Special Relativity and Superluminal Motions: 
a Discussion of Some Recent Experiments†

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Abstract

Some experiments, performed at Berkeley, Cologne, Florence, Vienna, Orsay and
Rennes led to the claim that something seems to travel with a group velocity larger than the
speed $c$ of light in vacuum. Various other experimental results seem to point in the same
direction: For instance, localized wavelet–type solutions of Maxwell equations have been
found, both theoretically and experimentally, that travel with Superluminal speed. Even
muonic and electronic neutrinos – it has been proposed – might be “tachyons”, since their
square mass appears to be negative.

With regard to the first–mentioned experiments, it was very recently claimed by
Guenter Nimtz that those results with evanescent waves or “tunneling photons” – implying
Superluminal signal and impulse transmission – violate Einstein causality. In this note, on the
contrary, we want to stress that all such results do not place relativistic causality in jeopardy,
even if they refer to actual tachyonic motions: In fact, Special Relativity can cope even with
Superluminal objects and waves. For instance, it is possible (at least in microphysics) to solve
also the known causal paradoxes, devised for “faster than light” motion, even if this is not
widely recognized. Here we show, in detail and rigorously, how to solve the oldest causal
paradox, originally proposed by Tolman, which is the kernel of many further tachyon
paradoxes. The key to the solution is a careful application of tachyon mechanics, as it
unambiguously follows from Special Relativity.

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1. Introduction

Superluminal propagation seems to have been observed in several areas of physics[1]. In electromagnetism, in particular, some recent experiments performed at Berkeley[2], Cologne[3], Florence, Vienna, Orsay and Rennes with evanescent waves (or “tunnelling photons”) led to the claim that evanescent modes can travel with a group velocity larger than the speed \( c \) of light in vacuum, thus confirming some older predictions[4]. Even more recently, some of the main experimental claims (e.g., in [3, 2]) have been shown to be theoretically sound just by solving the (classical) Maxwell equations with the requested boundary conditions[5], or by analysing the corresponding (quantum) tunnelling problems[5].

Various other experimental results seem to point in the same direction: For instance, localized wavelet-type solutions of Maxwell equations have been found, both theoretically[1, 5] and experimentally[6], that travel with Superluminal speed. Even muonic and electronic neutrinos —it has been proposed— might be “tachyons”, since their square mass appears to be negative[7]. As to the apparent Superluminal expansions observed in the core of quasars[8] and, recently, in the so-called galactic microquasars[9], we shall not deal here with that problem, too far from the topics of this paper: without mentioning that the astrophysical data are often the most difficult to be interpreted.

Let us confine our attention to the first-mentioned experiments[2, 3, 10]. From a historical point of view, let us recall that for long time the topic of the electromagnetic wave propagation velocity was regarded as already settled down by the classical works of Sommerfeld[11] and Brillouin[12]. A few authors, however, studying the propagation of light pulses in anomalous dispersion (absorbing) media both theoretically[13] and experimentally[14], had found their envelope speed to be the group velocity \( v_g \), even when \( v_g \) exceeds \( c \), equals \( \pm \infty \), or becomes negative! In the meantime, evanescent waves were predicted[15] to be faster-than-light just on the basis of Special Relativistic considerations.

But evanescent waves in suitable (“undersized”) waveguides, in particular, can be regarded also as tunnelling photons[16], due to the known formal analogy[17] between the Schroedinger equation in presence of a potential barrier and the Helmholtz equation for a wave-guided beam. And it was known since long that tunnelling particles (wave packets) can move with Superluminal group velocities inside opaque barriers[18]; therefore, even from the quantum theoretical point of view, it was expected[18, 16, 15] that evanescent waves could be Superluminal.

The point we are more interested in is just the propagation in waveguides of pulses obtained by amplitude modulation of a carrier-wave endowed with an under-cutoff frequency; and the experiments —for instance— in refs.[2, 3, 19, 20] seem to have detected in such a case a Superluminal group-velocity, \( v_g > c \), in agreement with the classical[15] and the quantum[18] predictions. For example, the work in refs.[3, 21] put in particular evidence the fact that the segment of “undersized” (= operating with under-cutoff frequencies) waveguide provokes an attenuation of each spectral component,
without any phase variation. More precisely, the unique phase variation detectable is due to the discontinuities in the waveguide cross-section (cf. also refs.[18] and [5]). As stressed, e.g., by Barbero et al.[5], the spectrum leaving an undersized waveguide segment (or photonic barrier) is simply the entering spectrum multiplied by the transfer function \( H(\omega) = \exp[i\beta L] \), with \( \beta(\omega) = \omega \sqrt{1 - (\omega_c/\omega)^2}/c \). For \( \omega > \omega_c \), the propagation constant \( \beta(\omega) \) is real, and \( H(\omega) \) represents a phase variation to be added to the outgoing spectrum. However, for \( \omega < \omega_c \), when \( \beta(\omega) \) is imaginary, the transfer function just represents an additional attenuation of the incoming spectrum.

In a sense, the two edges of a “barrier” (undersized waveguide segment) can be regarded as semi-mirrors of a Fabry-Perot configuration. The consequent negative interference processes can lead themselves to Superluminal transit times. These points have been exploited, e.g., by Japha and Kurizki[22] (who claimed the barrier transit mean-time to be Superluminal provided that the coherence time \( \tau_c \) of the entering field \( \psi_m(t) \) is much larger than \( L/c \)).

2. Transients and Signals

With regard to the same experiments, Guenter Nimtz claimed very recently[10] that those results with evanescent waves, or “tunnelling photons”, do really imply Superluminal signal and impulse transmission.

A common objection consists in recalling that, on the other hand, the speed of the precursors cannot be larger than \( c \) (a fact that appears to be enough for satisfying the requirements of the so-called, naive Einstein causality). Actually, every perturbation passes through a transient state before reaching the stationary regime; this happens also when transmitting any kind of wave. In the case of electromagnetic waves, such a transient state is ordinarily associated with the propagation of precursors, arriving before the principal signal. For instance, the existence of Sommerfeld’s and Brillouin’s precursors (the so-called first and second precursors) has been recently stressed in refs.[23], while studying the transients in metallic waveguides.

To investigate the interplay between Einstein causality and the fact that \( v_p \gg c \) when a signal is transported in a metallic waveguide by a carrier-wave with \( \omega_p < \omega_c \), one has to examine simultaneously those two effects. To be clear about such two effects, let us recall (at the cost of repeating ourselves) that: (i) in Sect.2 of ref.[5] the results of computer simulations (based on Maxwell equations only) have been presented, which show how the first electric perturbation, reaching any point \( P \), always travels with the speed \( c \) of light in vacuum, independently of the medium; (ii) in Sect.3 of the same ref.[5], however, it has been shown, by further computer simulations based on Maxwell equations only, that the evanescent guided-waves are endowed with a Superluminal group velocity.

Actually, as already mentioned, the propagation constant \( \beta(\omega) \) is imaginary for the under-cutoff frequencies, so that the transfer function \( H(\omega) \) works only as an attenuation factor for such (evanescent) frequencies. However, the higher (non-evanescent) frequencies
will be phase shifted, in such a way that $\beta(\omega)$ will tend to its free-space value $\omega/c$ for $\omega \to \infty$. In other words, the higher spectral components travel with speed $c$; they are the responsible both for the finite speed of the evanescent beams, and for the appearance of the precursors.

At this point, one can accept —following Nimtz— that a signal is really carried (not by the precursors, but) by well-defined amplitude bumps, as in the case of information transmission by the Morse alphabet, or the transmission of a number e.g. by a series of equal (and equally spaced) pulses. In such a case, the “signal” can travel even at infinite speed, in the considered situations; and, by the above-quoted computer simulations[5], it has been verified the important fact that the width of the arriving pulses does not change with respect to the initial ones. The signal, however, seems to be unable to overcome the transients, “slowly” travelling with speed $c$. In the (theoretical) case that a pulse were constituted by under-cutoff frequencies only, the situation could however be rather different, since the precursors would not exist; a crucial question being: Is that theoretical case experimentally realizable?

Even if the AM signal were totally constituted by under-cutoff frequencies, when the experiment is started (e.g., by switching on the carrier wave) one does necessarily meet a transient situation, which generates precursors. One might think, therefore, of arranging a setup (permanently switched on) for which the precursors are sent out long in advance, and of awaiting afterwards for the moment at which the need arises of transmitting a signal with Superluminal speed (without violating the naive “Einstein causality”, as far as it requires only that the precursors do not travel at speed higher than $c$).

The authors in refs.[3, 21, 24], do actually claim that they can build up (smooth) signals by means of under-cutoff frequencies only, without generating further precursors: in such a case one would be in presence, then, of Superluminal information transmission. On the basis of other calculations (which imply the existence also of above-cutoff frequencies in any signal) this does not seem to be true in practice. But perhaps this is not so important, in the case when a carrier-wave is permanently switched on; in fact, due to the linearity of the Maxwell equations, one can expect to meet solutions in which we have both $c$-speed precursors and faster-than-light signals: the precursors constituting no impediment to the passage of the Superluminal signals.

Such critical issues deserve further investigation. [For instance, a problem is whether one must already know the whole information content of the signal when starting to send it; in such a case, it would become acceptable the mathematical trick of representing any signal by an analytical function[25].] But here we want to exploit the fact that, even if Nimtz[10] is right and we are in presence of really tachyonic motions, nevertheless nobody will be able to take advantage of such motions for sending signals backwards in time and killing his parents before his own birth...
3. G. Nimtz’s recent claims

Let us re-emphasize that Nimtz has very recently[10] declared that the cited results with evanescent waves do really imply Superluminal signal and impulse transmission. Nimtz bases his argument on the fact that—on the claim—he can have recourse to a finite frequency band (entirely under cutoff), so to have no frequencies above cutoff, and therefore no precursors (and no front-wave) travelling at the speed of light, which could be an obstacle to the transmission of Superluminal signals. An important consequence is that Nimtz may then claim to be in presence of something (information, and impulse) which travels faster than light.

To show that he can use a finite frequency-band, Nimtz argues that his frequency-band must be finite, since the radiated energy is of course finite and therefore the beam cannot contain at all photons too much energetic, i.e. with too high frequency.

Such a “quantum” consideration is not as obvious as it could seem, and should be revised and deepened since in quantum physics the Heisenberg uncertainty relations are known to allow, in a sense, energy violations for sufficiently short times. But this is not so essential to the conclusions in ref.[10] for the fact that, as we have already commented, the precursors do not seem to be able to prevent the passage of Superluminal signals in the case when a standing wave is permanently switched on.

One could then tentatively accept the revolutionary claim by Nimtz.

However, in ref.[10] it is moreover stated that one should then accept that Einstein causality, and Special Relativity (SR), have been violated. In this note we want to object to such a last conclusion, since in our opinion the existence of something travelling really faster-than-light does not rule out the ordinary postulates of SR. In fact, when Special Relativity (SR) is not restricted to subluminal speeds, one ends up with an “extended relativity” which—on the basis of the ordinary postulates—can cope with actually Superluminal objects or waves without abandoning the principle of retarded causality: Namely, without allowing one to take advantage of those objects or waves for killing his parents before his own birth...

We are going to show, in other words, that all the mentioned experimental results do not place relativistic causality in jeopardy, even if they refer to actual tachyonic motions. As far as the foundations of Extended Relativity are concerned, we shall here confine ourselves only to quote ref.[26] and references therein; by contrast, we shall address our attention to the fact that it is possible to solve the known causal paradoxes, devised for “faster than light” motion.

4. Superluminal motions and relativistic causality

Claims exist since long that all the ordinary causal paradoxes proposed for tachyons can be solved[27, 28, 29] (at least “in microphysics”) on the basis of the “switching procedure” (swp) introduced by Stückelberg[30], Feynman[30], Sudarshan[27], and Recami et al.[26,
28], also known as the reinterpretation principle: a principle which in refs.[26, 28] has been given the status of a fundamental postulate of special relativity, both for bradyons [slower-than-light particles] and for tachyons. Schwartz,[31] at last, gave the SWP a formalization in which it becomes “automatic”.

However, the effectiveness of the SWP and of that solution is often overlooked, or misunderstood. Here we want therefore to show, in detail and rigorously, how to solve the oldest “paradox”, i.e. the antitelephone one, originally proposed by Tolman[32] and then reproposed by many authors. We shall refer to its recent formulation by Regge,[33] and spend some care in solving it, since it is the kernel of many other paradoxes. Let us stress that: (i) any careful solution of the tachyon causal “paradoxes” has to make recourse to explicit calculations based on the mechanics of tachyons; (ii) such tachyon mechanics can be unambiguously and uniquely derived from SR, by referring the Superluminal \(V^2 > c^2\) objects to the class of the ordinary, subluminal \(u^2 < c^2\) observers only (i.e., without any need of introducing “Superluminal reference frames”); (iii) moreover, the comprehension of the whole subject will be substantially enhanced if one refers himself to the (subluminal, ordinary) SR based on the whole proper Lorentz group \(\mathcal{L}_+ = \mathcal{L}_+^1 \cup \mathcal{L}_+^2\), rather than on its orthochronous subgroup \(\mathcal{L}_+^1\) only [see refs.[34], and references therein]. At last, for a modern approach to the classical theory of tachyons, reference can be made to the review article[26] as well as to refs.[28, 29].

Before going on, let us mention the following. It is a known fact that in the time-independent case the (relativistic, non-quantistic) Helmholtz equation and the (non-relativistic, quantum) Schroedinger equation are formally identical[35] [in the time-dependent case, such equations become actually different, but nevertheless strict relations still hold between some solutions of theirs, as it will be explicitly shown elsewhere[36]]: one important consequence of this fact being that evanescent wave transmission simulates electron tunnelling. On the other side, a wave-packet had been predicted[37] since long to tunnel through an (opaque) barrier with Superluminal group-velocity. Therefore, one could expect evanescent waves too to be endowed with Superluminal (group) speeds.[38] The abovenamed experiments[1, 2, 3], which seem to have actually verified such an expectation, are the ones that most attracted the attention of the scientific press.[39] But they are not the only ones which seem to indicate the existence of Superluminal motions.[40]

5. Tachyon mechanics

In refs.[41] the basic tachyon mechanics can be found exploited for the processes: a) proper (or “intrinsic”) emission of a tachyon T by an ordinary body A; b) “intrinsic” absorption of a tachyon T by an ordinary body A; where the term “intrinsic” refers to the fact that those processes (emission, absorption by A) are described as they appear in the rest-frame of A; particle T can represent both a tachyon and an antitachyon. Let us
recall the following results only.

Let us first consider a tachyonic object $T$ moving with velocity $V$ in a reference frame $s_0$. If we pass to a second frame $s'$, endowed with velocity $u$ w.r.t. (with respect to) frame $s_0$, then the new observer $s'$ will see —instead of the initial tachyon $T$— an antitachyon $\bar{T}$ travelling the opposite way in space (due to the swp: cf. Appendix A), if and only if

$$u \cdot V > c^2. \quad (1)$$

Recall in particular that, if $u \cdot V < 0$, the “switching” does never come into play.

Now, let us explore some of the unusual and unexpected consequences of the trivial fact that in the case of tachyons it is

$$|E| = +\sqrt{p^2 - m_0^2} \quad (m_0 \text{ real; } V^2 > 1), \quad (2)$$

where we chose units so that, numerically, $c = 1$. We are using the quantities $(E, p)$, referring to the case of elementary particles; but all our equations in the electromagnetic case can be rewritten in terms of $\omega$ and of the wave-number $k$. For instance (besides $\omega^2 - k^2 = 0$ for the light-like case) we shall have $\omega^2 - k^2 = +\Omega^2 > 0$ in the subluminal case, and $\omega^2 - k^2 = -\Omega^2 < 0$ in the Superluminal case: cf. Appendix B.

Let us, e.g., describe the phenomenon of “intrinsic emission” of a tachyon, as seen in the rest frame of the emitting body: Namely, let us consider in its rest frame an ordinary body $A$, with initial rest mass $M$, which emits a tachyon (or antitachyon) $T$ endowed with (real) rest mass $m \equiv m_0$, four-momentum $p^\mu \equiv (E_T, p)$, and velocity $V$ along the $x$-axis. Let $M'$ be the final rest mass of body $A$. The four-momentum conservation requires

$$M = \sqrt{p^2 - m^2} + \sqrt{p'^2 + M'^2} \quad \text{(rest frame)} \quad (3)$$

that is to say $|V| = |\bar{V}|$:

$$2M|p| = [(m^2 + \Delta)^2 + 4m^2 M'^2]^{1/2}; \quad V = [1 + 4m^2 M^2/(m + \Delta)^2]^{1/2}, \quad (4)$$

where [calling $E_T \equiv +\sqrt{p^2 - m^2}$]:

$$\Delta \equiv M'^2 - M^2 = -m^2 - 2ME_T, \quad \text{(emission)} \quad (5)$$
so that

\[-M^2 < \Delta \leq -|p|^2 \leq -m^2.\]  
(emission) \hspace{1cm} (6)

It is essential to notice that $\Delta$ is, of course, an invariant quantity, which in a generic frame $s$ writes

\[\Delta = -m^2 - 2p_\mu P^\mu,\]  
(7)

where $P^\mu$ is the initial four-momentum of body $A$ w.r.t. frame $s$.

Notice that in the generic frame $s$ the process of (intrinsic) emission can appear either as a $T$ emission or as a $T$ absorption (due to a possible “switching”) by body $A$. The following theorem, however, holds:

Theorem 1: << Necessary and sufficient condition for a process to be a tachyon emission in the $A$ rest-frame (i.e., to be an intrinsic emission) is that during the process the body $A$ lowers its rest-mass (invariant statement!) in such a way that $-M^2 < \Delta \leq -m^2.$ >>

Let us now describe the process of “intrinsic absorption” of a tachyon by body $A$; i.e., let us consider an ordinary body $A$ to absorb in its rest frame a tachyon (or antitachyon) $T$, travelling again with speed $V$ along the $x$-direction. The four-momentum conservation now requires

\[M + \sqrt{p^2 - m^2} = \sqrt{p^2 + M^2},\]  
(rest frame) \hspace{1cm} (8)

which corresponds to

\[\Delta \equiv M'^2 - M^2 = -m^2 + 2ME_T,\]  
(absorption) \hspace{1cm} (9)

so that

\[-m^2 \leq \Delta \leq +\infty.\]  
(absorption) \hspace{1cm} (10)

In a generic frame $s$, the quantity $\Delta$ takes the invariant form

\[\Delta = -m^2 + 2p_\mu P^\mu.\]  
(11)
It results in the following new theorem:

**Theorem 2**: \(<\text{Necessary and sufficient condition for a process (observed either as the emission or as the absorption of a tachyon T by an ordinary body A) to be a tachyon absorption in the A-rest-frame —i.e., to be an intrinsic absorption— is that } \Delta \geq -m^2.>\rangle\>

We now have to describe the *tachyon exchange* between two ordinary bodies A and B. We have to consider the four-momentum conservation at A and B; we need to choose a (single) frame relative to which we describe the whole interaction; let us choose the rest-frame of A. Let us explicitly remark, however, that —when bodies A and B exchange one tachyon T—the tachyon mechanics is such that the “intrinsic descriptions” of the processes at A and at B can a priori correspond to one of the following four cases[41]:

\[
\begin{align*}
1) & \quad \text{emission—absorption ,} \\
2) & \quad \text{absorption—emission ,} \\
3) & \quad \text{emission—emission ,} \\
4) & \quad \text{absorption—absorption .}
\end{align*}
\]

(12)

Case 3) can happen, of course, only when the tachyon exchange takes place in the receding phase (i.e., while A, B are receding from each other); case 4) can happen, by contrast, only in the approaching phase.

Let us consider here only the particular tachyon exchanges in which we have an “intrinsic emission” at A, and in which moreover the velocities \(u\) of B and \(V\) of T w.r.t. body A are such that \(u \cdot V > 1\). Because of the last condition and the consequent “switching” (cf. Eq.(1)), from the rest-frame of B one will therefore observe the flight of an antitachyon \(\overline{T}\) *emitted* by B and absorbed by A (a necessary condition for this to happen, let us recall, being that A, B *recede* from each other).

More generally, the dynamical conditions for a tachyon to be exchangeable between A and B can be shown to be the following:

I) Case of “intrinsic emission” at A:

\[
\begin{align*}
\text{if } u \cdot V < 1 , & \quad \text{then } \Delta_B > -m^2 \quad (\rightarrow \text{intrinsic absorption at B}); \\
\text{if } u \cdot V > 1 , & \quad \text{then } \Delta_B < -m^2 \quad (\rightarrow \text{intrinsic emission at B}).
\end{align*}
\]

(13)
II) Case of “intrinsic absorption” at A:

\[
\begin{cases}
\text{if } u \cdot V < 1, & \text{then } \Delta_B < -m^2 \quad (\text{intrinsic emission at } B) ; \\
\text{if } u \cdot V > 1, & \text{then } \Delta_B > -m^2 \quad (\text{intrinsic absorption at } B).
\end{cases}
\]

Now, let us finally pass to examine the Tolman paradox.

6. The paradox

In Figs.1, 2 the axes \( t \) and \( t' \) are the world-lines of two devices A and B, respectively, which are able to exchange tachyons and move with constant relative speed \( u, [u^2 < 1] \), along the \( x \)-axis. According to the terms of the paradox (Fig.1), device A sends tachyon 1 to B (in other words, tachyon 1 is supposed to move forward in time w.r.t. device A). The device B is constructed so as to send back tachyon 2 to A as soon as it receives tachyon 1 from A. If B has to emit (in its rest-frame) tachyon 2, then 2 must move forward in time w.r.t. device B; that is to say, the world-line \( BA_2 \) must have a slope lower than the slope \( BA' \) of the \( x' \)-axis (where \( BA' / x' \)): this means that \( A_2 \) must stay above \( A' \). If the speed of tachyon 2 is such that \( A_2 \) falls between \( A' \) and \( A_1 \), it seems that 2 reaches A (event \( A_2 \)) before the emission of 1 (event \( A_1 \)). This appears to realize an anti-telephone.

The solution

First of all, since tachyon 2 moves backwards in time w.r.t. body A, the event \( A_2 \) will appear to A as the emission of an antitachyon 2. The observer “\( t' \)” will see his own device A (able to exchange tachyons) emit successively towards B the antitachyon 2 and the tachyon 1.

At this point, some supporters of the paradox (overlooking tachyon mechanics, as well as relations (12)) would say that, well, the description put forth by the observer “\( t' \)” can be orthodox, but then the device B is no longer working according to the stated programme, because B is no longer emitting a tachyon 2 on receipt of tachyon 1. Such a statement would be wrong, however, since the fact that “\( t' \)” observes an “intrinsic emission” at \( A_2 \) does not mean that “\( t' \)” will see an “intrinsic absorption” at B! On the contrary, we are just in the case considered above, between eqs. (12) and (13): intrinsic emission by A, at \( A_2 \), with \( u \cdot V_7 > c^2 \), where \( u \) and \( V_7 \) are the velocities of B and 2 w.r.t. body A, respectively; so that both A and B experience an intrinsic emission (of tachyon 2 or of antitachyon 2) in their own rest frame.

But the tacit premises underlying the “paradox” (and even the very terms in which it was formulated) were “cheating” us ab initio. In fact, Fig.1 makes it clear that, if \( u \cdot V_7 > c^2 \), then for tachyon 1 a fortiori \( u \cdot V_1 > c^2 \), where \( u \) and \( V_1 \) are the velocities of B and 1 w.r.t. body A. Therefore, due to the previous consequences of tachyon mechanics,
observer “t’” will see B intrinsically emit also tachyon 1 (or, rather, antitachyon T). In conclusion, the proposed chain of events does not include any tachyon absorption by B (in its rest frame).

For body B to absorb (in its own rest frame) tachyon 1, the world-line of 1 ought to have a slope higher than the slope of the x'-axis (see Fig. 2). Moreover, for body B to emit (“intrinsically”) tachyon 2, the slope of 2 should be lower than the x'-axis’. In other words, when the body B, programmed to emit 2 as soon as it receives 1, does actually do so, the event A2 does happen after A1 (cf. Fig. 2), as requested by causality.

The moral

The moral of the story is twofold: i) one should never mix the descriptions (of one phenomenon) yielded by different observers; otherwise —even in ordinary physics— one would immediately meet contradictions: in Fig. 1, e.g., the motion direction of 1 is assigned by A and the motion-direction of 2 is assigned by B: this is “illegal”; ii) when proposing a problem about tachyons, one must comply [27] with the rules of tachyon mechanics [38]; this is analogous to complying with the laws of ordinary physics when formulating the text of an ordinary problem (otherwise the problem in itself will be “wrong”).

Most of the paradoxes proposed in the literature suffered the above shortcomings.

Notice once more that, in the case of Fig. 1, neither A nor B regard event A1 as the cause of event A2 (or vice-versa). In the case of Fig. 2, on the other hand, both A and B consider event A1 to be the cause of event A2: but in this case A1 does chronologically precede A2 according to both observers, in agreement with the relativistic covariance of the law of retarded causality.

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APPENDIX A: The Stueckelber–Feynman–Sudarshan “Switching Principle”

What follows refers equally well to bradyons and to tachyons. For simplicity, then, let us fix our attention in this Appendix only to the case of bradyons. Let us start from a positive-energy particle P travelling forward in time. As well-known, any orthochronous LT, $L^\dagger$, transforms it into another particle still endowed with positive energy and motion forward in time. On the contrary, any antichronous (=non-orthochronous) LT, $L^\dagger = -L^\dagger$, will change sign among the others— to the time-components of all the four-vectors associated with P. Any $L^\dagger$ will transform P into a particle P’ endowed in particular with negative energy and motion backwards in time (Fig.3). We are of course assuming that negative-energy objects travelling forward in time do not exist. (Elsewhere this Assumption has been given by us the status of a fundamental postulate).

In other words, SR together with the natural Assumption above, implies that a particle going backwards in time (Gödel[42]) (Fig.3) corresponds in the four-momentum space, Fig.4, to a particle carrying negative energy; and, vice-versa, that changing the energy sign in the latter space corresponds to changing the sign of time in the former (dual) space. It is then easy to see that these two paradoxical occurrences (“negative energy” and “motion backwards in time”) give rise to a phenomenon that any observer will describe in a quite orthodox way, when they are—as they actually are— simultaneous (Recami[26, 27, 28, 29] and refs. therein).

Notice, namely, that: (i) every observer (a macro-object) explores space-time, Fig.3, in the positive t-direction, so that we shall meet B as the first and A as the last event; (ii) emission of positive quantity is equivalent to absorption of negative quantity, as $(-) \cdot (-) = (+) \cdot (+)$; and so on.

Let us now suppose (Fig.5) that a particle P’ with negative energy (and, e.g., charge $-\epsilon$), travelling backwards in time, is emitted by A at time $t_1$ and absorbed by B at time $t_2 < t_1$. Then, it follows that at time $t_1$ the object A “looses” negative energy and negative charge, i.e. gains positive energy and positive charge. And that at time $t_2 < t_1$ the object B “gains” negative energy and charge, i.e. loosizes positive energy and charge. The physical phenomenon here described is nothing but the exchange from B to A of a particle Q with positive energy, charge $+\epsilon$, and travelling forward in time. Notice that Q has, however, charges opposite to P’; this means that the present “switching procedure” (previously called also “RIP”) effects a “charge conjugation” $C$, among the others. Notice also that “charge”, here and in the following, means any additive charge; so that our definitions of charge conjugation, etc., are more general than the ordinary ones (see Recami and Mignani,[43] hereafter called Review I; and refs.[26, 28]). Incidentally, such a switching procedure has been shown[44] to be equivalent to applying the chirality operation $\gamma_5$.

*Matter and Antimatter from SR*— A close inspection shows that the application of any antichronous transformation $L^\dagger$, together with the switching procedure, transforms P into an object
Q \equiv \overline{P}

(15)

which is indeed the antiparticle of P. We are saying that the concept of antimatter is a purely relativistic one, and that, on the basis of the double sign in \[ c = 1 \]

\[ E = \pm \sqrt{p^2 + m_0^2}, \]

(16)

the existence of antiparticles could have been predicted already in 1905, exactly with the properties they actually exhibited when later discovered, provided that recourse to the “switching procedure” had been made. We therefore maintain that the points of the lower hyperboloid sheet in Fig.4—since they correspond not only to negative energy but also to motion backwards in time—represent the kinematical states of the antiparticle \( \overline{P} \) of the particle P represented by the upper hyperboloid sheet).

Let us stress that the switching procedure not only can, but must be enforced, since any observer can do nothing but explore spacetime along the positive time direction. That procedure is an improved translation into a purely relativistic language of the Stückelberg–Feynman[30] “Switching principle”. Together with our Assumption above, it can take the form of a “Third Postulate”: \( \langle \langle \langle \text{Negative-energy objects travelling forward in time do not exist; any negative-energy object} \rangle \rangle \rangle \). Together with our Assumption above, it can take the form of a “Third Postulate”: \( \langle \langle \langle \text{Negative-energy objects travelling forward in time do not exist; any negative-energy object} \rangle \rangle \rangle \). Cf. e.g. refs.[45, 26, 28] and references therein.

Concluding remarks—Let us go back to Fig.3. In SR, when based only on the two ordinary postulates, nothing prevents a priori the event \( A \) from influencing the event \( B \). Just to forbid such a possibility we introduced our Assumption together with the Switching procedure. As a consequence, not only we eliminate any particle-motion backwards in time, but we also “predict” and naturally explain within SR the existence of antimatter.

In the case of tachyons the Switching procedure was first applied by Sudarshan and coworkers[27]; see e.g. ref.[28] and refs. therein.

At last, it is necessary—however—to observe the following: Whenever it is met an object (or wavelet), \( \mathcal{O} \), travelling at Superluminal speed, negative contributions should be expected to the “tunnelling” times[46, 5]: and this ought not to be regarded as unphysical or at variance with the expectations of SR. In fact we have just seen above that, whenever an “object” \( \mathcal{O} \) overcomes the infinite speed[26, 28] with respect to a certain observer, it will afterwards appear to the same observer as its “anti-object” \( \overline{\mathcal{O}} \) travelling in the opposite space direction. For instance, when passing from the lab to a frame \( \mathcal{F} \) moving in the same direction as the waves (or particles) entering the undersized waveguide (or the barrier region), the objects \( \mathcal{O} \) penetrating through that waveguide or barrier (with almost infinite speeds[5, 46]) will appear in the frame \( \mathcal{F} \) as anti-objects \( \overline{\mathcal{O}} \) crossing the waveguide or barrier in the opposite space–direction. In the new frame \( \mathcal{F} \), therefore, such anti-objects \( \overline{\mathcal{O}} \) would yield a negative contribution to the traversal time: which could even
result, in total, to be negative.

**APPENDIX B: Group velocity of the evanescent waves.**

Let us here define the group velocity of an electromagnetic signal in the evanescent region, as well as in a normal (not undersized) waveguide. We shall follow ref.[47]. Let us start with a normal waveguide. The wave equation will be:

\[ \phi(t, x, y, z) = \phi_0(t, x) e^{i\omega t - \gamma z}, \quad (17) \]

where \( \gamma \) is the propagation constant inside the waveguide

\[ \gamma^2 \equiv k_z^2 \equiv -\beta^2, \quad (18) \]

so that

\[ \beta = ik_z = \pm i \sqrt{k_c^2 - k^2} = \pm \sqrt{k_c^2 - k^2} \quad (19) \]

where \( k_c^2 = -k_x^2 - k_y^2 \). The cutoff frequency (i.e., the lowest frequency that can propagate, in the normal way, along a waveguide having width \( a \)) is given by the condition \( \beta^2 = 0 \); which yields:

\[ \sqrt{k_c^2 - k^2} = 0 \quad \Rightarrow \quad k_c^2 - k^2 = 0 \quad \Rightarrow \quad k^2 = k_c^2. \quad (20) \]

Let us now recall that

\[ k_x = -\frac{i m \pi}{a} \quad \text{e} \quad k_y = -\frac{i n \pi}{b} \]

and that

\[ -k_c^2 = k_x^2 + k_y^2 \quad \Rightarrow \quad k_c^2 = \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2. \quad (21) \]

If we adopt the propagation mode TE\(_{10}\), relation (21) reduces to

\[ k_c^2 = \left( \frac{\pi}{a} \right)^2 \]

which (since \( \omega^2 = k_c^2 c^2 \)) yields the critical frequency:

\[ \frac{\omega^2}{c^2} = \left( \frac{\pi}{a} \right)^2 \quad \Rightarrow \quad \omega_c = \frac{\pi}{a} c \quad (22) \]

representing the minimum frequency that a (carrier)-wave can possess in order to be able to carry a signal, for the mentioned mode, along the waveguide with width \( a \). The wavepacket (group)-velocity is obtained through its dispersion relation, which in the considered case is

\[ \beta^2 = k^2 - k_c^2 \]
and can be rewritten as
\[ \beta^2 = \frac{\omega^2}{c^2} - \frac{\pi^2}{a^2} \, . \]

By definition, the group velocity is given by \( \frac{d\omega}{d\beta} \). Quantity \( \beta(\omega) \) is known. One gets:
\[ \frac{d\beta}{d\omega} = \frac{\omega}{c^2} \sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2}} \]
and, by use of relation (22),
\[ \frac{d\beta}{d\omega} = \frac{\omega}{c \sqrt{\omega^2 - \omega_c^2}} \, . \]

Finally, by inverting, we obtain
\[ \frac{d\omega}{d\beta} = v_{gr} = c \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} \quad \text{with} \quad \omega > \omega_c \]
which shows that the packet propagates in the waveguide with a subluminal velocity.

**Velocity of an evanescent wave** — If at a certain point the signal meets a “barrier”, i.e. a waveguide segment with a smaller width \( a' \), such that the lowest propagating frequency is larger than the carrier’s, the dispersion relation does then change, assuming (as requested by Extended Relativity [26]) the form:
\[ \beta^2 = \frac{\omega^2}{c^2} + \frac{\pi^2}{a'^2} \, . \]

Following the same procedure as before, we arrive at
\[ v_{gr} = c \sqrt{1 + \left(\frac{\omega_c}{\omega}\right)^2} \quad \text{with} \quad \omega < \omega_c \, . \]
wherefrom it is evident that the signal is now endowed with a superluminal velocity.

Just for completeness’ sake, let us add that simple simulations, based on Maxwell equations (by using Mathematica\textsuperscript{TM}), have been performed also at Milan university [47] confirming these evaluations. For instance, a numerical experiment was performed by R. Garavaglia for microwave frequencies between 5 and 10 GHz in a rectangular waveguide. Let us examine the case with a single barrier (region II). In the normal barrier segments (regions I and III, respectively), the propagating wave is described by the equation
\[ \phi = \phi_0 \cos \left(\frac{\pi}{a} x\right) e^{i(\omega t - kz)} \, , \]
while, in the undersized (evanescent) region, that equation becomes
\[ \phi = \phi_0 \cos \left(\frac{\pi}{a'} x\right) e^{i\omega t e^{-k'z}} \, . \]
with real $k' = ik, k$ and $k'$. When the system is in a stationary state, the complete set of equations is:

\[
\begin{align*}
\psi_I &= A \cos \left( \frac{\pi}{\lambda} x \right) e^{ik_1 z} + R \cos \left( \frac{\pi}{\lambda} x \right) e^{-ik_1 z} ; \\
\psi_{II} &= B \cos \left( \frac{\pi}{\lambda'} x \right) e^{ik_2 z} + C \cos \left( \frac{\pi}{\lambda'} x \right) e^{ik_2 z} ; \\
\psi_{III} &= T \cos \left( \frac{\pi}{\lambda} x \right) e^{ik_1 z} + E \cos \left( \frac{\pi}{\lambda} x \right) e^{-ik_1 z} .
\end{align*}
\]  

(23)

If the entering waves come from $z = -\infty$ (and none come from $z = +\infty$), one has $A = 1$ and $E = 0$. The continuity conditions, which allow determining the coefficients $R, T, B, C$, are (for the regions I and II)

\[
\begin{align*}
\{ \psi_I (0) = \psi_{II}(0) ; \quad \left. \frac{d\psi_I}{dz} \right|_0 &= \left. \frac{d\psi_{II}}{dz} \right|_0 ,
\end{align*}
\]  

(24)

and (for the regions II and III), quantity $L$ being the evanescence region length,

\[
\begin{align*}
\{ \psi_{II} (L) = \psi_{III}(L) ; \quad \left. \frac{d\psi_{II}}{dz} \right|_L &= \left. \frac{d\psi_{III}}{dz} \right|_L
\end{align*}
\]  

(25)

For a carrier-wave frequency of 7 GHz, the group velocity in the normal and in the evanescent regions resulted to be 0.7c and 1.7c, respectively.
References


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[25] Cf., e.g., Low F.E.: (private communication).


[38] Cf., e.g., page 158 in ref.[26], and references therein.


Fig. 1 – The apparent chain of events, according to the terms of the Paradox.
Fig. 2 – The solution of the Tolman paradox: see the text.
Fig. 3 – Special Relativity, when not restricted to subluminal motions (i.e., “Extended Relativity”; see the text), implies that any particle travelling backwards in time (from A to B) has to carry negative energy. The simultaneous occurrence of such two unorthodox situations will necessarily lead to an orthodox reinterpretation, in terms of an antiparticle regularly travelling from B to A (with positive energy and forward in time): cf. Fig.5.
Fig. 4 – The two hyperboloid sheets, representing the kinematical states of free particles (upper sheet) and antiparticles (lower sheet), in the ordinary subluminal case: see the text.
Fig. 5 – The reinterpretation (or switching) procedure, mentioned in the caption of Fig.3. The application of such a procedure is not only possible, but compulsory: see the text.