A FIBER OPTIC AIR BACKED MANDREL HYDROPHONE TO DETECT HIGH ENERGY HADRONIC SHOWERS IN THE WATER

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Abstract

We have studied the design of an air-backed optic fiber hydrophone. With respect to the previous models, this prototype is optimized to provide a band width sufficiently large to detect acoustic signals produced by high energy hadronic showers in water. After a discussion on the geometrical configuration and on the choice of the materials we evaluate the expected performances on the basis of simple analytical calculations.

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1 INTRODUCTION

The study of the highest energy region of the cosmic rays (UHECR) represents one of the most challenging fields of modern physics. It was recently pointed out that a substantial UHE neutrino component is produced by UHE protons and nuclei interacting with the interstellar medium [1]. This production is predicted to occur in the fireball models of the gamma ray bursts, in the active galactic nuclei and from the Greisen-Zatsepin-Kuzmin (GZK) effect [2] where the inelastic scattering of UHE protons off the Cosmic Microwave Background Radiation (CMBR) should suppress the proton flux to generate neutrinos from the decay pions. These neutrinos could, unlike UHE gamma rays and protons, reach us from distant and powerful sources, opening a deeper horizon for astrophysics.

In general the study of the UHECR is limited by the very low flux at these extreme energies. At $10^{18}$ eV typical values are $100 \text{ km}^{-2}\text{year}$ [3] and different methods have been proposed and realized to detect these events. Since 1957 A. Askarian [4] has suggested the detection of high energy showers by acoustic signals of thermoelastic origin due to the energy deposit in a dense medium. In particular, the hadronic shower is produced during the interaction of the Ultra High Energy (UHE) neutrinos ($10^{18}$ eV) in water and a large surface underwater hydrophone arrays has been proposed to detect these events [5].

In this paper we describe the prototype of a hydrophone based on the optic fiber technology. Unlike the conventional piezoelectric sensors, this hydrophone is completely passive being read out from the shore without any amplifier and ADC converter directly connected to it. This feature is particularly important in the design of large underwater arrays where reliability is of the utmost importance and the financial aspect determinant.

Fiber optic air-backed mandrel hydrophones have been developed since the 80s [6]. However these sensors have been mainly used for military applications where a bandwidth up to 5 kHz was required [7]. In our prototype we have investigated the possibility to increase this range in order to match the required 10 - 20 kHz interval [5] where the spectral maximum of the pressure signal from the hadronic shower is expected.

In the following we describe the principle and the design of the hydrophone. The realization of the prototype and the corresponding measurements in the air will be discussed in a forthcoming publication.

2 DESIGN PARAMETERS

A fiber optic air-backed mandrel hydrophone is obtained by wrapping a fiber, slightly strained during the winding process, into a thin-walled, end-capped, air-filled tube. Here the high responsivity is obtained because the circumferential strain induced in the tube by the pressure wave is transferred into the axial fiber strain.

In this design an ‘active’ cylinder is mounted outside a ‘passive’ hollow inner tube with sufficient clearance to provide air-backing as shown in fig.1: the pressure signal is then translated into a small variation of the length of the fiber in the coil. The coil is then spliced into the sensor arm of a Michelson interferometer to obtain an easy to fabricate and extremely sensitive hydrophone.

Different aspects have to be considered in the design of the hydrophone:
The static responsivity obtained from the evaluation of the stress, strain and displacement of a symmetrically pressure loaded cylinder.

The trade-off between the sensitivity and the maximum water depth at which the hydrophone can still operate.

The cylinder end-constraints and their influence on the responsivity.

The natural mechanical resonances which can appear at low frequencies and their variation from the air to the water.

The directionality which becomes important when the acoustic wavelength in water approaches the dimension of the hydrophone.

All these characteristics essentially depend on few parameters, namely the characteristics of the materials and in particular the Young Modulus $E$ and the Poisson ratio $\nu$, the radius $R$, the length $L$ and the thickness $t$ of the ‘active’ cylinder and the number of optic fiber coils. Most of the evaluations reported in the following paragraphs are obtained from the analyses of different Authors [8,9] based on the linear theory of elasticity applied to a cylindrical shell. A detailed analysis can be found in the PHD thesis of S. Knudsen [10] which has been the guideline of our analysis.

![FIG. 1: The cross section of an air-backed cylindrical optic fiber hydrophone](image)

### 3 THE STATIC RESPONSIVITY

This quantity which relates the interferometer performances to the external pressure $P$ is expressed as a phase responsivity $\frac{d\phi}{\phi}$. Only the hydrostatic pressure load is considered, the result being valid at low frequencies where the acoustic wavelength is long compared to the dimensions of the cylinder. The calculation assumes an infinite length cylinder and according to Love [8] determines the deformation in a pressure loaded cylinder and its propagation to the axial fiber strain.

The expression for $\epsilon_{\phi\phi} = \frac{d\phi}{\phi}$
reads
\[
\varepsilon_{\varphi\phi} = 0.71 \frac{1 - \nu^2}{E} \frac{R}{t} P
\]  

(1)

where \( t \) is the thickness of the tube, \( E, \nu \) are the effective Young Modulus and Poisson ratio and the coefficient 0.71 is used to take into account the damping due to the photoelastic effect [11]. \( d\phi \) is measured in \( 1/\text{Pa} \) or, for the range of acoustic intensities considered, in the units of \( \text{dB re } 1/\mu\text{Pa} \) (value in dB referred to 1 \( \mu\text{Pa} \)).

In this model the cylinder and the layers of optic fibers wrapped around it are considered as a unique isotropic medium and therefore effective values of \( E \) and \( \nu \) need to be used. Using the so called ‘rule of mixture’ [12] effective elastic parameters can be derived.

In our case we have at least four different material components: 1) the cylinder and, in the fibers, 2) the core/cladding part, 3) the coating and 4) the glue which is used to bond the fibers into the tube.

In table 1 we report the values of \( E \) (in GPa) and \( \nu \) for typical material of the cylinder (aluminium, ULTEM) and for the fiber.

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>Young Modulus (GPa)</th>
<th>Poisson Ratio</th>
<th>Density (gr/cm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>70</td>
<td>0.33</td>
<td>2.27</td>
</tr>
<tr>
<td>ULTEM 1000</td>
<td>3.3</td>
<td>0.44</td>
<td>1.28</td>
</tr>
<tr>
<td>ULTEM 2400</td>
<td>11.7</td>
<td>0.37</td>
<td>1.61</td>
</tr>
<tr>
<td>Core/cladding</td>
<td>72</td>
<td>0.17</td>
<td>1.43</td>
</tr>
<tr>
<td>Coating</td>
<td>1.1</td>
<td>0.17</td>
<td>1.2</td>
</tr>
<tr>
<td>Glue (typical)</td>
<td>0.014</td>
<td>0.43</td>
<td>-</td>
</tr>
</tbody>
</table>

**TABLE 1:** Specifications for some common materials used in the fabrication of the hydrophone

The effective elastic parameters of the hydrophone will then depend on the material and thickness of the cylinder, the cladding/coating diameters of the optic fiber (80/135 \( \mu\text{m} \) or 80/165 \( \mu\text{m} \)) and the number of layers.

As an example in table 2 we report the value of the responsivity for different possible configurations (here an 80/165 \( \mu\text{m} \) optic fiber is used).
<table>
<thead>
<tr>
<th>Configuration</th>
<th>Material</th>
<th>Radius (mm)</th>
<th>Thickness (mm)</th>
<th>N.layers</th>
<th>Responsivity (dB re rad/µPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Aluminum</td>
<td>8</td>
<td>0.5</td>
<td>3</td>
<td>-318</td>
</tr>
<tr>
<td>2</td>
<td>ULTEM 1000</td>
<td>8</td>
<td>0.5</td>
<td>3</td>
<td>-305</td>
</tr>
<tr>
<td>3</td>
<td>ULTEM 1000</td>
<td>25</td>
<td>0.5</td>
<td>3</td>
<td>-295</td>
</tr>
<tr>
<td>4</td>
<td>ULTEM 1000</td>
<td>8</td>
<td>0.5</td>
<td>6</td>
<td>-310</td>
</tr>
<tr>
<td>5</td>
<td>ULTEM 1000</td>
<td>8</td>
<td>1</td>
<td>6</td>
<td>-311</td>
</tr>
<tr>
<td>6</td>
<td>ULTEM 2400</td>
<td>8</td>
<td>1</td>
<td>5</td>
<td>-309</td>
</tr>
<tr>
<td>prototype</td>
<td>ULTEM 1000</td>
<td>8.5</td>
<td>1</td>
<td>5</td>
<td>-309</td>
</tr>
</tbody>
</table>

**TABLE 2:** The static responsivity for different geometrical configurations

From the table above the advantage of hard plastic material like ULTEM 1000 with respect to the metals like aluminium is evident: in fact, due to the value of the Young Modulus, in the same geometrical configuration ULTEM 1000 provides an additional sensitivity exceeding 10 dB (see configurations 1, 2).

In general, a large diameter of the hydrophone favours the responsivity and the low bending attenuation in the optic fibers but, as we shall see in chap.4, it reduces the bandwidth due to the mechanical resonances at low frequencies (see configurations 2, 3).

When using low Young modulus materials like ULTEM 1000, the number of layers of optic fibers becomes important as the stiffness of these layers is comparable or larger than the stiffness of the cylinder (see configurations 2, 4).

The effect of the finite length of the cylinder is discussed in detail in [10]. It gives the reduction of the responsivity of the hydrophone considering both the cases when the ‘active’ cylinder is bonded in the ‘passive’ support (see conditions 6.35 reported in [10]) or it is simply supported at the two ends (free). In table 3 we consider few cases where it is evident that this correction is more pronounced as the length of the cylinder decreases and becomes comparable to its diameter.

<table>
<thead>
<tr>
<th>Config.</th>
<th>Material</th>
<th>Radius (mm)</th>
<th>Thick. (mm)/layers</th>
<th>Length (mm)</th>
<th>Responsivity (dB re rad/µPa)</th>
<th>Responsivity (dB re rad/µPa)</th>
<th>Responsivity (dB re rad/µPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Infinite length</td>
<td>Free edges</td>
<td>bonded</td>
</tr>
<tr>
<td>2a</td>
<td>ULTEM 1000</td>
<td>8</td>
<td>0.5/3</td>
<td>40</td>
<td>-305</td>
<td>-305</td>
<td>-306</td>
</tr>
<tr>
<td>2b</td>
<td>ULTEM 1000</td>
<td>8</td>
<td>0.5/3</td>
<td>15</td>
<td>-305</td>
<td>-306</td>
<td>-308</td>
</tr>
<tr>
<td>3a</td>
<td>ULTEM 1000</td>
<td>25</td>
<td>0.5/3</td>
<td>10</td>
<td>-295</td>
<td>-301</td>
<td>-310</td>
</tr>
<tr>
<td>Protot.</td>
<td>ULTEM 1000</td>
<td>8.5</td>
<td>1.0/5</td>
<td>20</td>
<td>-309</td>
<td>-311</td>
<td>-313</td>
</tr>
</tbody>
</table>

**TABLE 3:** The finite length effect of the cylinder on the responsivity
4  MECHANICAL RESONANCES AND THE DYNAMICAL SENSITIVITY IN
AIR AND IN WATER

The air-backed mandrel in the hydrophone presents mechanical resonances which
can occur in the band width of the pressure signal. A resonance changes the phase and the
amplitude of the response of the hydrophone and it is therefore important to predict in
advance at what frequency the resonance peak will appear and, if a fairly uniform response
is desired, how the geometry has to be defined to avoid that a resonance occurs within the
band width.

The calculations for a thin walled cylinder can be found in [8] and [13] where the
displacements u,v,w respectively in the axial, circumferential and radial directions are
expressed in terms of n, the nodes of circumferential waves and m, the number of axial
half periods.

Only few modal resonances associated with pure radial motion will correspond to
extensional vibrations and therefore to changes in the length of the optic fiber: in particular
the radial, ‘breathing’ mode with n=0 and m=1 results to be the most important normal
mode which affects and modifies the response of the hydrophone.

The resonance frequency associated with this mode is:

\[ \omega_{res}^2 = \frac{E}{\rho(1-v^2)R^2} \left(1 + \frac{t^2 \pi^4 R^2}{12L^2}\right) \]  

(2)

The frequency responsivity of the hydrophone has been obtained in [10] for the
lowest, radial, ’breathing’ mode in vacuum and in fluids. The fluid will in general lower
the resonance frequency due to the added mass effect and it will also decrease the
amplitude at the peak due to re-radiation of acoustic energy.

The response is expressed as the normalized radial displacement, \(W_{1.0}(\omega)/R\),
produced by a pressure wave \(\Delta p_\omega\) of frequency \(\omega\):

\[ \frac{W_{1.0}(\omega)}{R} = \frac{\Delta p_\omega}{E} \frac{R(1-v^2)}{Et} \left(1 + \frac{1}{K_0(\gamma R)}\right) \frac{1}{1 + \frac{\rho_{fluid}}{\rho_{hyd}} K_1(\gamma R)} - 1 - \frac{t^2 \pi^4 R^2}{12L^2} \]  

(3)

This response contains the static strain induced by \(\Delta p_\omega\) in the first term while the
denominator represents the responsivity of the mandrel due to the radial resonance of
eq(2).

The correction factor \(1 + \frac{\rho_{fluid}}{\rho_{hyd}} K_0(\gamma R)\) describes the effect of the fluid.

Here \(K_0(\gamma R), K_1(\gamma R)\) are the modified Bessel function and

\[ \gamma^2 = \frac{\pi^2}{L^2} - \frac{\omega^2}{v_{\text{fluid}}} \]  

(4)
\( v_{\text{fluid}}^2 \) being the velocity of sound in the fluid. The parameter \( \gamma \) is imaginary for frequency \( f \) above \( v_{\text{fluid}} / 2L \) indicating that the cylinder radiates sound into the fluid.

In the following table we have applied eq. (3) to determine the resonance frequency in vacuum (air) and in the fluid (water) of different hydrophones made in ULTEM 1000.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Radius (mm)</th>
<th>Thickness/#layers</th>
<th>Length (mm)</th>
<th>( f_{\text{air}} ) (kHz)</th>
<th>( f_{\text{water}} ) (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2a</td>
<td>8</td>
<td>0.5/3</td>
<td>40</td>
<td>54.7</td>
<td>17.4</td>
</tr>
<tr>
<td>3a</td>
<td>25</td>
<td>0.5/3</td>
<td>10</td>
<td>21.0</td>
<td>11.9</td>
</tr>
<tr>
<td>3b</td>
<td>25</td>
<td>0.5/3</td>
<td>20</td>
<td>17.4</td>
<td>7.4</td>
</tr>
<tr>
<td>prototype</td>
<td>8.5</td>
<td>1/5</td>
<td>20</td>
<td>49.7</td>
<td>28.1</td>
</tr>
</tbody>
</table>

**TABLE 4:** Resonance frequencies in air and water for different geometrical configuration of the mandrel.

The results of the previous tables can be summarized as follows:

- to improve the static sensitivity, plastic materials like ULTEM 1000 are preferable with respect to metals due to their lower Young modulus.
- the number of layers of optic fiber can contribute to the stiffness of the hydrophone and should be carefully tuned. The higher is this number, the better is the amplification in the sensor arm of the Michelson interferometer, but the less is the responsivity.
- the edge of the ‘active’ shell should not preferably be glued on the ‘passive’ hollow cylinder. However this effect becomes negligible when the ratio between length and the radius of the hydrophone increases.
- the resonance peak in water may occur at low frequencies. In order to obtain peak frequencies above 20 kHz the configuration where the total thickness of the shell is increased and the radius decreased is preferable.

These considerations lead us to the design of a prototype where a compromise between the sensitivity, the minimum allowable bending radius of the fiber and a bandwidth up to 20 kHz has been obtained. This configuration has been labelled ‘prototype’ in the previous tables.

The corresponding plot of the responsivity as a function of the frequency in air and water is reported in fig.2.
5 THE DESIGN OF THE PROTOTYPE

In fig 3 we report the design of our prototype. As discussed above the design is a result of a compromise between various effects which guarantee a sufficiently wide band width necessary for our purposes.

In our approach the mandrel (‘active’ cylinder) is realized in ULTEM 1000, it is 1mm thick, 17 mm of external diameter and 20 mm long. In this design the mandrel is suspended in the support via two O-rings which allow a 0.5 mm air gap between them. The mandrel, which can be easily mounted, is free to move at the two ends and, according to table 3, a slightly improved sensitivity is obtained. The layers of optic fibers are wound outside while a plastic cone is used to adapt the fiber from the mandrel to the axis of the hydrophone.

For ease of handling and final protection the hydrophone will be coated in polyurethane: to limit the formation of bubbles during this process we have avoided carved surfaces and minimized sharp edges.

From the analysis described above we have designed an air backed mandrel hydrophone to detect high energy hadronic showers in the water. The choice of the material and the dimensions were determined to optimize the sensitivity and to guarantee a sufficiently large band width to detect frequencies as high as 20 kHz.
This analysis was based on theoretical models which have shown to be adequate when compared to Finite Element Analysis calculations: we are therefore confident that this prototype will fit the requirements.

![FIG. 3: The design of our prototype: the support(cyan and magenta), the two o-rings(grey), the mandrel (light yellow) and the fibers (green).](image)

6 REFERENCES

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