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**NUMERICAL SIMULATIONS FOR THE ARES SUPERCONDUCTING LOW EMITTANCE INJECTOR**
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1 - Introduction

A big effort has been done for some years in several laboratories to design and build electron sources capable to deliver a few hundreds of Ampères beams with a transverse normalized emittance in the range 5+10 mm mrad. We recall the Los Alamos RF-gun experiment, where beams of 100+150 A peak current with a norm.emittance, en, of 20+30 mm mrad were measured(1,2), and the Brookhaven S-band laser driven RF-gun, with an anticipated output beam current of 150 A at $\varepsilon_n = 7$ mm mrad(3). In the context of the ARES project(4) one of the primary goal is the production of high quality beams at high repetition rate (from few kHz up to tens of MHz) in order to make possible both the X-UV FEL(5-8) experiments and accelerator physics experiments in the main-streams of the Φ-B Factories (LINAC-ring collider, etc.) and TeV colliders(9). The low RF frequency (500 MHz) selected for the SC cavities of the LINAC constitutes, in this sense, a basic choice to assure a good beam quality (lower RF and wake-field induced effects, i.e. lower deteriorations of the beam dynamics properties through the acceleration).

The injector system for the LINAC must be compatible to this choice. Therefore, a design based on a superconducting RF gun equipped with a laser driven photocathode(10) becomes, in our opinion, mandatory to satisfy the requirements on the repetition rate and on the beam quality. A
magnetic compressor must be foreseen to grow up the peak current to the level of some hundreds of Ampères: the current delivered by the SC RF gun cannot be in fact higher than a few tens of Ampères if the norm. emittance at the injector exit must be kept below a few mm mrad, as will be shown later. It is important to notice that the gun must provide excellent beam properties not only in the transverse phase space (i.e. a low transv. emitt.) but also in the longitudinal one, since non-linearities in this space affect the full exploitation of the magnetic compression and limit the peak current at the exit of the injector.

2 - Basic Theory

We recall now and discuss the main effects which govern the beam quality behaviour from the cathode up to the injector exit. All the process listed below tend to heat the electron bunch, growing up its transverse temperature and increasing the normalized transverse emittance from its initial value at the photocathode. The design criteria for the gun cavity geometry as well as the requirements on the cathode and on the laser pulse characteristics will be dictated by the preservation of the beam quality through the whole injection system.

Transverse emittance on the photocathode.

According to the literature\(^{11,12}\) we will define the normalized (or invariant) transverse emittance, \(\varepsilon_n\), of a bunch - referred to the transverse phase space \((x,p_x)\) - as given by the quantity:

\[
\varepsilon_n = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \langle p_x^2 \rangle - \langle x p_x \rangle^2
\]

where: \(\rho(x,p_x,z,p_z)\) is the momentum-position distribution and

\[
\langle x^2 \rangle = \frac{\int x^2 \rho(x,p_x,z,p_z) \, dx \, dp_x \, dz \, dp_z}{\int \rho(x,p_x,z,p_z) \, dx \, dp_x \, dz \, dp_z}
\]

\[
\langle p_x^2 \rangle = \frac{\int p_x^2 \rho(x,p_x,z,p_z) \, dx \, dp_x \, dz \, dp_z}{\int \rho(x,p_x,z,p_z) \, dx \, dp_x \, dz \, dp_z}
\]

\[
\langle x \, p_x \rangle^2 = \left(\frac{\int x \, p_x \rho(x,p_x,z,p_z) \, dx \, dp_x \, dz \, dp_z}{\int \rho(x,p_x,z,p_z) \, dx \, dp_x \, dz \, dp_z}\right)^2
\]
It is well known that for a Kapchinsky-Vladimirsky distribution $4\pi \varepsilon_n$ gives the emittance associated to 100% of the distribution. Under the approximation that all the photo-electrons emerge from the cathode surface with the same energy (i.e. neglecting the straggling inside the cathode) and supposing that this energy is given by the difference between the laser photon energy and the work function of the cathode material, $\varepsilon_n$ at the cathode surface will be given by the formula:

$$\varepsilon_{n\text{-cath}}[m\text{ rad}] = \sqrt{\frac{W}{3 m_0 e^2}} \cdot \sigma_z[m]$$

where: $W = h\nu_{\text{laser}} - W_1 [\text{eV}]$ and $W_1$ is the $e^-$ work function.

We have considered a laser pulse with a double gaussian distribution in radius and time (of widths $\sigma_r$ and $\sigma_t$ respectively). To minimize this quantity one can decrease the laser spot, but the limit on the maximum current density delivered by the cathode (typically 500 A/cm$^2$) is quickly reached, giving a minimum laser spot radius of the order of 1 mm if some tens of Ampère have to be extracted from the cathode. The "tunable lasers" (see below for details) would be surely quite useful in decreasing the photon energy just above the work function of the cathode material, in order to minimize the starting energy of the photoelectrons: however that energy difference cannot be pushed below a few tenths of eV, since the transit time of the photoelectrons through the cathode thickness must be less than a few picoseconds to avoid an important bunch lengthening in the photo-emission process. It follows that the typical values of $\varepsilon_n$ at the cathode ranges between .8 up to 2 mm mrad.

The normalized brightness, $B_n$, is usually defined as the ratio between the peak current, $I$, of the bunch and the square of $4\pi \varepsilon_n$:

$$B_n = \frac{I}{(16 \pi^2 \varepsilon_n^2)}$$

Such a quantity gives a figure of merit for the current density of the beam since it accounts not only for the beam transverse size but also for its divergence, setting up an estimation of the beam current per unit surface and per unit solid angle. An increase of the norm. emittance gives rise - being square-weighted in the formula - to a serious deterioration of the beam brightness.

**RF linear effects**

The radio-frequency field resonating inside the gun cavity, usually a TM$_{010-\pi}$ mode of a multicell structure (as shown in Fig.1), acts on the electrons emerging from the photo-cathode with longitudinal and transverse forces which are time dependent. This implies a coupling between the transverse and the longitudinal phase space if the photo-electron bunch has a finite time length, "transverse" and "longitudinal" being referred to the symmetry axis of the cavity. In other words, the bunch layers emitted at different times (i.e. RF phases) from the cathode experience different
transverse kicks along the acceleration inside the gun cavity, giving rise to a typical transverse phase space at the gun exit as the one shown in Fig. 2. The different correlation of each segment of the phase space, corresponding to each layer, from the tail to the head of the bunch, causes an effective increase of the normalized transverse emittance\(^{13}\), since that is computed integrating all over the bunch density distribution, as earlier shown.

Later it will be shown that the emittance increase due to the linear RF effects is minimum at a given injection phase, i.e. at a specified incidence time of the laser pulse on the cathode. Moreover, it turns out that shorter bunch lengths (in unit of RF degrees) correspond to lower emittance increase: that is the main advantage of low frequency RF guns.

![Figure 1](image)

**Fig. 1** - The superconducting RF cavity of the electron gun: the one and a half cell structure has the same geometry as the LEP SC cavity. The iso-\(rH_\phi\) lines, positive in the first half cell and negative in the second cell, give the electric field lines typical of a TM\(_{010,\pi}\) accelerating mode. The bunch just emitted by the laser pulse striking the photocathode is shown in the middle of the first half cell.

**RF non linear effects**

For a finite bunch transverse size the off-axis electrons experience transverse forces due to the RF field which contains higher order components in the transverse (radial) coordinate. That causes a filamentation of the phase space distribution which corresponds to an effective increase of the normalized transverse emittance. Usually such an effect is not so high since the number of RF periods elapsed during the acceleration in the gun cavity is small. However, to minimize this effect is recommended to adopt a low ratio between the rms transverse size of the bunch and the iris diameter of the RF cavity: that still claims for low RF frequencies.

A proper shaping of the iris profiles\(^{2}\) is also useful. Nevertheless it can be shown that its effect is a minor one and so it will be investigated at the end of the optimization process, taking into account the requirements dictated by the SC cavity performances. The eventual reduction of the obtainable electric field on the cathode surface would be much worst than the marginal effect of a transverse field linearization.
Space charge effects

The forces due to the space charge associated to the bunch have transverse and longitudinal components which are intrinsically non-linear both in the radial and in the axial coordinate, with the exception of the ideal case of an infinitely long bunch with uniform density distribution. That causes a further increase of $e_n$, associated with the filamentation process produced by the space charge force in the transverse phase space, as sketched in Fig. 3. Since such a force decreases as $1/\gamma^2$ along the acceleration process\(^{(12)}\) (I is the peak current of the bunch) a natural way to minimize the momentum transfer from the space charge field to the electrons is to accelerate as fast as possible the photoelectrons just emitted by the cathode.

Fig. 2 - Typical transverse phase space of a cylindrical bunch accelerated into an axi-symmetrical RF field (no space charge field). The bunch length is a few RF degrees and the transverse emittance, calculated in the $(x,x')$ space, is $1.5 \times 10^{-7}$ m rad.

Fig. 3 - Similar as the previous figure, but with the space charge forces taken into account, for a bunch charge of 20 nC. The transverse emittance, in the $(x,x')$ space, is now $1.2 \times 10^{-5}$ m rad.

Once the bunch has achieved a relativistic energy the space charge forces are rapidly switched off. As a matter of fact, relativistic electrons in parallel motion can interact each other very weakly, because, due to the relativistic time dilation, they exchange photons on a time scale going asymptotically to infinite. So that, to minimize the emittance increase due to the space charge effects, one must maximize the peak electric field on the photo-cathode and/or accelerate longer bunches (i.e. lower currents for a given bunch charge). The low frequency domain is surely less favourable to obtain high peak field, but it allows to compensate for that by the use of longer bunches.

Wake field effects

The current associated to the electron bunch excites, during the transit through the RF gun, several higher order resonating modes of the cavity: the superposition of the whole set of excited modes is the wake field associated to the bunch\(^{(14,15)}\). The wake field has generally a solenoidal
component, given by the sum of all the resonating modes coupling to the bunch current (i.e. the modes which account for the induced current on the cavity walls), and an irrotational component, given by the electric field associated to the bunch charge and to the image charges on the cavity surface (which is actually equivalent to the space charge field, although usually this field does not account for the image charges).

Due to the axi-symmetrical configuration of the whole system (RF cavity + bunch current and charge) the multipole higher order modes are usually neglected in all the computations and the wake field of the bunch is assumed to be given by a sum of TM\textsubscript{0np} modes, i.e. monopole modes. Inside the gun cavity the wake field is therefore essentially longitudinal: the transverse wake field being negligible if multi-bunch effects are not taken into account. The wake field spectrum contains frequencies much higher than the fundamental one (which is associated to the accelerating field), with typical wavelengths of the order of the bunch length\textsuperscript{(16)}. The forces by which it act on the electrons of the bunch are therefore highly non linear, giving rise again to an emittance deterioration.

While the wake field components of low frequency strongly depends on the shape of the cavity profile, scaling for example as the inverse cube of the iris aperture\textsuperscript{(15)}, the high frequency components are quite insensitive to the actual shape of the cavity, being essentially a quasi-optical field propagating through an array of diffraction apertures (the iris). In Fig.4. we show, as an example, the propagation surfaces of the wake field associated to a bunch of 1 nC emitted by a photo-cathode and accelerated in the Brookhaven RF gun. Since the longitudinal wake field amplitude scales with the square of the frequency and with the bunch current, again the larger transverse sizes of the resonating structures together with the possibility to accelerate longer bunches clearly show the advantage of the low RF frequencies.

K.J. Kim has made an estimation\textsuperscript{(13)} of the transverse normalized emittance deterioration suffered by a bunch emitted and accelerated in a RF gun, under the following approximations:

- the electric RF field on the axis is a pure sinusoid (only the first spatial harmonic of a standard S-band multi-cell structure is taken into account)
- the RF field off axis is given by a first order expansion (only linear RF effects are taken into account)
- the space charge field of the bunch is computed neglecting charge images on the cavity walls (no wake field effects are considered); the charge density distribution of the bunch can be either gaussian or uniform.
- the emitted bunch is mono-energetic and the electrons trajectory are straight lines parallel to the axis (no beam envelope variation is taken into account through the gun, only momentum transfers, both transverse and longitudinal, are considered)
Fig. 4 - The wake-field associated to a gaussian bunch of 1 nC charge (200 A the peak current) accelerated through a Brookhaven RF-gun cavity. The relativistic contraction of the self-field is clearly visible (the bunch energy grows from zero up to 4 MeV).

The Kim result can be summarized in the following formula:

\[ \Delta \varepsilon_{TOT} = \Delta \varepsilon_{RF} + \Delta \varepsilon_{SC} + 0.2 J_x + \Delta \varepsilon_{RF} \Delta \varepsilon_{SC} \]

where \( J_x \) is a correlation factor (see ref. 13), \( \Delta \varepsilon_{SC} \) is the emittance increase due to the space charge forces and \( \Delta \varepsilon_{RF} \) is the emittance increase due to the RF linear effects, computed at the injection phase \( \phi_0 \) which minimizes the emittance increase. \( \Delta \varepsilon_{SC}, \Delta \varepsilon_{RF} \) and \( \phi_0 \) are defined - for a gaussian bunch - by the following equations:

\[ \Delta \varepsilon_{SC} [\text{m rad}] = \frac{5.7 \times 10^{-6} \cdot Q_b [\text{nC}]}{\sin \phi_0 \cdot E_0 [\text{MV/m}] \cdot (3\sigma_r [\text{m}] + 5\sigma_z [\text{m}])} \]

\[ E = E_0 \sin (\omega_{RF} t + \phi_0) \]

\[ \Delta \varepsilon_{RF} [\text{m rad}] = 0.69 \cdot E_0 [\text{MV/m}] \cdot \sigma_r \cdot (k_{RF} \cdot \sigma_z)^2 \]

\[ \left( \frac{\pi}{2} - \phi_0 \right) \sin \phi_0 = 5.1 \times 10^{11} \cdot \frac{E_0 [\text{MV/m}]}{k_{RF}} \]
The normalized transverse emittance at the gun exit will be given by a quadratic sum of the emittance at the cathode and of the total emittance increase:

\[ \epsilon_{n_{ax}} = \sqrt{\frac{2}{\epsilon_{n-cath}} + \frac{2}{\Delta \epsilon_{TOT}}} \]

Once the RF frequency is fixed, together with the maximum peak value expected for the electric field on the cathode, the value of the exit emittance is a function of the bunch charge and of its gaussian widths \( \sigma_f \) and \( \sigma_z \). It is worthwhile to adopt an operating diagram for the emittance increase which is able to give at a glance, once taken the bunch charge as a fixed parameter, the best bunch shape needed to achieve a given emittance value at the gun exit. Such an operating diagram consists essentially of an iso-emittance plot in which are traced the equi-emittance lines as functions of the bunch spot radius and length. In our case, taking 500 MHz for the RF frequency and 30 MV/m as the foreseen peak field on the cathode (corresponding to an accelerating gradient of about 13 MV/m), we have considered four possible bunch charges, namely .1, .5, 1 and 10 nC. Plotting the corresponding operating diagrams, as shown in Fig.5. (a,b,c,d), it can be easily seen that too short bunches are dominated by the space charge effect and too long bunches are dominated by the RF effects.

**Fig. 5a** - Iso emittance plot for a 100 pC bunch emitted and accelerated by a RF-gun: the RF frequency is 500 MHz, the peak field on the cathode 30 MV/m and the starting energy of the photo-electrons .2 eV. The plot shows the iso-emittance lines as functions of the bunch gaussian length (\( \sigma_z \)) and radial width (\( \sigma_f \)) in mm. The upper scale marks the laser pulse time-length (FWHM) in ps, while the arrows show the current at the gun exit. The values of the normalized transverse emittance, \( \epsilon_n \), written on the lines are in mm rad. The dot marked at the position (\( \sigma_z=6, \sigma_f=2 \)) shows the value of \( \epsilon_n \) computed by the PIC code ITACA.

**Fig. 5b** - Similar to the previous figure. The bunch charge is .5 nC. The emittance value computed by the PIC simulation is 5 mm rad at the position (\( \sigma_z=3, \sigma_f=2 \)). The line marked by \( Q_{max} \) fix the threshold on the maximum charge which can be extracted from the photo-cathode with a 30 MV/m peak field on the cathode surface. The region below the line requires bunch charge densities higher than the 10% of the maximum value (27 nC/cm²): in this region a significant bunch lengthening must be expected.
Fig. 5c - Similar to the previous figure. The bunch charge is 1 nC. The simulation value for the emittance is 4.2 mm mrad at the point (σx=7, σz=2). The region below the line marked by \( J_{\text{max}} \) requires current densities in excess of 500 A/cm².

Fig. 5d - Similar to the previous figure. The bunch charge is 10 nC.

The actual shape of the bunch must be chosen taking into account also the behaviour of the longitudinal phase space, which is not accounted in such an operating diagram. Long bunches (a few tenths of RF degrees) suffer in fact a curvature of the longitudinal phase space that strongly decrease the efficiency of the magnetic compressor. As will be shown later it can be preferable to use shorter bunches with lower charge content, taking constant the current: that usually gives rise to a higher peak current at the exit of the compressor.

3 - Computational tools

The numerical simulations (17,18) has indicated that, in all the present RF gun under design or in construction, the electron trajectory behaviour is essentially dominated by the RF field acceleration process: the field is so high (some tens of MV/m) that the photo-electrons emitted from the cathode becomes relativistic in a few cm beyond the cathode surface. Their transverse degree of freedom is quickly frozen and the forces due to the self-field of the bunch (always much lower in amplitude than the RF field in most cases under consideration) are not able to affect the transverse motion of the bunch electrons: the beam envelope is practically determined by the RF field transverse components. On the other side, the momentum transfers due to the self-field forces are of great relevance, especially in the radial direction, to determine the electron trajectory in the transverse
phase space. Hence, the behaviour of the transverse normalized emittance is dominated by the interaction of the bunch with its self-field, and of the self-field with the environment (i.e. the cavity surface). That naturally claims for a numerical simulation procedure able to describe the self-field propagation inside the cavity, coupled with the electrons dynamics in presence of the RF field and of the self-field.

That is, a numerical self-consistent procedure, of the PIC type, must be used if one needs to estimate with accuracy the behaviour of the emittance and all the other related beam dynamics quantities (energy spread, longitudinal emittance, etc.). In the Los Alamos case, the code MASK has been extensively run for the simulations, as in the Brookhaven case, where they used also the code PARLELA, able to take into account the space charge effects but not the wake-field effects, with a procedure which is not self-consistent. At Wuppertal, the new code TBCI-SF (a self consistent PIC code of the MAFIA group) has been adopted to simulate the electron dynamics inside their superconducting RF gun under construction. The importance of a good a priori numerical simulation of the whole injection system has been realized by all the involved laboratories. At the Milan University has been recently developed a new PIC code, named ITACA, by two of the authors (L.S. and A.P.). This code has been extensively checked against the numerical simulations made with PARMELA for the Brookhaven RF gun. Good agreement both between the two codes and the analytical estimations made by K.J. Kim has been met. The field equations and the integration algorithm used in the code have been reported elsewhere\(^\text{17}\). Here we recall only that the code is an axi-symmetrical one and it solves self-consistently a sub-system of the Maxwell + Newton Lorentz equations for cylindrical symmetric fields and sources. The bunch current is assumed to have radial and axial components and the self-field is assumed to be a monopole field (TM\(_{\text{mp}}\)). A special developed charge assignment algorithm allows to minimize the unphysical fluctuations in the driving term, and the fourth-order integration algorithm for the equations of the motion produces a great accuracy in the calculation of all the quantities related to the particle dynamics, especially the transverse and longitudinal emittance, the energy spread, the rms divergence, etc. An eigenvector finder, able to compute the TM\(_{\text{mp}}\) resonating modes of any axi-symmetrical structure, is included in the package, allowing to study the distribution of the accelerating RF field inside the gun cavity. At both Frascati and Milan laboratories we have installed also the Parmela-Superfish procedure, which allows to study the bunch dynamics in presence of only the RF and the space-charge field: we anticipate to study the magnetic compressor with such a procedure, which is adequate when the interaction with the environment is fairly negligible (i.e. inside a uniform beam pipe). The emittance deterioration due to the bunch length compression, although performed at relative high energy (7-10 MeV), has to be evaluated in order to find the minimum energy which gives rise to a tolerable emittance increase.
4 - Design criteria

On the basis of the previous considerations and looking at the experience with the numerical simulations done on various geometries, we can establish the following rules as the design criteria for the injector, submitted to the conditions imposed by the basic choices of the project on the RF frequency and the repetition rate.

The bunch charge.

With an expected peak field on the cathode surface of 30 MV/m the maximum charge density which can be extracted is 27 nC/cm². Taking as typical a laser spot radius, σ_r, of a few (2-3) mm, this means a maximum charge of about 8 nC. To avoid a serious lengthening of the extracted bunch one must limit the bunch charge to a few nC, typically 1 nC. Larger spots allow to extract a few tens of nC, making possible to generate high current (a few hundreds of A) beams with poor emittance (some tens of mm mrad).

Photo-cathode characteristics.

From the point of view of the beam dynamics the cathode should combine a short time response with a high current density disposable. The time response should be compatible with picoseconds pulse: since such a time scales like the thickness of the cathode and inversely with the velocity of the photo-electrons, the metallic photo-cathodes are preferable, but the semiconductor photo-emitting materials seem better both for the emitted current density and for the quantum efficiency. As earlier mentioned, the availability of tunable lasers in the visible and ultraviolet region should overcome the problem of metallic cathodes which have a higher work function: independently from that, the photon energy can be adjusted at a level just above the threshold and the energy of the photo-electrons should be tuned as one needs.

Laser pulse shape and distribution.

The laser pulse of a single mode laser (as a mode-locked laser) is naturally gaussian distributed in time and in radius. As earlier shown, the time length (FWHM) needed for the possible beams should range between 10 and 40 picoseconds and the spot radius (intended as the gaussian width σ_r) should be tunable in the range 1.5-3.5 mm. Laser pulses uniformly distributed in radius and time could be interesting: the emittance at the cathode is fairly lower (a factor of two is gained if the spot radius is equal to the gaussian width) and the different distribution for the space charge forces should be investigated, as some preliminary simulations suggest.

RF cavity shape.

Beside the standard iris profiling required to minimize the non linear RF transverse components near the axis (Proc. ICFA) some other precautions must be taken in designing the RF gun cavity.
The cut-off tubes at the gun-exit are quite critical in determining the fringing field distribution: the transverse emittance modulations are actually very sensitive to such a distribution as the injection phase which minimizes the RF emittance increase. One is of course interested to achieve the highest possible injection phase, in order to minimize the space charge contribution of the emittance increase: in general sharper fringing field distribution give higher injection phases but also higher RF induced emittance increases, since they require a lower cut-off tube radius. Due to the wake field induced effects all the beam pipe discontinuity in the cut-off tube must be as smooth as possible. Curve photo-cathodes should be investigated: a local increase of the electric field on the cathode, achieved with a proper convex shape of the cathode surface, is surely a benefit in reducing the space charge induced emittance increase. Unfortunately the highly non linear RF components generated on such a surface tend to lower this beneficial effect: careful simulations are needed to tailor the best shape for the cathode surface.

Number of cells.

A high number of cells in the RF gun cavity, which works in the π mode, would be desirable. The beam envelope is in fact better controlled and the rms divergence, which is maximum at the gun exit if the injection phase has been selected for the minimum emittance, is naturally damped by the successive focussing defocussing action of the RF field in the cells. For the same peak field on the cathode the bunch energy at the exit is higher, fact that guarantee a lower emittance increase during the magnetic compression. Unfortunately the reliability and the possibility to achieve a peak field on the cathode as high as possible is surely larger for a simpler RF structure, since in a Superconducting RF cavity the maximum achievable peak field is fairly inversely proportional to the cavity surface. From this point of view few cells work better! A good trade off between these antithetical requirements seems to be a cavity with one cell and a half, but also two cells and a half worth the work of a careful investigation.

5 - Preliminary design

Results of the numerical simulations

In designing the RF cavity of a superconducting electron gun one must satisfy, besides the criteria listed above (concerning mainly with the beam dynamics), all the requirements on the cavity shape needed to make a superconducting RF cavity operating with reliability at the maximum accelerating field.

We recall here just the main constraints: the rectangular (pill-box like) shape must be avoided due to the problems caused by the multipacting, the main and the HOM coupler cannot be placed on the inductive part of the cavity (due to the presence of the equatorial welding), the iris
profile must be designed with the main purpose of minimizing the ratio between the maximum magnetic field on the equator and the accelerating field.

The suggestion to avoid re-entrant cavities, which gives surely severe problems in the polishing operations of the cavity, must be also taken into account. In our case some main consequences follow from such constraints:

- enough space must be left free on the cut-off tubes to room the main coupler and the high order mode coupler: that implies a minimum radius and length of the cut-off tubes. As reported elsewhere in this paper the suggested values are: 85 mm for the minimum radius and 150 mm for the minimum length.

- the cells must be coupled in the pi-mode. That implies a minimum value for the coupling coefficient beta (usually a few percent): the thickness of the iris and its aperture must be properly fixed.

- since we are interested to obtain the maximum peak field on the cathode the iris profile must be shaped in order to achieve the minimum ratio between the maximum electric field on the boundary (usually on the iris) and the field on the cathode.

On the basis of the considerations listed above it comes clearly out that the trade off between the beam dynamics requirements and the RF superconductivity requirements is a quite complicate task. To make a preliminar attack to the problem we decided to choose a cavity geometry already tested and in operation and to cheek the peformances of such a geometry from the point of view of the beam dynamics. That choice has the advantage that the cavity geometry is fully compatible to an RF superconducting operation: each of the successive corrections to the geometry, needed to optimize the beam dynamics properties, can be checked if consistent with the RF superconductivity design criteria. The selected geometry is that one of the superconducting LEP cavity with only one cell and a half: the first half cell allows to put the photocathode in the center of its flat wall and the second cell is already compensated to resonate at the same frequency of the first half and, at the same time, ready to room inside the cut-off tube the main coupler and the HOM coupler. All the geometry is of course scaled down to resonate at 500 MHz. In all the simulations the peak field on the cathode has been taken at 30 MV/m, corresponding to an average accelerating field of about 13 MV/m. Such a field is the 30% higher than the nominal expected field of the ARES linac cavities: that seems to be a reasonable guess for a single dedicated structure of only two cells. The accelerating mode TM\(_{010-\pi}\) is presented in Fig.1, as computed by the code RELCAV, which is a part of the ITACA package. The energy stored in the cavity is about 86 J, with a R/Q of about 210 Ohm.

The beam dynamics simulations have been carried out for three different values of the bunch charge: 100 pC, .5 nC and 1 nC. The results are summarized in Table I. We note that the
injection phases are different: that causes the slight variations of the energy at the exit of the gun. The injection phases correspond to the minimum value for the transverse emittance at the exit: their variation is due to the different time lengths selected for the laser pulse.

<table>
<thead>
<tr>
<th>Table I - Preliminary SC Gun Results</th>
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<tbody>
<tr>
<td>Bunch charge [nC]</td>
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<tr>
<td>RF injection phase [Deg]</td>
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<tr>
<td>Laser spot (σr) [mm]</td>
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<tr>
<td>Laser length (2σl) [psec]</td>
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<tr>
<td>Output energy [MeV]</td>
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<tr>
<td>Energy spread (rms) [KeV]</td>
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<td>Bunch radius (rms) [mm]</td>
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<td>Bunch length (rms) [mm]</td>
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<tr>
<td>Bunch divergence (rms) [mrad]</td>
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<td>Bunch transv. norm. emiss. [m rad]</td>
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<tr>
<td>Bunch long. norm. emiss. [m rad]</td>
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<tr>
<td>Peak curr. (no mag. comp.) [A]</td>
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<tr>
<td>Peak current (mag. comp.) [A]</td>
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The behavior of the bunch energy and energy spread are presented in Fig.6 for the case of the .5 nC bunch, while the rms radius and length of the bunch are shown in Fig.7. The same quantities for the other two cases (100 pC and 1 nC) are quite similar. The rms divergence and radius at the exit of the gun are fully compatible with the expected acceptance of the magnetic compressor transport system. As deduced from Table I, the bunch lengthening is really negligible: the bunch length is actually slightly larger than the laser pulse length. Indeed, the bunch charge extracted in all the selected cases is much less than the maximum of 27 nC/cm²: for the 10 nC case some lengthening must be expected.

The current densities distribution of the bunch are shown in Fig.8: the gaussian shape in the longitudinal direction is conserved, while in the radial direction the axial current density shows a slight transition to a linear distribution, with some slight fluctuations. The radial current density is characterized by a gaussian shape longitudinally and by a Maxwellian shape radially (with a peak centered at about r=4 mm, which is fairly the same value of the rms radius of the bunch at z=317
mm). The associated self-field (space charge field + wake field) is visible in Fig. 9, where the total (RF + self field) electric field components are plotted as functions of z for different radii (from the axis up to the radius \( r = 6.5 \) mm). Only the radial component of the self field is clearly visible over the RF field, since in this region of the cavity (the middle point of the second cell) the longitudinal RF electric field is at its maximum, while the radial component is crossing a zero-point. The smoothness of the bunch self-field is the best prove of the absence of unphysical fluctuations in the driving-source term.

\[ \text{Fig. 6 - Bunch energy (left scale, solid line) and relative rms energy spread (right scale, dashed line) behaviors during the acceleration of a 0.5 nC bunch in the SC RF-gun. The final energy is 7.3 MeV and the final energy spread (rms) is \( \pm 30 \) Kev (4 % of the final energy). On the diagram, the boundary profile of the gun cavity is also plotted.} \]

The transverse emittance is plotted in Fig. 10 as a function of the bunch centroid z-position along the acceleration, for the case of the 100 pC bunch. While the transverse emittance (solid line marked by \( \varepsilon \)) is naturally damped by the growing of the gamma-factor, the normalized transverse emittance (which should be invariant in absence of self-fields and Rf-field couplings) exhibits large fluctuations. As earlier explained, such fluctuations are due to the coupling between the transverse and the longitudinal motion, which is given by the previous mentioned linear RF effects: a proper injection phase guarantees that the emittance at the gun exit corresponds to a minimum in the fluctuations.

On the contrary the slow growing of the minima is due to both non linear RF effects and self-field (space charge and wake-field) effects. To better visualize the effect of the self-field on the normalized emittance we plotted, in the same figure, the dashed curve marked by \( \varepsilon_{\text{core}} \), which
represents the normalized transverse emittance associated to the central layer of the bunch. That consists of a thin layer centered around the bunch centroid and extending radially along the whole bunch transverse size: the axial thickness of such a layer is tailored in order to contain roughly the 10% of the bunch charge (for a gaussian distribution ±σ/8). Due to its small extension in z, the linear RF effects on the central layer are negligible: any increase of its normalized transverse emittance must therefore caused by the self-field effects (and for a little amount by non linear RF effects). The comparison between Fig. 10 and Fig. 11, in which the emittance curves for the 1 nC bunch are plotted, is in this sense really clarifying. In the first case the self-field effects are small: indeed, εcore stays fairly constant all along the gun and the small increase of εn (from .8·10⁻⁶ mrad on the cathode up to 1.4·10⁻⁶ at the exit) is due almost completely to the RF effects. In the second case εcore grows quickly just after the bunch photo-emission from the cathode and then saturates at a constant level: recalling that the space-charge transverse field goes like 1/γ², this is just the behavior we may expect for the emittance increase. Here again the difference between εn and εcore gives an estimation of the increase due to RF effects. For reasons of space we don't present the plot for the .5 nC bunch: the emittance behavior is very similar to the 1 nC case. The charge indeed has been halved, but the laser pulse length too, giving a current slightly higher: the emittance at the gun exit comes out to be fairly larger (5.1 versus 4.2 mm mrad) than the 1 nC case. Since the best case, with respect to the output current, is the .5 nC bunch (as can be deduced by TABLE I), we present in Fig. 12 and 13 the transverse (x,x') phase space and the longitudinal phase space respectively.

![Graph showing bunch rms length, rms radius behaviors](image-url)

**Fig. 7** - Bunch rms length (left scale, solid line) and rms radius (right scale, dashed line) behaviors during the acceleration of a .5 nC bunch in the SC RF-gun. The values are in mm.
Fig. 8 - Bunch current densities. The solid lines represent, at the indicated radii, the axial current density associated to the bunch as a function of the axial coordinate (left scale, values in A/mm²). The dashed lines represent, at the indicated radii, the radial current density associated to the bunch as a function of z (right scale). The densities functions are plotted in the middle of the second cell: since the bunch is being focussed by the RF field transverse component, the radial current densities assume negative values.

Fig. 9 - Total electric field components plotted at the center of the second cell during the passage of the bunch. The dashed lines show the radial electric field component, at the indicated radii, as a function of z: the superposition of the RF radial field, which has a zero value at all radii for z=307 (symmetry point of the second cell), with the self-field of the bunch, whose behavior is gaussian as the source density distribution, is clearly visible. The linear behavior of the radial RF field, as a function of both z and r, away from the symmetry point and off axis is also evident. The solid lines show the behavior of the axial electric field. Since the longitudinal RF field has in this region a maximum, the self-field of the bunch (axial component) is hidden. The fields are in V/m.
Fig. 10 - Transverse emittance behavior during the acceleration of a 100 pC bunch in the SC RF-gun. The solid line (left scale) gives the transverse emittance [m rad] as a function of the average bunch position $z$ during the acceleration. The short-dashed line gives the normalized transverse emittance along the acceleration in m rad (right scale), while the long-dashed line gives the normalized transverse emittance associated to the central core of the bunch. See text for details. The final value of the normalized transverse emittance is $1.4 \times 10^{-6}$ m rad.

Fig. 11 - Transverse emittance behavior during the acceleration of a 1 nC bunch in the SC RF-gun. The solid line (left scale) gives the transverse emittance [m rad] as a function of the average bunch position $z$ during the acceleration. The short-dashed line gives the normalized transverse emittance along the acceleration in m rad (right scale), while the long-dashed line gives the normalized transverse emittance associated to the central core of the bunch. See text for details. The final value of the normalized transverse emittance is $4.2 \times 10^{-6}$ m rad.
The longitudinal one is slightly curved, especially on the bunch head: that was due to the RF acceleration process. The bunch head enters in fact the second cell too early, while the RF field is not yet reversed in sign, and gets a slight deceleration. This effect is more pronounced for longer bunches, giving rise to much more curved phase spaces. One must worry about such a curvature, because it decreases strongly the efficiency of the bunching action performed by the magnetic compressor\(^{(1)}\). Such a device, usually constituted by four dipoles, causes a rotation of the longitudinal phase space of fully relativistic beams (i.e. single velocity beams with a momentum spread) which leaves unaltered the momentum spread of the beam but decreases the bunch-length. Since the bunching effect is due to the path length differences for particles of different momenta, a correlation must exist between energy and phase, as clearly visible in Fig.13. The path length difference produced for particles with different momenta is represented in TRANSPORT notation by the R56 matrix element. In our case, applying a transport matrix with \(R56 = -0.73\) cm/percent, one obtains the longitudinal phase space at the exit of the compressor shown in Fig.14. The output current is about 200 A, with a rms bunch length reduced to .8 mm. As evident from the figure, the availability of a fully linear phase space would strongly increase the output current. On the contrary the output current of the other two cases are very low, as can be seen from TABLE I, due to the strong curvature of their phase space, which affects the efficiency of the magnetic compressor. A possible correction could be the insertion of a third harmonic cell after the gun cavity, needed to achieve a flat-topping of the RF field\(^{(1)}\): this possibility is still to be investigated.

**Fig.12** - Transverse (\(x[\text{mm}]\), \(x'[\text{rad}]\)) phase space at the exit of the gun for the case of the .5 nC bunch.

**Fig.13** - Longitudinal (\(z[\text{mm}]\), \(\Delta p_z\)) phase space at the exit of the gun for the case of the .5 nC bunch. The momentum deviation is in unit of \(m_o c\). The peak current in the bunch is 20 A.

From the numerical simulations it has been observed that higher injection phases, for a fixed gun cavity geometry, give rise to better longitudinal phase spaces: since higher injection phases (up to 90 degrees, which gives the electric field peak) reduce also the transverse emittance increase due to the space-charge forces, one is interested to push up the injection phase as much as possible. Unfortunately the injection phase which gives rise to the minimum emittance increase due to the
RF field is fixed, once chosen the laser pulse length, and its value depends on the cavity geometry. As an example, advancing the cathode position toward the first iris - as shown in Fig.15, where a modified geometry is plotted together with the energy gain and the energy spread behaviors - one can increase the minimum injection phase, but, at the same time, one increases the non linear RF effects on the emittance, decreases the output energy (about 5 MeV) and obtains a worse longitudinal phase space. The trade off between all such requirements will guide the cut-and-try job which will be developed in the near future to accomplish the best cavity geometry.

![Graph](image)

**Fig.14** - Longitudinal $(z[mm], \Delta p_z)$ phase space after the magnetic compressor for the case of the .5 nC bunch. The magnetic compression is achieved applying in the transport matrix, representing the magnetic compressor, $R56 = -.73 \text{ cm/percent}$. The peak current is 200 A.

![Graph](image)

**Fig.15** - Bunch energy (left scale, solid line) and relative rms energy spread (right scale, dashed line) behaviors during the acceleration of a .5 nC bunch in a SC RF-gun with a modified geometry. The final energy is 5. MeV and the final energy spread (rms) is $\pm 40 \text{ Kev} (8 \% \text{ of the final energy})$. On the diagram, the boundary profile of the gun cavity is also plotted: the main difference with the LEP geometry is the bump in the middle of the first half cell, carrying the photo-cathode. See text for details.
One solution which could be considered is the insertion of a third harmonic cell, after the gun cavity, to achieve a flat-topping of the RF field. This possibility is still to be investigated in detail, because, a preliminary study of this solution shows that the price paid is a severe degradation of the transverse phase space due to a significant beam pipe restriction (the iris of a 1500 MHz cell being three times smaller with respect to those of a 500 MHz cavity).

Another interesting idea, which gave very promising results, is to join the RF gun cavity with a single first harmonic cell fully decoupled from the previous cavity, as shown in Fig.16. If the length of the cell is not equal to half of the RF wavelength, some net effect (increasing or decreasing) should be met on the transverse emittance, since the transverse RF kicks will have a non-zero integral along the cell length.

That could permit to exit from the gun at a maximum of the emittance fluctuation and to recover such a maximum down to a minimum by means of the de-coupled cell, which can be phased independently from the gun cavity. The injection phase should be in this case a free parameter, to be optimized according to the other requirements on the longitudinal phase space: the goal is still the highest beam current at the lowest transverse emittance! An optimization work on the decoupled cell geometry is in progress. The higher energy of the beam, delivered by the gun+decoupled cell system, has the other advantage to lower the emittance increase in the magnetic compressor. Some recent numerical simulations have indeed shown that a beam of 12 MeV, magnetically compressed up to a current of 200 A, suffers a rms emittance increase of about 1.8 \(10^{-6}\) [m rad], due to the space-charge forces: that claims therefore for a beam energy at the compressor not lower than 10 MeV.

![Fig.16 - Boundary profile of the SC RF-gun attached to a single full-decoupled cell. See text for details.](image)

A part from the solution of the de-coupled cell, it would be of course possible to add a first section of four cell cavity to boost up the beam energy before to inject it into the magnetic compressor.
However, since the preliminary results on the basic geometry seem to be quite promising, especially in terms of the emittance achieved, we feel confident on the data given in TABLE II, which represents the major beam properties that can be reached with the proposed injector, for two typical bunch charge. The envisaged design is an optimization of that sketched above, including the first harmonic de-coupled cell. The beam parameters presented in Table II are, in our opinion, a good basis for the exploitation of significant experiments in the fields of collider-grade beams and of X-UV FEL's. The current output is that anticipated as the result of an optimized longitudinal phase space, while the normalized transverse emittance could be, in principle, lowered increasing the peak field on the cathode (or by means of some exotic idea).

<table>
<thead>
<tr>
<th>Table II. - SC Gun Expected Performances</th>
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<tbody>
<tr>
<td>Bunch charge</td>
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<td>Output energy</td>
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<td>Energy spread Δγγ</td>
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<td>Bunch transv. norm. emitt.</td>
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<td>Peak curr. (no mag. comp.)</td>
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<tr>
<td>Peak current (mag. comp.)</td>
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</table>

References


(4) ARES Design Study (The Machine), INFN Report, LNS-90/005(R).


