ABSTRACT. -

A study of $\pi^+ n \rightarrow p \pi^+ \pi^+ \pi^- \pi^-$ reaction at 5.1 GeV/c is reported. $A_{33}^{++}$ production characteristics are discussed and evidence of structures in the $(4\pi)^0$ system is presented.

1. - INTRODUCTION. -

In this paper we report results relative to the process:

(1) $\pi^+ d \rightarrow p p \pi^+ \pi^+ \pi^- \pi^-$

obtained in the study of $\pi^+ d$ interactions at 5.1 GeV/c with the 80 cm DBC exposed at the CERN proton-synchrotron.

We have selected interactions with five or six outgoing prongs where respectively one or two positive tracks could be recognized by ionization as proton candidates. From the scanning of about 450,000
pictures we selected about 8000 good topologies. After measuring and processing through the Thresh-Grind chain, 1572 of these candidates were unambiguously classified as belonging to channel (1). The corresponding cross-section is $339 \pm 9 \mu b$.

In the following state we discuss the general features of the process. In particular results on the $\Lambda^{++}_{33}$ and $(4 \pi)^0$ system are presented. Preliminary results on reaction (1) were reported in ref. (1)).

2. - MASS SPECTRA. -

The final state (1) is dominated by the production of the $\Lambda^{++}_{33}$ and $\eta^0$ resonances as shown in Fig. 1a) and 1b).

The cross sections obtained from a Breit-Wigner fit with hand-drawn phase space are respectively $39.0 \pm 3.0 \mu b$ for $\pi^+ n \rightarrow \Delta^{++}_{33} \pi^+ \pi^- \pi^-$ and $40.0 \pm 2.9 \mu b$ for $\pi^+ n \rightarrow p \pi^+ \pi^- \eta^0$.

The $p \pi^-$ mass spectrum (not shown) gives evidence for $\Lambda^{9}_{33}$ production corresponding to a cross section of $22.0 \pm 2 \mu b$ for the final state $\pi^+ n \rightarrow \Delta^{9}_{33} \pi^+ \pi^+ \pi^-$. The $(3 \pi)^+$ mass spectra are reported in Fig. 1c) 1f). There is some indication of $A^+_1$, $A^+_{12}$ especially when we select events in the $\eta^0$ mass region in association with the $\Lambda^{++}_{33}$. It is interesting to remark that in these final states the $A^+_1$ cannot be produced diffractively.

2.1. - $\Delta^{++}_{33}$ production mechanisms. -

The presence of $\Delta^{++}_{33}$ is surprising as the neutron-$\Lambda^+_1$ vertex is allowed only by an $|I_{33}|=2$ exotic exchange.

In the following stage we discuss the different mechanisms which could explain the observed $\Delta^{++}_{33}$ production:

1) higher isobars production decaying in ($\Delta^{++}_{33} \pi^-$).
2) kinematical reflections due to the decay of $(4 \pi)^0$ system produced in a single particle exchange (Fig. 2a) or interaction in the final state.
3) exotic exchange ($|I_{33}|=2$) in a single particle exchange diagram (Fig. 2b).
4) double exchange of particles having $|I_{33}|=1$ in a diagram of the type of Fig. 2c.

(x) - The well known impulsive approximation allows us to consider the process (1) as interaction on the free neutron with the slower proton as spectator ($p_s$).
FIG. 1 - Mass spectra for reaction $\pi^+n \rightarrow p\pi^+\pi^+\pi^-\pi^-$.

a) $m(p\pi^+)$: the curve reported is the results of the fit with hand drawn phase space plus one Breit-Wigner for the $\Delta^{++}$. b) $m(\pi^+\pi^-)$: the curve reported is the result of the fit with hand drawn phase space plus one Breit-Wigner for the $\phi$. 
FIG. 1 - Mass spectra for reaction $\pi^+ p \rightarrow p \pi^+ \pi^+ \pi^- $.

- c) $m(\pi^+ \pi^+ \pi^-)$: $\rho^0$ with $\phi^0$ selection
- d) $m(\pi^+ \pi^+ \pi^-)$: $\rho^0$ and $\Delta_{33}^0$ selections
- e) $m(\pi^+ \pi^- \pi^-)$: $\rho^0$ with $\phi^0$ selection
- f) $m(\pi^+ \pi^- \pi^-)$: $\phi^0$ and $\Delta_{33}^{++}$ selection.

$\phi^0 \equiv (0.65-0.85 \text{ GeV}/c^2)$; $\Delta_{33}^{++} \equiv (1.12-1.32 \text{ GeV}/c^2)$. 
FIG. 2 - Exchange mechanisms discussed for the $\Delta^{++}$ production: a) Kinematical reflections of the $(4\pi)^0$ decay. b) Exotic exchange in a single particle exchange diagram. c) Double exchange of particles having $|I_z| = 1$.

We can exclude a higher isobar production: there is no indication of higher isobars in the $p\pi^+\pi^-$ mass-combinations in such an amount to justify the $\Delta^{++}$ production.

In order to investigate the kinematical reflections, we consider the Jackson angle $\theta^4\pi$ of the $\pi^+$ in the $(4\pi)^0$ rest frame (2).

Fig. 3 shows the mass distribution ($p\pi^+$) for three different regions of $\cos \theta^4\pi$ [respectively $\cos \theta^4\pi < -0.5$ (Fig. 3a), $|\cos \theta^4\pi| < 0.5$ (Fig. 3b), $\cos \theta^4\pi > 0.5$ (Fig. 3c)]. The $\Delta^{++}$ observed in Fig. 3a could be interpreted by the diagram of Fig. 2a or interactions in the final state. But this interpretation is no longer valid if we consider Fig. 3b which still shows the $\Delta^{++}$ production (x). For comparison in Fig. 3e and 3f we report the $(p\pi^+\pi^+\pi^-)$ mass spectra obtained with similar cuts in the $\cos \theta^3\pi$ angle in the channel

(2) $\pi^+p \rightarrow p\pi^+\pi^+\pi^-$

at the same energy, where, as it is well known, the $\Delta^{++}$ is mainly

(x) - A clear $\Delta^{++}$ production is also observed in the higher multiplicity process $\pi^+n \rightarrow p\pi^+\pi^+\pi^-\pi^-\pi^0$ at the same energy. The production characteristics are the same as in reaction (1).
FIG. 3 - \( (p \pi^+) \) mass combinations in the final state \( \pi^+ n \rightarrow p \pi^+ \pi^+ \pi^- \pi^- \) with different cuts on \( \cos \theta_j^{4\pi} \). a) \( \cos \theta_j^{4\pi} \leq -0.5 \), b) \( |\cos \theta_j^{4\pi}| < 0.5 \), c) \( \cos \theta_j^{4\pi} > 0.5 \), -(p \pi^-) mass combinations in the final state \( \pi^+ p \rightarrow p \pi^+ \pi^+ \pi^- \) with different cuts on \( \cos \theta_j^{3\pi} \). d) \( \cos \theta_j^{3\pi} \leq -0.5 \), e) \( |\cos \theta_j^{3\pi}| < 0.5 \), f) \( \cos \theta_j^{3\pi} > 0.5 \).
due to OPE. The behaviour of the spectra in Fig. 3b and 3c are very similar and support the idea that the neutron $\Delta_{33}^{++}$ vertex really could exist. This vertex is extremely important as only permitted by a double charge exchanges in Fig. 2b and 2c.

Regarding the exotic exchange of an $|I_2|=2$ meson, we remember that the exotic meson proposed by Rosner(3) is not coupled to the incoming meson and therefore the graph of Fig. 2b does not explain the $\Delta_{33}^{++}$ production. On the other hand considering the cross sections Fig. 4 for the reactions $\pi^+ n \rightarrow \Delta_{33}^{++} \pi^+ \pi^- \pi^-$ and the charge symmetric(4) $\pi^- p \rightarrow \Delta_{33}^{++} \pi^+ \pi^+ \pi^-$ obtained in hydrogen we find an energy dependence $[\sigma \sim s^{-1.1 \pm 0.5}]$ which is very different from the ones found for other exotic exchange reactions(5) (the exponent of the energy square $s$ is of the order $4.0 - 6.0$).

Therefore it seems that we can exclude a single exchange $|I_2|=2$ graph as dominant. One possible mechanism to explain the observed $\Delta_{33}^{++}$ production could be the double exchange of Fig. 2c).

For what concerns the other characteristics of the $\Delta_{33}^{++}$ in(1) we observe that the $\phi_{\pi^+}$ distribution is quite flat and the $\cos \theta_{\pi^+}$ asymmetric(6) (Fig. 5a and 5b). The corresponding distributions for the $\Delta_{33}^{++}$ decay in reaction (2) are in Fig. 5c and 5d. They are in agreement with the OPE dominant mechanism. The low $e^{-\Delta t'/(t' - t_{\min})}$

\begin{itemize}
  \item We select the $\Delta_{33}^{++}$ with the cut $1.12 - 1.32$ GeV/c$^2$ in the $p\pi^+$ mass combination, $\phi_{\pi^+}$ and $\theta_{\pi^+}$ are the decay angles of $\Delta_{33}^{++}$ in the Jackson's frame of the isobar. The dashed distribution corresponds to positive pions with lower longitudinal momentum.
\end{itemize}
FIG. 5 - $\Delta_{33}^{++}$ decay in reaction $\pi^+n \rightarrow p\pi^+\pi^+\pi^-$.

a) $\cos \theta_j^{\pi^+}$, b) $\phi_j^{\pi^+}$. $\Delta_{33}^{++}$ decay in reaction $\pi^+p \rightarrow p\pi^+\pi^+\pi^-$. c) $\cos \theta_j^{\pi^+}$, d) $\phi_j^{\pi^+}$.

Distributions with the $(p\pi^+)$ combination corresponding to the $\pi^+$ with lower longitudinal momentum.
has been fitted to the differential cross sections \( \frac{dN}{d\Omega'} \) of the \( A_{33}^{++} \) produced in reaction (1) and (2) (Fig. 6a and 5b). We obtain the results reported in Table I. The \( A_{33}^{++} \) produced in reaction (1) is less peripheral than the \( A_{33}^{++} \) produced in the final state (2).

2.2. - \((4\pi)^0\) mass system.

Fig. 7 shows the total \((4\pi)^0\) mass spectrum. Different attempts have been made to obtain a "real" phase space reproducing the general shape of the spectrum.

The statistical phase space is absolutely not able to describe the total mass spectrum. No big improvement is obtained if the \( q^0 \) is taken into account. In both the cases indicated as curves A and B in Fig. 7a the shape does not fit the general behaviour. At the end we have tried to parametrize the phase space with a polynomial formula. A good fit of the total mass spectrum is obtained with a polynomial phase space background (the 4th degree is the one with a reasonable number of parameters) and four Breit-Wigner for the structure indicated by the arrows. The results obtained are reported in Table II.

By using higher degree polynomials and cutting the events with the \((p\pi^+)\) mass combination in the \( A_{33}^{++} \) region we do not change sensitively the previous results. The mass spectra \((q^0\pi^+\pi^-)\), \((A_2^+\pi^+)\) and \((q^0q^0)\) are reported in Fig. 8. The more interesting result concerns the \((q^0q^0)\) spectrum obtained when we eliminate the \( A_{33}^{++} \) events (Fig. 8d). The statistical significance of the narrow peak at \( \sim 1.6 \) GeV/c with respect to the phase space is at a limit of \( \sim 3 \) standard deviation. As it is evident from the Glebsch-Gordon coefficient the \((q^0q^0)\) decay mode is permitted only for \( I^G=0^+ \) state. Such a structure, if it exists, could not therefore correspond to the \( q^0 \) observed in the same mass region. Some indications of similar peaks are reported in \( pp \) reactions. We have studied also the \((\pi^+\pi^-)\) and \((3\pi)^\pm\) mass distribution in the three \((4\pi)^0\) mass regions corresponding to our structure (x) (Fig. 9). The \( q^0 \) is

(x) - With the Monte Carlo program we have generated events of type (1) and introduced the percentage of \( q^0 \) events produced in this final state. All the other experimental conditions have been taken into account i.e. Fermi motion of the nucleons in the deuterium target, cuts on the fast proton etc.

(x) - These regions are defined as follows: A-region (1.55–1.65 GeV/c\(^2\)), B-region (1.65–1.85 GeV/c\(^2\)), C-region (1.85–2.0 GeV/c\(^2\)).
TABLE I

Slopes A for differential cross sections in the reaction
\[ \pi^+ n \rightarrow \Delta_{33}^{++} \pi^+ \pi^- \] and \[ \pi^+ p \rightarrow \Delta_{33}^{++} \pi^+ \pi^- \] at 5.1 GeV/c
(0. < t' < 0.3 GeV/c^2/c^2).

<table>
<thead>
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<th>Final state</th>
<th>( \chi^2/ND )</th>
<th>A</th>
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<tbody>
<tr>
<td>( \pi^+ n \rightarrow \Delta_{33}^{++} \pi^+ \pi^- )</td>
<td>6.1/13</td>
<td>5.1+0.6</td>
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<tr>
<td>( \pi^+ p \rightarrow \Delta_{33}^{++} \pi^+ \pi^- )</td>
<td>39.8/13</td>
<td>8.7+0.3</td>
</tr>
</tbody>
</table>

TABLE II

Fit of the \( (4\pi)^0 \) mass spectrum with a 4th degree polynomial and four Breit Wigner

<table>
<thead>
<tr>
<th>Peak</th>
<th>1\textsuperscript{st}</th>
<th>2\textsuperscript{nd}</th>
<th>3\textsuperscript{th}</th>
<th>4\textsuperscript{th}</th>
<th>( \chi^2/ND )</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1.242( \pm 0.012 )</td>
<td>1.600( \pm 0.015 )</td>
<td>1.719( \pm 0.012 )</td>
<td>1.904( \pm 0.010 )</td>
<td>37./59.</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>0.065( \pm 0.015 )</td>
<td>0.096( \pm 0.015 )</td>
<td>0.088( \pm 0.015 )</td>
<td>0.040( \pm 0.010 )</td>
<td></td>
</tr>
</tbody>
</table>
\[ \pi^+ n \rightarrow A_{13}^{++} \pi^+ \pi^- \pi^- \]

\[ A = 5.1 \pm 0.6 \]

\[ \pi^+ p \rightarrow A_{33}^{++} \pi^+ \pi^- \pi^- \]

\[ A = 8.7 \pm 0.3 \]

**FIG. 6** - a) Differential cross section \( dN/dt' \) in reaction \( \pi^+ n \rightarrow A_{13}^{++} \pi^+ \pi^- \pi^- \). b) Differential cross section \( dN/dt' \) in reaction \( \pi^+ p \rightarrow A_{33}^{++} \pi^+ \pi^- \).

**FIG. 7** - \((4\pi)^0\) total mass spectrum.

For the meaning of the curves see the text.
FIG. 8 - a) $(\rho^0 \pi^+ \pi^-)$ combinations with defined as the $\pi^+ \pi^-$ mass region $0.65 - 0.85$ GeV/$c^2$.
b) $(A_2^+ \pi^+)$ with $A_2^+$ obtained selecting the $(3\pi)^+$ in the mass region $1.25 - 1.35$ GeV/$c^2$. In dashes the contributions of the $A_2^+ \rightarrow \rho^0 \pi^+$ decay are indicated.
c) $(\rho^0 \rho^0)$ mass spectrum.
d) $(\rho^0 \rho^0)$ mass spectrum anti-selecting the events with $(p\pi^+)$ combination in the $\Delta^{++}$ mass region.
The curve reported is the statistical phase space.
FIG. 9 - Dipion (π⁺π⁻) and (3π)± mass system in the A(1.55-1.65 GeV/c²) B(1.65-1.85 GeV/c²) and C(1.85-2.0 GeV/c²) (4π)⁰ mass region.

a) b) c) (π⁺π⁻) distributions in A, B, C respectively.
d) e) f) (3π)⁺ distributions in A, B, C respectively.
g) h) i) (3π)⁻ distributions in A, B, C respectively.
j) k) l) (3π)± distributions in A, B, C respectively.
In dashes the (3π)± mass spectra obtained selecting the q⁰ events are reported.
present in all the three regions and $A_2$ seems to be chiefly associated with the central region ($\sim 1.720$ GeV/c$^2$). The difficulty in drawing a correct phase space does not allow us to evaluate branching ratios and to estimate how much of the structures could be to kinematical selections.

3. - CONCLUSIONS. -

Our analysis of reaction (1) indicate that

1) the process is dominated by $\Delta_3^{++}$, $\Delta_3^{0}$, $\phi^0$ resonances production,
2) the discussion of the production mechanism for the $\Delta_3^{++}$ favours the presence of double particle exchange contribution,
3) the $(4\pi)^0$ system has indications of structures and the more interesting of these is the peak at $1.6$ GeV/c$^2$. We clearly want to point out that our effect is at the limit of three standard deviation and its significance is quite poor. Nevertheless its width and decay mode, seem not to correspond to the $\phi'$ meson.

We would like to thank all the members of Bari-Bologne-Florence-IPN Collaboration to allow us to use the data for the present analysis.
REFERENCES.


(4) - The following energies in the CM are used in the fit:


