target.  

We have:

\[ I_n(\psi, x) = \left[ \frac{1}{x} + (1-x)^2 \right] \left[ \psi_1^c(\frac{\psi}{\psi_1}) + \frac{\psi_1^c}{\psi_1} \right] + \sum_{h \neq 1} \psi_h^c(\psi, \frac{\psi}{\psi_1}) - \frac{2}{3} (1-x) \left[ \psi_2^c(\frac{\psi}{\psi_1}) + \right. \]

\[ + \frac{\psi_2^c}{\psi_1} + \sum_{h \neq 1} \psi_h^c(\psi, \frac{\psi}{\psi_1}) \]  

where

\[ x = k/E; \quad E = 1 \text{ Gev} \quad \text{Electron energy} \]

\[ \frac{\psi}{\psi_1} = (mc^2/2E)(x/1-x) \quad \text{Minimum momentum transferred to the nucleus in units of me.} \]

The numerical values involved in formula (4) are the same as those used in ref. 1, with the only difference that now we take the lattice spacing relative to the axis [111].

We computed the series \[ \sum_{h \neq 1} \psi_h^c + \sum_{h \neq 1} \psi_h^c \] only for the case \[ \psi = 0 \].

In Fig. 1(a) we have:

\[ k = 240 \text{ MeV} \quad \quad k_o = 910 \text{ Mev} \]

The experimental data show two symmetrical peaks at the left and right side of \[ \psi = 0 \] and a central minimum in
a good qualitative agreement with the Überall's calculations obtained in the Born approximation. Formula (4) results from an integration over the angles of the emitted photon and of the scattered electron; but the coherence effect among nuclei shows a strong dependence on the angle of the emitted photon. This enables us to explain the fact that the maximum value of the measured effect is larger than the theoretical one; this might be due to the enrichment suffered by the X-ray beam in photons emitted at small angles, owing to the sharp collimation employed.

In Fig. 1(b) we have:

\[ k = 30 \text{ Mev} \quad k_o = 865 \text{ Mev}. \]

In the measurements shown in this figure the localization of the central minimum is very critical. The spacing between the two maxima is about 2 mrad and the curve is very sharp, while the goniometric device has an angular reproducibility of ± 0.5 mrad. We then proceed in the following manner. For small angles we have:

\[ \Theta = \left( \frac{\Theta^2}{\Theta_h^2} + \frac{\Theta^2}{\Theta_v^2} \right)^{1/2}, \]

where \( \Theta_h \), \( \Theta_v \) are the angles of rotation of the crystal about a horizontal and a vertical axis respectively.

After a preliminary approximate alignment of the crystal axis with the electron beam, we rotate the crystal about a horizontal axis until we find a relative minimum, for which we put \( \Theta_h = 0 \). We then rotate it about the vertical axis by an angle \( \Theta_v = \frac{\Theta}{\Theta_v} = 1 \) mrad, to find the absolute minimum for which we put \( \Theta_v = 0 \) (remember the effect has an axial symmetry).
In Fig. 1(b) we plot the values \( \rho_{lm} \) obtained by rotation about the horizontal axis (crosses) versus the angle
\[
\Theta = \left( \frac{\Theta_s^2 + \Theta_v^*}{2} \right)^{1/2}
\]
and the ones obtained by rotation about the vertical axis (circles) versus \( \Theta = \Theta_v \).

Since the error \( \Delta \Theta / \Theta \) is very small for \( |\Theta_s / \Theta_v| << 1 \), we have a good angular resolution for the crosses near the maxima.

The analysis of the data plotted in Figs. 1(a) and 1(b) shows that the central minimum does exist, even if less marked than the theoretical one, especially as far as the Fig. 1(b) is concerned. At this date we do not know if this difference between theory and experiment really exists or if it is due to the insufficient angular resolution of the experimental arrangement. A better goniometric device is under construction for a more precise analysis of the effect.

At these energies it seems that the correction to the Born approximation near the minimum is smaller than the one predicted by Schiff, according to which the central minimum would be completely washed out for \( \hbar = 100 \text{ MeV} \).

In Fig. 2 we plot the quantity \( \tilde{W}(x, \Theta) \) given by formulas (3) and (4), as a function of \( x \), for \( \Theta = 6 \text{ mrad} (\text{solid curve}) \) and the bremsstrahlung intensity from a silicon non-crystalline target for comparison (dot-dash curve). We also plot the experimental quantity
\[
\lambda \frac{\tilde{W}(x, \Theta)}{\delta x'(x)},
\]
where \( \lambda \) is a normalization factor, so chosen that the
experimental value is coincident with the theoretical one at 900 Mev. The data for 6 mrad confirm that the collimation operates an angular selection of the photons. For $\Theta \approx 1$ mrad we do not draw the theoretical curve because at these angles the dependence on $\Theta$ is very strong, and for the experimental data we have: $\Theta = 1 \pm 0.5$ mrad.

The proceeding experiments $^6,^7$ have shown a dependence on $\Theta$ of the bremsstrahlung intensity without showing any central minimum. We think this is due to insufficient angular resolution, with respect to the low-energy detected photons.

We thank Dr. G. Barbiellini for the collaboration given in this work.

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Figure Captions

Fig. 1
Intensity of the bremsstrahlung produced in a Si single crystal \( T = 293^\circ \text{K} \) versus \( \Theta \) (the angle between the 1 GeV-electron beam and the crystal axis \([111]\))

The solid curves of the figures represent \( R_{th}(\Theta) \) given by formula (2), while the experimental points represent \( R_{ex}(\Theta) \) given by formula (1).

The dashed curves are simply drawn for visualizing the behaviour of the experimental points.

The statistical error \((\sim 2\%)\) is indicated for some points.

Fig. 1(a) \( k = 240 \text{ Mev}; \quad k_o = 910 \text{ Mev} \)
Fig. 1(b) \( k = 380 \text{ Mev}; \quad k_o = 865 \text{ Mev} \).

Fig. 2
Intensity of the bremsstrahlung produced in a Si single crystal \( T = 293^\circ \text{K} \) versus \( x = k/E \) (the fractional energy of the photon with respect to the electron energy \( E = 1 \text{ Gev} \)).

The solid curve represents \( I(x, \Theta) \) given by formula (3) for \( \Theta = 6 \text{ mrad} \).

The dash-dotted line represents the same quantity for a new crystalline target (Bothe-Heitler + Wheeler-Lamb).

The experimental points represent the quantity \( (5) \).

The circles are relative to \( \Theta = 6 \pm 0.5 \text{ mrad} \), while the crosses are relative to \( \Theta = 1 \pm 0.5 \text{ mrad} \).
Fig. 1(a) - Bremsstrahlung intensity from Si single crystals
(T = 293°C)

$\mathbf{K} = 240$ MeV
$\mathbf{K}_0 = 910$ MeV

$$R_{\text{in}}(\theta) = \frac{I_n(\theta, x) + I_o(x)}{I_n(\theta, x_0) + I_o(x_0)}$$

$$R_{\text{ex}}(\theta) = \frac{N(\theta, x)}{N(\theta, x_0)} \frac{\frac{dK_0}{dx}}{\frac{dK}{dx}}$$

$I = \frac{k}{m}$

Angle between the 1 GeV electron beam and the crystal axis (111)
Fig. 1(b) - BREMSSTRAHLUNG INTENSITY FROM Si SINGLE CRYSTAL
(T = 293° K)

\[ R_{th}(\theta) = \frac{I_n(\theta, x) + I_e(x)}{I_n(\theta, x_0) + I_e(x_0)} \]

\[ R_{ex}(\theta) = \frac{N(\theta, K)}{N(\theta, x_0)} \frac{p(x_0)}{p(K)} \]

ANGLE BETWEEN THE 1 GeV ELECTRON BEAM AND THE CRYSTAL AXIS (111)
\[ I(x, \varepsilon) \]

--- Theoretical (Überall+Wheeler-Lamb; \( \Theta = 6 \text{ mrad} \))

+ Experimental; \( \Theta = 6 \text{ mrad} \)

\( \cdot \) Experimental; \( \Theta = 150.5 \text{ mrad} \)

--- Theoretical (Bethe-Heitler+Wheeler-Lamb)

\[ K \text{ = Photon Energy} \]

\[ E \text{ = 1 Gev-Electron Energy} \]

\[ x = \frac{I}{I_0} \]

*Fig. 2 - BREMSSTRAHLUNG INTENSITY FROM Si SINGLE CRYSTALS (T = 293° K)*