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A MEASUREMENT OF THE PARTIAL HADRONIC WIDTHS OF THE Z° USING NEURAL NETWORKS
A MEASUREMENT OF THE PARTIAL HADRONIC WIDTHS OF THE Z\(^0\) USING NEURAL NETWORKS

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ABSTRACT

From the data collected by the DELPHI detector at LEP during 1990, the hadronic branching fractions of the Z\(^0\) boson into all five known quarks have been measured for the first time in the literature.

A classifier based on a feed-forward neural network has been used for separating the hadronic decays of the Z\(^0\) into four classes, corresponding to \((u\overline{u}+d\overline{d})\) unresolved, \(s\overline{s}, c\overline{c}, b\overline{b}\). Data on the hard final-state photon radiation have been then used to resolve the decay rate into \(u\overline{u}\) pairs from the decay rate into \(d\overline{d}\) pairs.

The preliminary results are consistent with the predictions from the Standard Model. Problems related to the estimate of systematics are discussed.

1. INTRODUCTION

Measurements of the partial widths of the Z\(^0\) into charged leptons were available from LEP since 1989, and presently the relative accuracy on their determination is of the order of 1\% [1]. Despite the primary importance of the knowledge of the hadronic branching fractions, after two years of running of LEP, only the partial widths into \(b\overline{b}\) pairs and into \(c\overline{c}\) pairs have been measured, with accuracies in the range 5 – 10\% and 10 – 20\% respectively [2]. The lack of experimental information in the hadronic sector is mainly due to the difficulty of separating events in which the Z\(^0\) decays into a pair of light quarks.

A powerful probe for the classification of events can be given by feed-forward neural networks [3], that can map a set of variables calculated from the topology of

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the event onto a feature space in which the different species are well separated. The possibility of using a feed-forward neural network for this purpose was explored in [4], for the problem of the classification of decays into $b\bar{b}$ pairs. The result of this study was that, in the case of a perfect detector, a separation could be achieved with a higher efficiency with respect to traditional separation variables [5]. Further studies [6] demonstrated that, also in the presence of detector effects, feed-forward neural networks could be a useful tool for the classification of $b\bar{b}$ events.

In what follows it is tested whether topological properties of the event (i.e., properties related to the structure of multiparticle production) can be used by a feed-forward neural network to classify not only $b\bar{b}$ events, but also $s\bar{s}$, $c\bar{c}$ and $(u\bar{u}+d\bar{d})$ unresolved events. The robustness of the separation against a wide range of systematic uncertainties related to the model-dependence of the classification has been investigated. As a result, it has been possible to measure, from the data collected by the DELPHI detector [7] at LEP during 1990, the rates of the hadronic decays into the four classes listed above.

For the sake of completeness, the LEP measurements of the rate of final state radiation from $q\bar{q}$ pairs have been used to compute the relative probabilities of decay into $u\bar{u}$ and $d\bar{d}$ pairs. This last separation is based on the assumption that the absolute value of the charge of the $u$ quark is double with respect to the absolute value of the charge of the $d$ quark, and thus, the probability for the photon bremsstrahlung process is four times larger.

2. EXPERIMENTAL PROCEDURE AND EVENT SAMPLE

This analysis is based on a data sample collected by the DELPHI detector at LEP during 1990.

The components of the DELPHI detector, relevant for this analysis, have already been described in Ref. [8], as well as the trigger for the hadronic events.

Charged tracks are measured in a 1.2 Tesla magnetic field by a set of three cylindrical tracking detectors: the Inner Detector (ID) (inner radius = 12 cm, outer radius = 28 cm, covering polar angles between $29^\circ$ and $151^\circ$), the Time Projection Chamber (TPC) (inner radius = 30 cm, outer radius = 122 cm, covering polar angles between $21^\circ$ and $159^\circ$) and the Outer Detector (OD) (inner radius = 198 cm, outer radius = 206 cm, covering polar angles between $42^\circ$ and $138^\circ$).

TPC, ID and OD provide a complete coverage of the region between $25^\circ$ and $155^\circ$ in the polar angle $\theta$, with reconstruction efficiency near to 1. The average momentum resolution is $\Delta p/p \simeq 0.005$ ($p$ in GeV/c).

Only charged particles fulfilling the following criteria were used in the analysis: (a) impact parameter at the nominal primary vertex below 5 cm in radius from the beam axis and to within 10 cm of the nominal crossing point in z; (b) momentum $p$ larger than 0.1 GeV/c; (c) measured track length in TPC above 50 cm; (d) polar angle $\theta$ between $25^\circ$ and $155^\circ$.

All particles were assumed to be pions. Hadronic events were then selected by
requiring that: (a) each of the two hemispheres \( \cos \theta < 0 \) and \( \cos \theta > 0 \) contained a total energy of the charged particles \( E_{ch} = \Sigma E_i \) larger than 3 GeV, where \( E_i \) are the particle energies; (b) the total energy of the charged particles seen in both hemispheres together exceeded 15 GeV; (c) there were at least 5 charged particles with momenta above 0.2 GeV/c; (d) the polar angle \( \theta \) of the sphericity axis was in the range \( 40^\circ < \theta < 140^\circ \) (this cut ensures that the retained events were well contained inside the TPC).

The resulting data sample contains 79015 hadronic events. The contamination from beam-gas scattering, \( \gamma \gamma \) interactions and \( \tau^+ \tau^- \) events is negligible (\(< 0.3\%)\).

3. VARIABLES USED FOR THE SEPARATION

Eighteen variables were used as an input for the separation.

Their choice came from the examination of the literature, and from a study of flavour-dependent distributions based on the JETSET 7.2 Parton Shower Monte Carlo [9] (JETSET PS in the following), that has proven, after two years of activity of LEP, to reproduce well the main features of the hadronic decays of the \( Z^0 \) [8, 10]. The particles in the event were clustered in jets according to the JADE/E00 algorithm [11], with \( y_{\text{cut}} = 0.05 \). In the following, the most energetic jet will be called "first jet", and indicated by the superscript \((f)\); the second most energetic jet will be called "second jet", and indicated by the superscript \((s)\).

The variables used are:

1. The sphericity \( S^{(f)} \) of the first jet, calculated after a boost \( \beta = 0.96 \) along its axis. The axis of the jet was defined by the sum of the momenta of the particles belonging to it.

2. The directed sphericity \( S^{(f)}_{123,4} \) of the 4 most energetic particles in the first jet.
   For a set \( Q \) of tracks in a jet, this variable is defined as
   \[
   S_Q = \frac{\sum Q p_i^2}{\sum Q |p|^2}
   \]
   where the \( |p| \)'s are the momenta in the rest frame of the set \( Q \) and the \( p_i \)'s are their components perpendicular to the original jet direction in the laboratory frame.

3. The directed sphericity \( S^{(s)}_{123,4} \).

4. The invariant mass \( M^{(f)}_{123,4} \) of the 4 most energetic particles in the first jet.

5. The invariant mass \( M^{(s)}_{123,4} \) of the 4 most energetic particles in the second jet.

6.9. The products of the homologue direct sphericities for triplets of particles in the first and second jet, \( S^{(f)}_{ijk} \times S^{(s)}_{ijk} \).
10..13. The products of the homologue invariant masses for triplets of particles in the first and second jet, \( M_{ij}^{(f)} \times M_{ij}^{(s)} \).

14. The momentum of the slowest pion of the jet 1, after a boost along the jet axis corresponding to a \( D^- \) energy equal to one half of the beam energy.

15. Same as 14., for the second jet.

16. The momentum of the most energetic \( K^0 \) in the event (0 if no kaons reconstructed).

17. The momentum component perpendicular to the axis of the nearest jet of the most energetic \( K^0 \) in the event (0 if no kaons reconstructed).

18. The sum over the jets of the ratios between the momentum of the leading particle and the momentum of the jet.

All variables were rebinned in such a way that they were ranging from 0 to 1. Examples of the distributions of the variables can be seen in Fig. 1, compared with a simulation based on JETSET PS plus the full detector simulation DELSIM [12].

4. THE NEURAL NETWORK CLASSIFIER

Four independent feed-forward neural networks have been used (one for each class that had to be separated) with 18 nodes in the input layer, associated with the input variables \( z_i \), defining the pattern space \( P \); a variable number of nodes in the hidden layer; and one output node, associated with the output value \( \Theta \), belonging to the feature space \( F \). The number of nodes in the hidden layer of each network was chosen by making different trials with an increasing number of nodes, and stopping when the classification efficiency was reaching a plateau.

In the structure chosen, each neuron (node) performs a weighted sum of the output values from all the nodes of the previous layer; the node output is then computed via a sigmoid function

\[
g_T(z) = \frac{1}{1 + e^{-z/T}}
\]

at a "temperature" \( T^3 \). The output \( o_i \) of the \( i \)-th neuron of a layer (starting from the second) is then

\[
o_i = g_T(\sum_j w_{ij} o_j),
\]

where the sum is made over the nodes of the previous layer.

\[\text{The sigmoid function squeezes the node output between 0 and 1; the final output becomes more peaked at 0 or 1 as the temperature decreases, becoming exactly 0 or 1 in the limiting case in which the temperature is zero.}\]
Figure 1: Individual input variables (simulation). (a) Variable 4 (see text) for $b\bar{b}$ events (crosses), and for non-$b\bar{b}$ events (solid); (b) Variable 14 for $c\bar{c}$ events (crosses), and for non-$c\bar{c}$ events (solid). The distributions are normalized to the same area.
The network training procedure fixes the values of the weights associated with the node interconnections. The aim is to realize a mapping of the input pattern space \((z, \epsilon \in P)\) to the feature space \((\Theta \in F)\), such that a good separation of events belonging to a class \(A\) from events belonging to the complementary class \(\overline{A}\) is obtained by a simple cut in \(F\).

In the “back-propagation” learning algorithm the output feature values obtained from the “training” input patterns are computed and compared with the corresponding desired “target” values. A least mean square error function \(E\) is computed to quantify the difference between the obtained output \(\Theta\) and the desired target \(t\).

\[ E = \frac{1}{2} \sum_{\text{patterns}} (t - \Theta)^2. \]

This function is minimized by changing (“updating”) the weights by an amount computed from the error function by the gradient descent method [3]. The process is controlled by the “learning strength parameter” \(\eta\) and the “momentum” \(\alpha\) [3]: each updating step in the space of weights, computed by gradient descent, is multiplied by \(\eta\) and added to the previous step, multiplied by \(\alpha\). To smooth out fluctuations, weights are updated using the cumulative error from a number of input training patterns (10 in our case). A sequence of 10 patterns will be simply called an “update” in the following.

For the training of the system, a set of 100,000 events generated by the Monte Carlo JETSET PS, plus a detector simulation, was used. The training was made with an equal number of “signal” (class \(A\)) and background (class \(\overline{A}\)) events, because it has been verified that this improves the performance of the network [6].

The training was done using the simulator JETNET [13] and the simulator developed in [6]. No differences in the performance were noted.

Two symmetric target values (0 for class \(A\) and 1 for class \(\overline{A}\)) were used.

At each step of the learning procedure, an indication on the performance of the network can be inferred from the error function. A more reliable evaluation is obtained by testing the response of the network on a set of input patterns independent of the training set. The test sample was generated by the Monte Carlo program JETSET PS, plus the full simulation of the detector DELSIM [12]. The test sample was made by 40,000 events.

The “signal efficiency” \(\epsilon_S\) was measured as the ratio:

\[ \epsilon_S = \frac{N^S_S}{N_S}, \]

where \(N^S_S\) is the number of patterns accepted by the \((\Theta > \Theta_{\text{cut}})\) criterion in a sample of \(N_S\) input patterns of “\(A\)” type. The “purity” \(p\) was defined as:

\[ p = \frac{N^S_S}{[N^S_S + N_B^S]}, \]

where \(N^B_B\) is the number of patterns, accepted by the same criterion, from a sample of \(N_B\) background patterns. The purity can be interpreted as the fraction of “\(A\)”
Table 1: Characteristics of the four neural networks

<table>
<thead>
<tr>
<th>Nodes in the hidden layer</th>
<th>Network 1 ((u\bar{u} + d\bar{d}))</th>
<th>Network 2 ((s\bar{s}))</th>
<th>Network 3 ((c\bar{c}))</th>
<th>Network 4 ((b\bar{b}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(_{layer 1} - layer 2)</td>
<td>1.0</td>
<td>2.0</td>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>T(_{layer 2} - layer 3)</td>
<td>0.33</td>
<td>0.67</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>(\alpha) (training)</td>
<td>0.1 - 0.8</td>
<td>0.5 - 0.8</td>
<td>0.1 - 0.9</td>
<td>0.5 - 0.9</td>
</tr>
<tr>
<td>(\eta) (training)</td>
<td>0.1 - 0.015</td>
<td>0.2 - 0.011</td>
<td>0.05 - 0.001</td>
<td>0.01 - 0.001</td>
</tr>
</tbody>
</table>

events to be found in a mixed sample, selected by the criterion \(\Theta > \Theta_{cut}\), if the input events are a mixture of signal and background in the proportion predicted by the Standard Model.

In the training phase, the four networks were specialized in such a way that network “1” was designed to be more performant for separating \(Z^0\) decays into \(u\bar{u}\) or \(d\bar{d}\), network “2” for separating decays into \(s\bar{s}\) pairs, network “3” for separating decays into \(c\bar{c}\) pairs, and network “4” for separating decays into \(b\bar{b}\) pairs.

The architecture of each network is summarized in Table 1, together with the parameters used in the training phase. For the four neural networks, purity from the test sample is plotted versus efficiency in Fig. 2.

5. ANALYSIS AND RESULTS

From each of the four networks \((i = 1...4)\), the fraction of events \(\beta_j^{(i)}\) of each class \(j\) \((j = 1...4)\) was determined by means of a \(\chi^2\) fit to the form

\[
\mathcal{R}^{(i)}(t) = \sum_j \beta_j^{(i)} a_j^{(i)}(t),
\]

where \(\mathcal{R}^{(i)}(t)\) is the map of the data through the network \(i\) into the feature space, and \(a_j^{(i)}(t)\) are the distributions for each class \(j\) in the feature space, determined in the test sample. All distributions were normalized to unity.

The four networks are constructed in such a way that each network provides a fit with small (in module) correlation coefficients between the class that the network itself was taught to distinguish and the other classes.

The determinations of the four branching fractions from each of the four networks are listed below, with the correlation coefficients \(C^{(i)}\) from the fits. The four networks have been checked to be statistically independent inside the accuracy of the measurement.

\[
\bar{\beta}^{(1)} = (0.455 \pm 0.034, 0.198 \pm 0.036, 0.155 \pm 0.041, 0.186 \pm 0.030)
\]
Figure 2: Purity of the sample selected and efficiency for background selection for each of the four networks, as a function of the efficiency for the selection of signal.
\[
C^{(i)} = \begin{pmatrix}
1.000 & -0.785 & -0.059 & -0.086 \\
-0.785 & 1.000 & -0.380 & 0.206 \\
-0.059 & -0.380 & 1.000 & -0.834 \\
-0.086 & 0.206 & -0.834 & 1.000 \\
\end{pmatrix}
\]
\[
\chi^2_{(i)}/NDF = 122/(92 - 4)
\]
\[
\vec{\beta}^{(2)} = (0.421 \pm 0.036, 0.183 \pm 0.036, 0.198 \pm 0.046, 0.193 \pm 0.013)
\]
\[
C^{(2)} = \begin{pmatrix}
1.000 & -0.670 & -0.477 & -0.271 \\
-0.670 & 1.000 & -0.276 & 0.337 \\
-0.477 & -0.276 & 1.000 & -0.332 \\
-0.271 & 0.337 & -0.332 & 1.000 \\
\end{pmatrix}
\]
\[
\chi^2_{(2)}/NDF = 122/(97 - 4)
\]
\[
\vec{\beta}^{(3)} = (0.435 \pm 0.052, 0.197 \pm 0.043, 0.104 \pm 0.015, 0.257 \pm 0.040)
\]
\[
C^{(3)} = \begin{pmatrix}
1.000 & -0.589 & -0.269 & -0.569 \\
-0.589 & 1.000 & -0.086 & -0.258 \\
-0.269 & -0.086 & 1.000 & 0.069 \\
-0.569 & -0.258 & 0.069 & 1.000 \\
\end{pmatrix}
\]
\[
\chi^2_{(3)}/NDF = 130/(100 - 4)
\]
\[
\vec{\beta}^{(4)} = (0.411 \pm 0.056, 0.196 \pm 0.043, 0.180 \pm 0.043, 0.208 \pm 0.010)
\]
\[
C^{(4)} = \begin{pmatrix}
1.000 & -0.634 & -0.578 & -0.285 \\
-0.634 & 1.000 & -0.233 & 0.186 \\
-0.578 & -0.233 & 1.000 & -0.045 \\
-0.285 & 0.186 & -0.045 & 1.000 \\
\end{pmatrix}
\]
\[
\chi^2_{(4)}/NDF = 115/(100 - 4)
\]

A graphical output of the fits is displayed in Fig. 3.

Finally, the expression
\[
\chi^2 \approx \sum_i (\vec{\beta}^{(i)} - \vec{\beta}^*)^T C^{(i)-1} (\vec{\beta}^{(i)} - \vec{\beta}^*)
\]

where \(C^{(i)}\) is the covariance matrix in the fit from the i-th network, was minimized with respect to \(\vec{\beta}^*\), under the constraint that the sum of the branching fractions is equal to 1. This led to the determinations
\[
\frac{\Gamma_{\mu + d \ell}}{\Gamma_h} = 0.417 \pm 0.015
\]
\[
\frac{\Gamma_{e\tau}}{\Gamma_h} = 0.233 \pm 0.016
\]
\[
\frac{\Gamma_{e\tau}}{\Gamma_h} = 0.139 \pm 0.010
\]
\[
\frac{\Gamma_{e\epsilon}}{\Gamma_h} = 0.211 \pm 0.006,
\]
where the error quoted gives the size of the minimal hypercube that contains the error hyperellipse, and thus keeps into account correlations.
Figure 3: Output of the four neural networks on real data (open circles), compared to the Monte Carlo predictions in the cases that the species on which the network is specialized is absent (solid line) or corresponds to the full sample (dashed line). Networks 1 (a), 2 (b), 3 (c), 4 (d).
5.1 Study of Systematics

To study the systematics, the uncertainties in the best tuning of parameters in JETSET PS as parametrized in [10] were kept into account. In addition, a detailed study of the effect of fragmentation parameters was done.

Due to the amount of computer time required to provide a full simulation of events, systematics were studied by comparing approximate fast simulations with varying parameters (taken as “test data”) with approximate fast simulations using JETSET PS with parameters tuned as in [10], taken as “real data”.

The sources of systematic uncertainties which have been considered are listed below, and the effects are summarized in Table 2. The ranges of variation correspond to 1σ uncertainties, mainly from the determinations of [10].

1. $F_{u\bar{u}}/F_{d\bar{d}}$. The ratio between the branching fractions into $u\bar{u}$ pairs and the fraction into $d\bar{d}$ pairs is not known a-priori, and was assumed in the Monte Carlo as in the Standard Model. The effect of this assumption was checked by allowing a variation between a situation in which the $u\bar{u}$ events are completely absent and a situation in which the $d\bar{d}$ events are completely absent.

2. $A$. The parameter $A$ in JETSET PS was allowed to vary between 0.28 and 0.31 GeV.

3. $Q_0$. The cutoff on the parton evolution was allowed to vary between 0.7 and 1.8 GeV.

4. Fragmentation. Systematic effects from fragmentation were checked:
   - In the Lund fragmentation scheme, by allowing a variation of the $a$ parameter between 0.13 and 0.30 (the $b$ parameter was fixed at 0.34 GeV$^{-2}$).
   - In the Peterson fragmentation scheme, by allowing a variation of $\varepsilon_b$ between $2 \times 10^{-3}$ and $7 \times 10^{-3}$, and a variation of $\varepsilon_c$ between $15 \times 10^{-3}$ and $40 \times 10^{-3}$.

The average of the systematics in the two cases listed above was taken as estimator of the systematic uncertainty due to fragmentation.

5. $\gamma_s/\gamma_n$. A range of variation between 0.25 and 0.30 was allowed, consistent with the averages from [14].

6. $Br(D^{*\pm} \rightarrow D^0\pi^{\pm})$. A range of variation between 0.49 and 0.66 was allowed.

Comparisons with three different models for the decay of the $Z^0$ were tried: ARIADNE[15], HERWIG [16] and the JETSET Monte Carlo with QCD 2nd order generation of the initial state at the parton level (JETSET ME). The Monte Carlos ARIADNE and HERWIG were tuned as in [10]; for JETSET ME, both the tunings suggested in [10] and [17] were used. In all cases, the comparison was impossible due
5.2 Separation of $\gamma$ from $dD$

These results, using the determination of the hadronic branching fractions of $\gamma$ from $Z^0$, provide a fifth independent equation relating the hadronic branching fraction of $dD$ to the emitted photon, but is proportional to the square of the charge of the quark.

The cross section for this process is approximately independent of the quark mass and is given by DELPHI [18]:

$\Gamma_{\gamma} = 1728 \pm 126 \text{ MeV}$

$\Gamma_{d} = 1728 \pm 126 \text{ MeV}$

Due to the effect of systematic uncertainties, the range appeared to be overestimated. The inverse spread of the momentum of hadrons in the JETSET PS parameter $\Delta$ is assigned in [10]. This range could reproduce the data within 2%.

Due to the fact that no values of the Monte Carlo simulation of such Monte Carlo should be done before they are useful in an analysis. A range of uncertainty at $\Delta = 0.22$ and $0.40 \text{ GeV}$ is assigned.

The result for the branching fraction of $dD$ with a $\chi^2/NDF < 3$ (the cut was made at $\chi^2/NDF = 2$) means that a better model of this branching fraction could reproduce the data within the confidence interval, it was not possible to find a set of values for the branching fractions for which the network could be fitted with a $\chi^2/NDF < 3$.
From the average of Ref. [19], one obtains
\[
\frac{\Gamma_{ uu}}{\Gamma_h} + \frac{\Gamma_{ dd}}{\Gamma_h} = 0.44 \pm 0.08
\]
that, together with the determinations of the hadronic branching fractions of the four classes separated by the networks, gives
\[
\frac{\Gamma_{ uu}}{\Gamma_h} = 0.30 \pm 0.10 \\
\frac{\Gamma_{ dd}}{\Gamma_h} = 0.12 \pm 0.10.
\]

The errors quoted above are of course almost completely anticorrelated.

6. CONCLUSIONS

The first determination in the literature of the hadronic branching fractions of the \( Z^0 \) boson into all five known quarks has been presented. To the knowledge of the authors, this is also the first time that results on a taxonomy problem have been obtained in high-energy physics using neural networks.

The results on all branching fractions are consistent with the predictions of the Standard Model.

Neural networks have shown the potentiality of a powerful tool for classification. The errors quoted will substantially decrease when variables related to impact parameters and lepton spectrum will be used as additional inputs [6].

The study of the effect of changing the Monte Carlo model used for computing efficiencies will require a more complete work of optimization of the other models available.

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