G. Sacerdoti: THE APPLICATION OF LIOVILLE'S THEOREM TO THE MOTION OF CHARGED PARTICLES IN TIME DEPENDENT ELECTROMAGNETIC FIELDS.

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G. Sacerdoti: THE LIOVILLE'S THEOREM APPLIED TO CHARGED PARTICLES MOTION IN PERIODIC IN TIME ELECTROMAGNETIC FIELDS.

As it is known the motion of particles in the phase space $x, y, z, p^x, p^y, p^z$ does not change the initial uniform density $f_0 = \frac{\partial N}{\partial p^x \partial p^y \partial p^z}(1)(x)$ if the only acting forces are electromagnetic.

We divide the phase space in two domains $D_1, D_2$. In the first $D_1$ there are the particles which remain at finite (in momentum and in coordinates); in the other $D_2$ the particles whose trajectories go to infinite.

The shape of $D_1$ and $D_2$ versus time is not variable if the electromagnetic field is stationary; otherwise it is in general variable by time.

To "capture" particles emitted from a source

(x) - That is not true when there are present radiation losses because in that case there are present also the internal forces of particle that forbid the particles to disintegrate.
these two conditions must be satisfied:
a) the particles have to remain at finite
b) the particles that originally are in the "source" must no anymore pass in the region where the source is. The source is represented as a portion of volume in the domain $D_1$.

If the volume of $D_1$ is finite (that is in the most of examples we have in practice) it is easy to demonstrate that capture is no possible also in periodic electromagnetic fields.

We call $V_1$ the volume of $D_1$. The total number of particles that remains always in $V_1$ is $N_0 = \int_0^1 \mathcal{S} \, dV$. The volume of the source $S$ be $V_S$ and the particles in the source region at time $t_0$ be $\int_{t_0}^{t_0 + \tau} \mathcal{S} \, dV$.

We consider the distribution of particles at time $t_0$, $t_0 + \tau$, $t_0 + 2 \tau$, ..., $t_0 + n \tau$ ($\tau$ period of electromagnetic fields).

If there were capture the particles at time $t_0$ in $V_S$ should pass in $V_1$ after a time $\tau < \kappa \tau$ has elapsed.

After the time $t_0 + n \tau$ if $u_0 > u_r = \frac{V_r}{V_S} \kappa$, in the volume $V_1 - V_S$ we should have a number of particles greater than $\frac{u_0 \mathcal{S} V_S}{\kappa} > \int_{t_0}^{t_0 + \tau} \mathcal{S} \, dV$ and the new density of particles should have been increased and this does not agree with the Lioville's theorem.

So it is impossible to have capture by periodic electromagnetic field. A little more complicated demonstration shows that the assumption $D_1$ of finite volume is not necessary for our conclusion.

**Bibliography**