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A CRYSTAL METHOD FOR MEASURING THE LINEAR POLARIZATION OF PHOTONS IN THE MULTI-GeV REGION.

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In a previous paper\textsuperscript{1} we presented experimental data concerning the linear polarization of 150-MeV photons. We obtained these data by measuring the asymmetry in electron pair production at certain angles, in an 'amorphous' converter.

This method is not feasible when the photon energy is larger than \(\approx 500\) MeV, because the angle of emission of the pair particles, with respect to the primary photon, becomes too small.

In this letter we propose a method for measuring linear polarization based again on asymmetry in pair production, but taking into account the pair particles emitted at any angle in a thin crystal converter\textsuperscript{2}.

The limitations in the use of this method are also examined.

Let us consider a high energy photon beam of momentum \(\vec{E} / c\) (\(c\) is the velocity of the light) linearly polarized in the direction \(\vec{E}\) and striking a single crystal at a small angle \(\bar{\theta}\) (\(\approx 0.1\) rad) with a reciprocal lattice axis \(\vec{b}_1\). We choose two other axes \(\vec{b}_2, \vec{b}_3\) perpendicular to each other and to \(\vec{b}_1\), in order to have a reference frame.

We started from May's\textsuperscript{3} electron pair production differential cross section from linearly polarized photons in the field of a nucleus and we used the same calculation technique employed in other works\textsuperscript{1,4,5}.

We considered the electron production cross sections \(d\sigma_\parallel\) and \(d\sigma_\perp\) for \(\vec{E}\) parallel and perpendicular, respectively, to the plane \((\vec{E}, \vec{b}_1)\), determined by the direction of the incident photons and the axis \(\vec{b}_1\). \(d\sigma_\parallel\) and \(d\sigma_\perp\) are differential in the energy \(E_e\) of the electrons and integrated over the emission angles of the pair electrons.

As a result of the calculation we obtained the asymmetry ratio
\begin{equation}
R = \frac{d\sigma_\parallel - d\sigma_\perp}{d\sigma_\parallel + d\sigma_\perp},
\end{equation}
where \(d\sigma = d\sigma_\parallel + d\sigma_\perp\) represents the cross section of unpolarized photons.
We have

\[
\left\{ \begin{array}{l}
\frac{\alpha_2(1-y)}{\alpha_1 y} - 2y(1-y) \mathcal{V}_1 (\theta, \phi) \\
\frac{\alpha_1(1-y)}{\alpha_2 y} + \frac{\alpha_1(1-y)}{\alpha_2 y}
\end{array} \right.
\]

(2)

where \(N\) is the number of atoms in the crystal, \(y = \frac{1}{2} \frac{c}{k}\), \(c = \frac{\hbar}{2k(1-y)}\) is the minimum momentum transferred to the nucleus in units of \(mc\) (\(m_0\) is the rest mass of the electron). \(Z\) is the atomic number and \(e\) the electron charge.

It results

\[
\mathcal{V}_1 (\theta, \phi) = \frac{\lambda_0^2}{N} \frac{\pi}{\Delta} \frac{\sum y^2 \delta^{l} \left[ \frac{\mathcal{V}_2 \mathcal{V}_3}{\delta^2} \right]^2}{\Delta + 2 \left[ \delta^2 \right]^2 + y^2 \left[ \delta^2 \right]^2 + \left( \delta^2 \right)^2}
\]

\[
\mathcal{V}_2 (\theta, \phi) = \frac{\lambda_0^2}{N} \frac{\pi}{\Delta} \frac{\sum y^2 \delta^{l} \left[ \frac{\mathcal{V}_2 \mathcal{V}_3}{\delta^2} \right] \left( \delta^2 \right)^2 \delta^2}{\Delta + 2 \left[ \delta^2 \right]^2 + \left( \delta^2 \right)^2}
\]

\[
\mathcal{V}_3 (\theta, \phi) = \frac{\lambda_0^2}{N} \frac{\pi}{\Delta} \frac{\sum y^2 \delta^{l} \left[ \frac{\mathcal{V}_2 \mathcal{V}_3}{\delta^2} \right] \left( \delta^2 \right)^2 \delta^2}{\Delta + 2 \left[ \delta^2 \right]^2 + \left( \delta^2 \right)^2}
\]

In these formulas \(\Delta\) is the volume of the fundamental cell, in units of \(\lambda_0^2\) (\(\lambda_0 = 2\pi \lambda_0\), \(\lambda_0\) is the Compton wave length of the electron), \(N\) is the number of fundamental cells in the crystal, \(S\) is the Bragg structure factor, and \(\mathbf{h}\) is a reciprocal lattice vector; \(g_j (j = 1, 2, 3)\) are its components in the reference frame \(\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\), and have the form

\[
g_j = \frac{2\pi}{\mathbf{b}_j} h_j \quad (h_j \text{ integer number});
\]

\(\alpha\) is the dyhedral angle between the planes \((\mathbf{h}_1, \mathbf{h}_2)\) and \((\mathbf{h}_1, \mathbf{h}_3)\).

The other quantities in Formulas (2) and (3) are defined and given by 'Überall'.

The triple sum in Formulas (3) is extended over the triplets \(\mathcal{T} = (g_1, g_2, g_3)\) satisfying the relationship

\[
\mathcal{T} = (g_1 + g_2 \cos \alpha + g_3 \sin \alpha) \mathcal{Q} \geq \mathcal{D}
\]

Note that \(S\) is defined with reference to the fundamental axes of the crystal, whilst \(g_j\) are referred to the axes \(\mathbf{b}_j\), not necessarily coincident with the fundamental ones. The introduction of the axes \(\mathbf{b}_j\) is convenient as the coherence effect takes place only at
small angles \( \theta \) between the photon direction and a lattice axis.

If we put \( \alpha_1 = 0 \) and \( \alpha = 0 \) (or \( \pi/2 \)), we find again the functions given by us in reference 4 (or 1) and by Überall\(^5\) for bremsstrahlung.

Let us consider a diamond-like crystal. We have \( N_o/N = 1/8 \) and \( \Delta = a^3 \), \( a \) being the edge of the fundamental cube in units of \( \lambda_c \).

The structure factor is given by

\[
S = \left\{ 1 + \exp \left[ i \pi (n_1 + n_2) \right] + \exp \left[ i \pi (n_2 + n_3) \right] + \exp \left[ i \pi (n_3 + n_1) \right] \right\}
\]

where \( (n_1, n_2, n_3) \) is any triplet of integers numbers.

Let us assume \( \mathbf{v}_1 = [110] \), \( \mathbf{v}_2 = [110] \), \( \mathbf{v}_3 = [001] \); it follows \( a_1 = a_2 = a/\sqrt{2} \), \( a_3 = a \).

The structure of the reciprocal lattice planes \( h_1 = 2l_1 \), \( l_1 \) being any integer number, is given in Fig. 1a of reference 4. The structure of the planes \( h_1 = 2l_1 + 1 \) is about the same, with the only difference that now the axis \([110]\) intersects these planes in the centre of a rectangle, the vertices of which are in the nearest four points having \( |S|^2 = 64 \) (Points enclosed in the circles in Fig. 1a of reference 4).

Numerical calculations were made for a diamond crystal at room temperature, assuming \( \Lambda = 129 \), \( a/2\pi = 147 \), \( \beta = 61.6 \), \( \alpha = \pi/2 \), with the choice of the preceding axes. These conditions are very convenient in order to obtain a large asymmetry ratio.

Some results are displayed in Fig. 1 as a function of the angle \( \theta \) between the direction of the primary photon and the axis \([110]\) for symmetrical pairs (\( y = 1/2 \)) and a photon energy \( k = 3 \text{ GeV} \). The continuous curve represents the asymmetry ratio \( R_{\text{c}} \) given by Formula (1) (read at the right scale); the dashed curve represents the cross section from unpolarized photons given by the second of Formulas (2) (read at the left scale). One can see that the largest value of \( R_{\text{c}} \) is 19.0%, for \( \theta = 49.5 \text{ mrad} \).

Let us choose now at each energy \( k \) the best value of \( \theta \), i.e., the value \( \theta_M \) for which the largest value \( R_{\text{cM}} \) of \( R_{\text{c}} \) is obtained.
$R_{CM}$ and $\Theta_{CM}$ are represented as a function of $k$ in Fig. 2. (The continuous curve and the left scale are for $R_{CM}$; the dashed curve and the right scale are for $\Theta_{CM}$.)

The contribution of the planes $h_1 \neq 0$ is negligible down to $k \geq 1 \text{ GeV}$. For smaller energies this contribution becomes important, but the value of $R_{CM}$ remains very small, as in the case of the plane $h_1 = 0$.

In order to measure the linear polarization of a photon beam (not necessarily monochromatic but with $k \geq 1 \text{ GeV}$) one can use a simple apparatus, similar to that employed by us in another work on coherent pair production. Let the directions of the photons be parallel and let the beam hit a thin analyzer diamond crystal at room temperature contained in a pair spectrometer. Let us choose a particular energy $k$ by fixing the spectrometer magnetic field and detect the symmetrical electron pairs emitted at any angle by means of a conventional system of detectors. The numbers of symmetrical pairs detected depends on the cross sections $d\sigma_{||}$ and $d\sigma_{\perp}$ already calculated and on the amount and direction of the photon beam polarization. Let us arrange the analyzer with the axis $[110]$ at an angle $\theta$ with $\vec{E}$ slightly larger than $\Theta_{CM}$ and place the plane of the axes $[110]$ and $[001]$ parallel to $\vec{E}$. Then rotate the analyzer around the direction of $\vec{E}$ until the largest counting rate is obtained. In this situation the polarization of the photon beam is parallel to the plane determined by $\vec{E}$ and $[110]$. Let $N_{||}$ be the number of symmetrical pairs detected per fixed number of photons. Then rotate the analyzer by 90° around $\vec{E}$; let $N_{\perp}$ be the correspondent numbers of pairs.

The quantity which is meaningful for the experimental possibilities is the relative difference between $N_{||}$ and $N_{\perp}$. It is given by

$$d = \frac{N_{||} - N_{\perp}}{N_{\perp}} = \frac{2R_{CM}P}{1 - R_{CM}P},$$

$R_{CM}$ being given in Fig. 2 and $P$ being the polarization of the photon beam, given by

$$P = \frac{n_{\perp} - n_{||}}{n_{\perp} + n_{||}}.$$
\( n_{\perp} \) and \( n_{\parallel} \) are the number of photons in the beam with the polarization perpendicular and parallel, respectively, to the plane \((\vec{E}, [110])\).

The measurement offers no difficulty whenever \( k \gg 1 \text{ GeV} \); below this value \( R_{\text{CM}} \) is too small, so that, with the usual values of \( P, D \), becomes smaller than \( 5\% \), which is the lowest measurable value. From Formula (4) we obtain

\[
P = \frac{1}{R_{\text{CM}}} \frac{\Delta}{2 + \Delta}.
\]

In order to obtain precise values of \( P \) it is necessary to measure \( D \) with a good statistical accuracy and to correct \( R_{\text{CM}} \) for the pair production in the field of the electrons, for the unavoidable angular divergence of the photon beam, and for the energy acceptance of the detectors. This is a straightforward matter in each particular experiment and we do not take it into account.

The error propagated on \( P \) by the statistical error of the counts is

\[
\mu_P = \frac{2}{R_{\text{CM}}(2 + \Delta)} \sqrt{\frac{1/\Delta}{(2 + \Delta) N_{\perp}}}
\]

Owing to the large values of the pair production cross sections, it seems reasonable to collect a number of counts \( N_{\perp} \approx 2 \times 10^4 \), provided that the photon beam intensity is not too small. Let us consider the favourable case \( D = 50.0\% \). From Formulas (5), (6) we obtain

\[
\frac{\mu_P}{P} = 2.2\%.
\]

As an unfavourable case we consider \( D = 5.0\% \). We have

\[
\frac{\mu_P}{P} = 20.0\%.
\]

We notice that this type of measurement is very much simpler than the measurement performed in the previous work, in which we had to operate an angular selection of one particle in the pair.

In reference 1 we gave the calculated polarization of \( k=150-\text{MeV} \) bremsstrahlung photons from \( E_1 = 1 \text{ GeV} \) electrons in a single diamond crystal.
We believe it is useful to present here some new numerical results for larger energies.

Let us consider a perfectly parallel collimated and monoenergetic electron beam whose energy is $E_1$. This beam hits a thin diamond crystal at room temperature at an angle $\theta_0$ with the crystal axis [110]; the direction of the electrons is parallel to the plane determined by the axes [110] and [001]. The entire bremsstrahlung X-ray beam has a polarization which we calculated by means of the formulas given in reference 1.

Among all the possible values of $\theta_0$, let us choose the angle $\theta_{\text{opt}}$ for which the polarization has its maximum value $P_M$ at a given photon energy $k$. The results are given in Fig. 3 as a function of $x = k/E_1$ (the fractional energy of the photons) for three representative values of $E_1$ (1, 6, 40 GeV), corresponding to existing or future accelerators. In the figure are represented the values of $P_M$ (read at the left scale) and $\theta_{\text{opt}}$ (read at the right scale).

Experimental difficulties arise for $E_1 = 40$ GeV, if one is interested in photon energies $k \ll 20$ GeV. In this case $\theta_{\text{opt}} \approx 1$ mrad, a very small angle.

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2. An alternative method based on photon absorption in a thick crystal converter was proposed by N. Cabibbo, G. Da Frate, G. De Franceschi, and U. Mosco, Phys. Rev. Letters 2, 270 (1962).
6. In reference 4 we called structure factor the quantity $(N_0/N)^{1/2}$.
7. H. Überall, Phys. Rev. 102, 1055 (1956)

9. We consider the crystal thin when the mean square angle of scattering of the pair electrons is much less than the vertical angle of acceptance of the detectors.
Figure captions.

Fig. 1 - Electron pair production from \( k = 3 \)-GeV linearly polarized photons in a diamond crystal at room temperature. The abscissa is the angle \( \phi \) (in milliradians) between the primary photon and the crystal axis \([110]\). The direction of the photons is parallel to the plane of the crystal axes \([110]\) and \([001]\). The continuous curve represents in \( \% \) the asymmetry ratio given by Formula (1) (read at the right scale) and the dashed curve represents the cross section from unpolarized photons (read at the left scale), given by the second of Formulas (2), for symmetrical pairs.

Fig. 2 - The abscissa is the energy \( k \) of the photons. The continuous curve gives the maximum value \( R_{\text{GM}} \) of the asymmetry ratio (the scale is at the left and is in \( \% \) and the dashed one gives the angle \( \Phi_{\text{GM}} \) (the scale is at the right and is in milliradians) for which \( R_{\text{GM}} \) is obtained. The other conditions are the same as in Fig. 1.

Fig. 3 - Polarization of the entire bremsstrahlung beam from electrons of three different energies \( E_i \) in a thin diamond single crystal at room temperature. The direction of the electrons is parallel to the plane of the crystal axes \([110]\), \([001]\). The abscissa is the fractional energy of the photons \( x = k/E_i \). The continuous curves represent the maximum value \( P_{\text{H}} \) of the polarization (the scale is at the left and is in \( \% \), as computed by means of the formulas given in reference 1, and the dashed curves represent the angle \( \Phi_{\text{b,H,M}} \) for which \( P_{\text{H}} \) is obtained (the scale is at the right and is in milliradians). \( \Phi_{\text{b,H,M}} \) is an angle between the direction of the primary electrons and the crystal axis \([110]\).