THE SEARCH FOR Z*-s, THE K+-NUCLEON RESONANT STATES AT
ABOUT 1800 MeV/c² MASS

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1. INTRODUCTION

A barion with positive strangeness cannot be made from three quarks; in fact five quarks at least (more precisely four quarks and one anti-quark) are necessary to build states with barion number $B=+1$ and strangeness $S=+1$.

If such a state really exists we begin to build-up a sort of spectroscopy of quarks; therefore the simplest SU(3) classification for mesons and barions must be considered no longer valid, since in the naive SU(3) model we need 2 quarks to form a meson and 3 quarks to form a barion.

Up to now no clear evidence has been found for meson and barion states which do not fit in the SU(3) scheme; it is therefore very important to establish the existence of these exotic states, which we call $Z$, following the literature.

The best way to reveal these states is the study of the $K^+$-nucleon interaction since the $K^+$-nucleon system has exactly the requested quantum numbers.

In the experiments, in which the $K^+p$ and $K^+d$ total cross sections versus energy have been measured, some structures have been found at around 1000 MeV/c $K^+$ incident laboratory momentum, corresponding to an invariant mass of $\approx 1800$ MeV/c$^2$ when the target nucleon is supposed to be at rest.

From the above discussion it is apparent that it is very important to determine if these structures are due to the formation of s-channel resonances.

As far as the production experiments for such exotic states are concerned, a limited work has been carried out; these states have been searched:

a) in the photoproduction reaction $\gamma + p \rightarrow K^+Z^{+++}$ (Mori 69);
b) in the reaction $K^+p \rightarrow K^N + \pi^+$ ($\pi^=1,2,3,4$) to be interpreted as $K^+p \rightarrow Z^+6\pi$ ($0 \leq 6\leq 4$) (Bassompierre 69);
c) in the reaction $\pi^-p \rightarrow K^-Z^{**}$ (Anderson 69)

but no evidence has been found; it has been possible to put only upper limits, which on the other hand are not very small, as it has been pointed out by Erne (70).

For this reason in this paper we discuss the formation experiments in the 0-2000 MeV/c range. We emphasize the situation of the I=0 channel, where in particular the Bologna-Glasgow-Roma-Trieste (BGET) collaboration has collected new data in the $K^+d$ experiment between 600 and 1500 MeV/c. We analyze the $K^+p$ and $K^+d$ total cross sections in Sect. 2 where the problem of the extraction of the I=0 total cross-section is also briefly discussed. A lot of new data on $K^+p$ differential cross section are now at our disposal; these data are commented in Sect. 3 while in Sect. 4 the I=1 inelastic cross-section is presented. The relevance of all these data on the general situation of the I=1 channel is discussed in Sect. 5.

In Sect. 6 we consider the problems with deuterium in connection with the extraction of the free-neutron differential cross sections. The data on the $K^+d$ inelastic cross-section are shown in Sect. 7; the deduction of the I=0 inelastic cross-section is studied in Sect. 8 while in Sect. 9 we illustrate the I=0 elastic cross-section, that shows a big bump at about 800 MeV/c. In the three following sections we show the last data on the $K^+n \rightarrow K^+p$ (Sect. 10), $K^+n \rightarrow K^+n$ (Sect. 11), and $K^+d \rightarrow K^+d$ (Sect. 12).
and \( K^+d \rightarrow K^+d \) (Sect. 12) differential cross-sections. The status of the \( I=1 \) and \( I=0 \) phase shift analyses is revised in Sect. 13 and 14 respectively. At the end in Sect. 15 we show that the \( Z^+ \) search is still a genuine search and we conclude with some comments on the experiments in progress.

2. - THE \( K^+p \) AND \( K^+d \) TOTAL CROSS-SECTIONS.

The possibility of doing experiment both on hydrogen and on deuterium target allows also the extraction of informations on \( K^+ \)-nucleon interaction in both \( I=0 \) and \( I=1 \) isospin states.

As far as the total cross sections are concerned we have for the \( K^+ \)-proton total cross-section

\[
\sigma_T(K^+p) = \sigma_1
\]

(1)

and for the \( K^+ \)-neutron total cross-section

\[
\sigma_T(K^+n) = \frac{1}{2} (\sigma_1 + \sigma_0)
\]

(2)

where \( \sigma_1 \) and \( \sigma_0 \) are the total cross-sections in \( I=1 \) and \( I=0 \) isospin state respectively.

For the total \( K^+ \)-deuterium cross section we have then

\[
\sigma_T(K^+d) = \sigma_T(K^+p) + \sigma_T(K^+n) - \delta = \frac{1}{2}(3\sigma_1 + \sigma_0) - \delta
\]

(3)

where \( \delta \) is the Glauber (59) screening correction. Here, for shortness, we suppose that, at least for the total cross-sections, all the difficulties, connected to the fact that a free-neutron target is not available, can be solved; therefore in the formula (3) the total cross-sections are to be considered on the respective target as free and at rest while all the effects due to the fact that the deuteron is a proton-neutron bound state are collected in \( \delta \). (For a more complete discussion of this problem see also for example Cool (70)). Then we simply obtain

\[
\sigma_1 = \sigma_T(K^+p)
\]

(4)

\[
\sigma_0 = 2\sigma_T(K^+d) - 3\sigma_T(K^+p) + 2\delta
\]

(5)

In Fig. 1 we show in the 0-2000 MeV/c range the total \( K^+p \) cross section versus incident \( K^+ \) momentum with the more recent data from Carroll (73), Adams (73) and BGRT collaboration (Cameron 74). In particular these recent data exclude the existence of a minimum at around 700 MeV/c, that appeared to be possible on the basis of previous experiments (Bugg 68, Bowen 70). Analogously in Fig. 2 the \( K^+d \) total cross-sections is plotted versus incident \( K^+ \) momentum with the more recent data from Carroll (73). Below 700 MeV/c the behaviour still shows some uncertainty. In
Table 1 the relevant informations for Fig. 1 and 2 are collected.

From the two $K^p$ and $K^d$ measured total cross-sections it is possible to obtain $\sigma_o$ by means of formula (5). In Fig. 3 the results of this extraction are shown in the 400 - 1400 MeV/c range; these $I=0$ cross section values have been computed from the data show in Fig. 1 and 2 by Carroll (73) using a technique that has been tested in earlier experiments (Abrams 69 and 70, Cool 70).

The experimental situation below 800 MeV/c appears not yet to be definitively clarified. Nevertheless the existence of two structures at about 750 MeV/c and 1100 MeV/c is confirmed but the exact shape of the first structure is not yet defined.

In Fig. 4 we show a sort of resumé of the situation concerning the $I=0$ and $I=1$ total cross-sections in the 0-2000 MeV/c range. It is obvious that the informations derived from the total cross-sections are not enough to establish the existence of one or more $Z^*$ resonances.

TABLE 1

<table>
<thead>
<tr>
<th>Reference</th>
<th>Channel</th>
<th>Momentum range MeV/c</th>
<th>$N^o$ of momenta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goldhaber 62</td>
<td>$K^p$</td>
<td>140 - 642</td>
<td>8</td>
</tr>
<tr>
<td>Bugg 68</td>
<td>$K^p$ and $K^d$</td>
<td>592 - 1945</td>
<td>21</td>
</tr>
<tr>
<td>Abrams 70</td>
<td>$K^p$ and $K^d$</td>
<td>1550 - 2000</td>
<td>9</td>
</tr>
<tr>
<td>Bowen 70</td>
<td>$K^p$ and $K^d$</td>
<td>366 - 717</td>
<td>12</td>
</tr>
<tr>
<td>Cool 70</td>
<td>$K^p$ and $K^d$</td>
<td>891 - 1996</td>
<td>20</td>
</tr>
<tr>
<td>Adams 73</td>
<td>$K^p$</td>
<td>432 - 689</td>
<td>7</td>
</tr>
<tr>
<td>Bowen 73</td>
<td>$K^p$ and $K^d$</td>
<td>569 - 1160</td>
<td>20</td>
</tr>
<tr>
<td>Carroll 73</td>
<td>$K^p$ and $K^d$</td>
<td>410 - 1065</td>
<td>19</td>
</tr>
<tr>
<td>Cameron 74</td>
<td>$K^p$</td>
<td>145 - 786</td>
<td>12</td>
</tr>
</tbody>
</table>

3. - THE $K^p$ ELASTIC CROSS-SECTION.

The elastic $K^p$ cross section have been extensively measured in this energy range (see Table 2). In the upper part of our interval very high statistics experiments have been made by Charles (72b) - 17 energies between 909 and 1907 MeV/c - and by Barber (73) - 27 energies between 1368 and 2259 MeV/c -. Although we deal with very precise and detailed experiments no new features have been found in the consequent phase shift analyses about the existence of $K^p$ resonant states.
### Table 2

<table>
<thead>
<tr>
<th>Reference</th>
<th>Independent normalization</th>
<th>Total elastic cross-sections</th>
<th>Incident K(^+) momentum (number of points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stubbs (61)</td>
<td>yes</td>
<td>yes</td>
<td>810(7)</td>
</tr>
<tr>
<td>Goldhaber (62)</td>
<td>yes</td>
<td>yes</td>
<td>1,105(6), 1,75(6), 2,05(6), 2,35(6), 2,65(6), 3,55(5), 5,20(5), 6,42(5)</td>
</tr>
<tr>
<td>Cook (63)</td>
<td>yes</td>
<td>yes</td>
<td>970(9), 1,710(16), 1,970(19)</td>
</tr>
<tr>
<td>Bettini (65)</td>
<td>yes</td>
<td>yes</td>
<td>1,455(40)</td>
</tr>
<tr>
<td>Chinowsky (65)</td>
<td>yes</td>
<td>yes</td>
<td>1,960(12)</td>
</tr>
<tr>
<td>Focardi (67)</td>
<td>yes</td>
<td>yes</td>
<td>778(20)</td>
</tr>
<tr>
<td>Bland (69a)</td>
<td>yes</td>
<td>yes</td>
<td>864(18), 969(18), 1,207(19)</td>
</tr>
<tr>
<td>Bland (69b)</td>
<td>yes</td>
<td>no</td>
<td>1,410(1), 1,477(1), 1,539(1), 600(1), 660(1), 721(1)</td>
</tr>
<tr>
<td>Caldwell (70)</td>
<td>yes</td>
<td>yes</td>
<td>900(19), 970(19), 1,060(19), 1,130(19), 1,210(19), 1,250(19), 1,320(19), 1,380(19), 1,450(19)</td>
</tr>
<tr>
<td>Giacomelli (70)</td>
<td>yes</td>
<td>yes</td>
<td>1,900(16), 910(27), 970(18), 1,090(19), 1,170(23), 1,220(20), 1,320(26), 1,370(19), 1,450(15), 1,530(21), 1,610(25), 1,710(23), 1,790(20), 1,850(23), 1,950(20), 2,050(12), 2,150(13)</td>
</tr>
<tr>
<td>Abrow (71)</td>
<td>no</td>
<td>no</td>
<td>1,900(16), 910(27), 970(18), 1,090(19), 1,170(23), 1,220(20), 1,320(26), 1,370(19), 1,450(15), 1,530(21), 1,610(25), 1,710(23), 1,790(20), 1,850(23), 1,950(20), 2,050(12), 2,150(13)</td>
</tr>
<tr>
<td>Charles (72a)</td>
<td>yes</td>
<td>yes</td>
<td>1,900(16), 910(27), 970(18), 1,090(19), 1,170(23), 1,220(20), 1,320(26), 1,370(19), 1,450(15), 1,530(21), 1,610(25), 1,710(23), 1,790(20), 1,850(23), 1,950(20), 2,050(12), 2,150(13)</td>
</tr>
<tr>
<td>Charles (72b)</td>
<td>yes</td>
<td>yes</td>
<td>1,900(16), 910(27), 970(18), 1,090(19), 1,170(23), 1,220(20), 1,320(26), 1,370(19), 1,450(15), 1,530(21), 1,610(25), 1,710(23), 1,790(20), 1,850(23), 1,950(20), 2,050(12), 2,150(13)</td>
</tr>
<tr>
<td>Adams (73)</td>
<td>yes</td>
<td>yes</td>
<td>4,328(15), 4,79(12), 5,25(14), 5,65(16), 6,03(16), 6,45(17), 6,83(17), 7,31(18), 7,72(17), 8,13(18), 8,57(18), 8,99(18), 9,39(18)</td>
</tr>
<tr>
<td>Barber (73)</td>
<td>yes</td>
<td>yes</td>
<td>1,368(33), 1,387(33), 1,414(33), 1,414(33), 1,472(33), 1,501(33), 1,530(33), 1,563(33), 1,594(33), 1,627(33), 1,665(33), 1,696(33), 1,728(33), 1,765(35), 1,799(35), 1,834(35), 1,873(35), 1,909(35), 1,946(35), 2,005(36), 2,087(36), 2,167(36), 2,237(37)</td>
</tr>
<tr>
<td>Barber (70)</td>
<td>no</td>
<td>no</td>
<td>1,368(33), 1,387(33), 1,414(33), 1,414(33), 1,472(33), 1,501(33), 1,530(33), 1,563(33), 1,594(33), 1,627(33), 1,665(33), 1,696(33), 1,728(33), 1,765(35), 1,799(35), 1,834(35), 1,873(35), 1,909(35), 1,946(35), 2,005(36), 2,087(36), 2,167(36), 2,237(37)</td>
</tr>
<tr>
<td>Cameron (74)</td>
<td>yes</td>
<td>yes</td>
<td>1,45(11), 1,75(12), 2,05(21), 2,35(21), 2,65(20), 2,95(20), 3,25(11), 3,55(11), 3,85(11), 5,00(20), 6,13(20), 7,26(20)</td>
</tr>
</tbody>
</table>
It is clear on the other hand that the phase shift analysis between 500 and 1500 MeV/c depends on the behaviour of the low energy phases-shifts and the situation was enough confused since in the region about 700 MeV/c the total cross-section measurements could indicate a deep. Such a minimum would be possible only if the phase behaviour should be relatively complicated; in particular the smooth and monotonic behaviour of the phase shifts below 1 GeV/c with the S, phase-shift dominant and negative (Goldhaber 62) was seriously questioned (Carreras 70a and 70b, Ferreira 71).

In order to clarify the situation the $K^+p$ elastic differential cross-section in the full angular range has been measured in bubble chamber for 13 values between 432 and 939 MeV/c by Adams (73) and for 12 values between 130 and 726 MeV/c by BGRT (Cameron 74). The threshold for one-pion production is ~ 510 MeV/c for a $K^+$ incident on a proton at rest, but there is practically no cross-section for one pion production below ~ 800 MeV/c, the $K^+p \rightarrow K\Delta$ threshold. Therefore we realize that the Adams (73) and BGRT (Cameron 74) measurements are also total cross-section measurements below 800 MeV/c. For this reason these points are shown in Fig. 1, and are in good agreement with the counter measurements (Carroll 73).

As we have already pointed out the result of all these experiments does not show any structure in the behaviour of the total cross-sections between 130 and 800 MeV/c. Moreover the old result of Goldhaber (62) is confirmed and the scattering amplitude at low energy is found to be dominantly repulsive s-wave. In Fig. 5 this result of the last phase shift analysis in the considered energy range is shown (Cameron 74).

The total elastic cross-section is shown in Fig. 6. It appears to be slowly falling down above 800 MeV/c.

### TABLE 3

<table>
<thead>
<tr>
<th>Reference</th>
<th>Momentum range MeV/c</th>
<th>number of momenta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stubbs (61)</td>
<td>810</td>
<td>1</td>
</tr>
<tr>
<td>Goldhaber (62)</td>
<td>642</td>
<td>1</td>
</tr>
<tr>
<td>Bettini (65)</td>
<td>1455</td>
<td>1</td>
</tr>
<tr>
<td>Chinowsky (65)</td>
<td>1960</td>
<td>1</td>
</tr>
<tr>
<td>Filippas (67)</td>
<td>735 - 785</td>
<td>2</td>
</tr>
<tr>
<td>Bland (69b)</td>
<td>864 - 1207</td>
<td>3</td>
</tr>
<tr>
<td>Giacomelli (70)</td>
<td>900 - 1480</td>
<td>9</td>
</tr>
<tr>
<td>Loken (72)</td>
<td>1370 - 1950</td>
<td>5</td>
</tr>
</tbody>
</table>

(only $K^p \rightarrow K\pi^+\pi^0$)
4. - THE \( K^+ \) INELASTIC CROSS-SECTION.

In this range some total inelastic cross-sections have been also measured. The data refer mainly to the 700-1500 range where we have practically only one-pion production. The data on the total inelastic cross-section are shown in Fig. 6; note their rapid rise in the 850-1250 MeV/c range. In Table 3 the relevant data for Fig.6 are collected.

5. - THE I=1 CHANNEL

In Fig. 4 we show a "resumé" of the situation. At this point some results are definitively confirmed:

1) The bump in the I=1 total cross-section cannot derive from the elastic channel.
2) The bump in the I=1 total cross-section could be understood very easily if the behaviour of the elastic and inelastic cross-sections are considered as a whole; the slow failing down of the first while the second rises abruptly can produce the bump. In this case we deal with a simple threshold effect.

From the first result we can state that the phase shift analyses based mainly on the elastic channel should not be in good position in order to reveal a possible resonance in the I=1 channel. Nevertheless, as we have already discussed, a lot of work has been done to measure these differential cross-sections for many \( K^+ \) incident momentum values and with very high statistics.

Moreover, because of the availability of polarized targets, it has also been possible to do polarization measurements with very high precision (Table 4).

---

**TABLE 4.**

<table>
<thead>
<tr>
<th>Reference</th>
<th>Incident ( K^+ ) momentum (Number of points) MeV/c.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andersson (69a)</td>
<td>870(17), 910(23), 970(19), 1090(22), 1170(23), 1220(21), 1320(28), 1370(22), 1450(21), 1540(25), 1640(25), 1740(24), 1790(25), 1890(24), 2150(13), 2740(12).</td>
</tr>
<tr>
<td>Andersson (69b)</td>
<td></td>
</tr>
<tr>
<td>Albrow (71)</td>
<td></td>
</tr>
<tr>
<td>Asbury (69)</td>
<td>1370(24), 1450(21), 1600(35), 1710(23), 1800(33), 1890(25), 2110(23), 2310(18).</td>
</tr>
<tr>
<td>Barnett (71)</td>
<td></td>
</tr>
<tr>
<td>Barnett (73)</td>
<td></td>
</tr>
<tr>
<td>Ehrlich (71)</td>
<td>1130(19), 1430(19), 1540(19), 1640(18), 1730(18), 1820(18), 1910(18), 2010(18), 2100(18), 2190(18), 2290(19), 2390(19), 2430(18), 2580(19).</td>
</tr>
</tbody>
</table>
Although the considerable quantity and quality of all these measurements the phase shift analyses carried out to now do not give a unique solution; this situation is summarised in Table 5. In order to distinguish between the various solutions the authors give some prescriptions. For example the discrimination could be possible if the R parameter of Wolfenstein (Albrow 71, Fish 73) or if the backward polarisation should be measured with high precision; nevertheless the difficulty of these experiments (for R a double scattering on a polarised target is necessary) and the previous experience induce a great caution on these previous.

Taking into account the difficulties of the elastic channel some attempt to clarify the situation in the s-channel has been done in the inelastic channel, where a priori the experimental conditions seen to be better. The reaction $K^+p \rightarrow K^0\pi^+$ has been studied by Bland (70) and BGR (Griffiths 72). This reaction shows a broad bump at $\approx$1100 MeV/$c$ (Fig.7); an energy dependent phase shift analysis has been carried out by Griffiths (72) using the Brody and Kernan (Brody 69) formalism. No unique solution was found; moreover the possible resonant amplitude is associated to the $P_{1/2}$ elastic wave - that is to possible resonant amplitude found in some phase shift analyses based on the elastic channel - but the change in phase from 900 to 1500 MeV/$c$ is very small and the speed plot shows a minimum rather than a maximum in the region of 1200 MeV/$c$. Therefore the conclusion is that a formation of an exotic resonance is not required in the s-channel for the quasi two-body reaction $K^+p \rightarrow K^0\pi^+\pi^-$.  

<table>
<thead>
<tr>
<th>Reference</th>
<th>Momentum range and type of analysis</th>
<th>Number and name of solutions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Les (68)</td>
<td>0-1500 MeV/$c$ e.d. (energy dependent)</td>
<td>4 types</td>
<td>Two of these groups of solutions show some evidence for a resonance in the $P_{1/2}$ partial wave.</td>
</tr>
<tr>
<td>Ayad (70)</td>
<td>0-2500 e.d. (energy independent)</td>
<td>3,3D,II</td>
<td>There is no evidence of a $Z^0$ in the $K^p$ elastic scattering.</td>
</tr>
<tr>
<td>Barber (70)</td>
<td>0-2500 500 - 1400 e.d. 1000 - 2000 e.d.</td>
<td>4,4,$a$,b</td>
<td>No solution provides evidence for a $Z^0$ resonance.</td>
</tr>
<tr>
<td>Cardenas (70)</td>
<td>1500-2000 e.i.</td>
<td>I,II</td>
<td>Two solutions with an attractive S-wave.</td>
</tr>
<tr>
<td>Blencoweall (70)</td>
<td>0-1500 e.d.</td>
<td>4 types</td>
<td>The preferred solution shows resonance-like behaviour of the $P_{3/2}$ amplitude.</td>
</tr>
<tr>
<td>Kato (70)</td>
<td>500-1500 e.i.</td>
<td>I,II,III,IV</td>
<td>All solutions indicate resonance-like behaviour in one partial-wave amplitude.</td>
</tr>
<tr>
<td>Kohri (70)</td>
<td>300-2000 e.i.</td>
<td>I,II,III</td>
<td>The $P_{3/2}$ wave displays some resonance-like behaviour, the data are not sufficient for a resolution of the ambiguities.</td>
</tr>
<tr>
<td>Albrow (71)</td>
<td>0-2500 e.i.</td>
<td>6,$d$,$y$</td>
<td>The uncertainty about a possible resonance in the $P_{3/2}$ wave has not been resolved.</td>
</tr>
<tr>
<td>Les (71)</td>
<td>0-2500 e.i.</td>
<td>12,12,12,III</td>
<td>Critical survey of the situation.</td>
</tr>
<tr>
<td>Lovelace (71)</td>
<td>0-2500 e.i.</td>
<td>12,12,12,III</td>
<td>Through an exotic resonance is not completely excluded, $K^p$ scattering can be accurately described without one.</td>
</tr>
<tr>
<td>Charles (71a)</td>
<td>900-1600</td>
<td>Check on previous solutions; preferred solutions: Albrow (71$b$), Albrow (71$a$), Albrow (71$c$).</td>
<td></td>
</tr>
<tr>
<td>Miller (72)</td>
<td>500-2000 600 e.i., e.t. and energy dependence</td>
<td>1,II (conventional)</td>
<td>There is still a large uncertainty in the values of the phase shifts which emerge, so it is not possible to account at this stage in the existence of a Z resonance. See also Albury (70), Gutowsky (70), Brodett (71) and Barrett (71).</td>
</tr>
<tr>
<td>Adams (72)</td>
<td>0-1500 e.d.</td>
<td>2 types</td>
<td>None of solutions show the type of speed variation that would be suggestive of a resonance.</td>
</tr>
<tr>
<td>Barber (73)</td>
<td>1368-2059</td>
<td>Check on previous solutions; preferred solutions: Albrow (71$b$), Albrow (71$a$), and Albrow (71$c$).</td>
<td></td>
</tr>
<tr>
<td>Borthin (73)</td>
<td>150-230 e.d.</td>
<td>$P_1, P_3, P_5$</td>
<td>It is not possible to discriminate between the current solutions obtained with the elastic data only. The S-wave phase remains always negative in the 0-170 MeV/c range.</td>
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<tr>
<td>Gonmor (71)</td>
<td>0-070 e.i. and c.d.</td>
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**Table 5**
More recently a simultaneous phase shift analysis in the $K^+p$, $K^*N$ and $K\pi$ final states has been carried out by Berthon (73) at the $\hbar$ values 1210, 1290, 1380 and 1690 MeV/c. Preliminary data have been presented at Aix-en-Provence (73); they do not show the possibility of discriminating between the various solutions obtained in the elastic channel. Moreover the data are not against the absence of exotic resonances in the $s$-channel. Finally we can conclude that also the inelastic channels in the $I=1$ state are up to now inconclusive.

From the preceding discussion both of the elastic and inelastic $I=1$ channel we can point out that the search for $Z^*$ has been up to now inconclusive; at the moment an inference that $K^+p$ resonant states would be elusive is more justifiable than the expectation of a clean solution of the problem.

6. - THE I=0 CHANNEL AND THE PROBLEMS WITH DEUTERIUM

The search of $K$-nucleon resonant states can be also carried out in the $I=0$ channel. The $I=0$ part of the interaction must be obtained from $K^+p$ and $K^+n$ data. The latter, implying the use of the deuteron target, involves the further complication of the neutron being bound in the deuteron.

As far as the total cross-sections are concerned we have already pointed out in Sect. 2 that the difficulties connected with the extraction of the $I=0$ total cross section have been successfully overcome. This is essentially due to the fact that in our energy range the behaviour of the total cross-section does not indicate at all sharp relative variation of the $K^+p$ and $K^+d$ data. From this we can safely assume that as a first approximation the relation $\sigma_\text{total}(d)=\sigma_\text{total}(p)+\sigma_\text{total}(n)$ is not so bad and that we are not in presence of narrow structures. With these hypotheses the derived $I=0$ total cross-section has a high confidence level.

On the contrary for the particular cross-sections and moreover for the differential cross-sections the problem presents a lot of difficulties.

As an example we discuss now the problem of obtaining the $K^+n+K^0\overline{p}$ charge exchange differential cross-section from the $K^+d-K^0\overline{p}$ measured events. It has been shown (Alberi 72; see also Stenger 64) that in the impulse approximation the charge exchange differential cross-section in the $K^+$-nucleon center of mass frame may be written

$$\frac{d\sigma}{d\theta^*} = \frac{1}{3} |g|^2 w^* + \left( |f|^2 + \frac{2}{3} |g|^2 \right) w^- \tag{6}$$

where $f=\frac{1}{2}(f_1-f_0)$ and $g=\frac{1}{2}(g_1-g_0)$, $f_1$ and $g_1$ being the standard non spin-flip and spin-flip amplitudes in $I=1$ and $I=0$ state, defined in the center of mass of the $K^+$-nucleon system. $w^*$ are the so called deuteron weight factors, functions of $\Theta^*$ (the $K^*$scattering angle in the $K^+$-nucleon center of mass frame) and $E$ (the total energy of the $K^+$-nucleon center of mass system). Often in the literature the formula (6) is
written in a different way, with the so called deuteron form factors $I_0$ and $J_o$ (Sten­
gger 64). The relation between the weight factors and the form factor is simply

$$W^+ = I_0 + J_o$$

and we then obtain the formula

$$\frac{d\sigma}{d\Omega^+} = |r|^2(I_0 - J_o) + |g|^2(I_0 - \frac{J_o}{3})$$

A typical behaviour of these weight factors is shown in Fig. 8; we realize that in the
forward direction $\frac{d\sigma}{d\Omega^+}=0$ because $W^+$ vanish and moreover $g_1(o)$, the spin-flip am­
plitudes, also vanish for $\cos^+ = +1$. Therefore for a whatever behaviour of the $K^+ n \rightarrow K^0 p$
differential cross-section in the forward direction the $K^0$ distribution in the
$K^+ d \rightarrow K^0 p p$ process vanishes when the $K^+$ goes in the same direction as the incident $K^+$;
this is essentially due to the Pauli principle.

But also in the case when a formula like (6) can be written and when all the
difficulties involved in obtaining a correct value for $W^+$ can be solved we realize
that the $K^+ n \rightarrow K^0 p$ cross-section on the free-neutron is not obtainable because of
the lack of knowledge of $f_1(o)$ and $g_1(o)$. Therefore this free-neutron cross-section
shall be obtained as a by-product for example of a complete phase shift analysis in
which the amplitudes are fitted directly to deuteron data rather than to free nu­
cleon data (Dean 72).

7. - THE $K^+ d$ PARTIAL CROSS SECTIONS

The BGRT collaboration has sistematically studied the $K^+ d$ interaction in the
bubble chamber for 13 momenta between 642 and 1510 GeV/c (Giacomelli 72a, Giacomel­
li 72b, Giacomelli 73, Giacomelli 74a). Previous results on this channel have
been published by Hirata (71a and 71b). A listing of all data on $K^+ d$ partial
and differential cross-sections is given in Table 6.

The first result of this experiment is the behaviour of the partial cross-sec­
tions for one-pion and two-pions production; it is shown in Fig. 9 and 10 respecti­
vely.

For the one-pion production we measured the cross-sections for the following
processes

$$K^+ d \rightarrow K^0 pn \pi^+$$
$$K^0 pp \pi^0$$
$$K^+ pp \pi^-$$

and, in order to obtain the cross-section for the process $K^+ d \rightarrow KNN\nu$, we used the
relation, obtained from I-spin conservation considerations (Hirata 71b and
references therein):
\[ \sigma(K^+d \to KN\pi) = \frac{3}{2} \left[ \sigma(K^+d \to K^+n\pi^+) + \sigma(K^+d \to K^0p\pi^+) + \sigma(K^+d \to K^0p\pi^-) \right] \]  

(10)

where we assume

\[ \sigma(K^+d \to K^+n\pi^+) \simeq \sigma(K^+p \to K^+n\pi^-) \]  

(11)

and this last cross section has already been measured in hydrogen (Giacomelli 70). With the same kind of considerations (for details see Giacomelli 72a) it is possible to obtain a good estimate of the cross-sections for the process $K^+d \to KN\pi\pi$. It is now necessary to point out that we are directly interested in the $\sigma_o(KN \to KN\pi)$ and $\sigma_o(KN \to KN\pi\pi)$ and not in the measured $\sigma(K^+d \to KN\pi\pi)$ and $\sigma(K^+d \to KNN\pi\pi)$ cross-sections. In fact in our energy range for the $I=0$ KN inelastic cross section we simply have

\[ \sigma_o(KN \text{ inelastic}) = \sigma_o(KN \to KN\pi) + \sigma_o(KN \to KN\pi\pi) \]  

(12)

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<th>$K^+d \to K^0p_{\text{spect.}}$</th>
<th>$K^0s \to K^+p_{\text{spect.}}$</th>
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8. THE $I=0$ INELASTIC CROSS-SECTION

The $I=0$ one pion production cross-section is obtainable with the method already mentioned in the preceding section from the relation

\[ \sigma_o(KN \to KN\pi) = 3\left[ \sigma(K^+n \to K^+n\pi^+) + \sigma(K^+n \to K^0p\pi^+) - \sigma(K^+p \to K^0p\pi^-) \right] \]  

(13)
The last cross-section is already known from hydrogen experiments (Giacomelli 70); for the two others it is necessary to know the cross-sections on free-neutron. We can note that the $K^+p \rightarrow K^0p\pi^+$ reaction measured in hydrogen is in principle obtainable also in deuterium, by means of the $K^0d \rightarrow K^0p\pi^+$ events. In order to do this we use the customary procedure, based on the simple impulse approximation. The neutron is taken as a spectator if

$$\begin{align*}
P_n &< P_p \\
P_n &< 250 \text{ MeV/c} \quad (14)
\end{align*}$$

where $P_n(p)$ is the neutron (proton) momentum. We indicate in brackets the nucleon spectator, that is in the nucleon that satisfies the two criteria (14). The measured $K^+p \rightarrow K^0p\pi^+$ and $K^0d \rightarrow K^0p\pi^+(n)$ cross-sections are shown in Fig. 11. We find

$$\frac{\sigma(K^+p \rightarrow K^0p\pi^+)}{\sigma(K^0d \rightarrow K^0p\pi^+(n))} = 1.26 \pm 0.04$$

constant with energy. This values disagrees with $\sim 1.12$, the ratio expected from the Glauber (59) theory; therefore in all the other channels we use this experimental number for the determination of the free-neutron cross-sections.

These cross-sections of course are always obtained in the spirit of a simple impulse approximation, using the two criteria mentioned above. Moreover the use of the experimental ratio in all channels is arbitrary, because the channel independence is simply assumed (*) .

With this assumption we can then compute $\sigma(K^+n \rightarrow K^0n\pi^+)$ from the $K^+d \rightarrow K^0n\pi^+(p)$ events and $\sigma(K^+n \rightarrow K^0\pi^-(n))$ from $K^+d \rightarrow K^0p\pi^-(n)$. Finally we obtain the $\sigma_o(KN \rightarrow KN\pi)$ cross-section. The $\sigma_o(KN \rightarrow KN\pi)$ cross-section is obtainable in an analogous way and finally we can compute the $\sigma_o(KN \text{ inelastic})$ cross-section by means of formula (12). This cross-section is plotted in Fig. 3 and the references are given in Table 6.

In the cross-section so obtained the smearing, due to the Fermi motion of the target nucleon, is not taken into account. Nevertheless this effect is small, and can be neglected, because of the still relatively large experimental errors, as can be seen in Carroll (73).

(*) Very recently it has been possible to estimate the different ratios for the various channels; in fact they are not the same for all the channels (Alberi 74).
9. - THE I=0 ELASTIC CROSS-SECTION

In Fig. 3 the I=0 total and inelastic cross-sections are shown. Subtracting the second from the first we understand immediately that the structure at about 800 MeV/c is essentially due to the elastic channel $K^+N \rightarrow K^+N$ in I=0, although its shape is not clearly defined. In Fig. 12 a resume of the situation is shown.

Because of the presence of such a maximum in the elastic cross-section it is clear that our hope of finally revealing a $Z^*$ state increases.

10. - THE $K^+ \rightarrow K^0 p$ CHARGE EXCHANGE DIFFERENTIAL CROSS-SECTION

At this point it is appropriate to remember that

$$A(K^+p \rightarrow K^0p) = A,$$
$$A(K^+n \rightarrow K^0n) = \frac{1}{2}(A_1 + A_0),$$
$$A(K^+n \rightarrow K^0p) = \frac{1}{2}(A_1 - A_0)$$

(16)

where $A_1$ and $A_0$ are the scattering amplitudes in I=1 and I=0 isospin states. If we assume that $A_1$ is well known (although not unique) then from the $K^+n \rightarrow K^0p$ process we can determine $A_0$. Therefore the $K^+d \rightarrow K^0pp$ (with the $K^0 \rightarrow 2\pi$ decay visible in the bubble chamber) has been extensively studied in our energy range (see Table 6) in order to extract informations on the $K^+n \rightarrow K^0p$ charge-exchange differential cross-section.

First of all in Fig. 13 we show the behaviour of the total cross-section $K^+d \rightarrow K^0pp$, with a beautiful maximum at about 800 MeV/c. If we interpret this cross-section as the $K^+n(p) \rightarrow K^0p(p) - (p)$ is the proton spectator - cross-section we can just say that the $K^+n \rightarrow K^0p$ total cross-section has in our interval a maximum at about 800 MeV/c.

In Fig. 14 we show the more recent and more detailed data on the $K^+n \rightarrow K^0p$ differential cross-section (Giacomelli 72b); the definition for the proton spectator is the one indicated in Sect. 8 and $\theta^*$ is the angle between the outgoing $K^0$ and the incident $K^+$ in the center of the mass frame defined by the $K^0$-proton not spectator final state system.

As we have already pointed out in Sect. 6 these differential cross-sections are not those one would obtain on a free neutron. Therefore these data have been used without any correction as input data in the phase shift analyses and formula (6) was used, in which $W^+$ and $W^-$ have been computed for each energy and with the cut for the nucleon spectator at 250 MeV/c.

(*) The BGRT collaboration has also collected some data on the $K^+d \rightarrow K^0pp$ reaction when the $K^0$ decay is not visible in the chamber (Giacomelli 72b).
The BGRT collaboration (Wilson 72) at this point attempts a phase shift analysis in which details we do not enter, because this analysis is now obsolete. We only emphasise that with these new data the situation has not been particularly exciting since 9 class of solutions have been found. Nevertheless a provision should be made: in the backward $K^+n \to K^+n$ elastic scattering the different solutions show very different behaviours; therefore a measurement of this process could clarify the situation.

11. - THE $K^+n \to K^+n$ ELASTIC DIFFERENTIAL CROSS-SECTION

The BGRT collaboration (Giacomelli 73) has then measured the $K^+n \to K^+n$ reaction from the analysis of the $K^+d \to K^+n(p)$ events, when the proton is visible.\(^(*)\) The extraction of the cross section for this channel presents a lot of problems for what concerns both event identification and normalisation. Only two-prong events, in which one particle stopped in the chamber, have been considered; a substantial ambiguity between the two hypotheses:

\[
\begin{align*}
K^+d &\to K^+np \\
K^+d &\to K^0pp
\end{align*}
\]

has been found not to be resolvable after ionization inspection.

We do not enter into details about this problem but we can say that this ambiguity has been resolved taking into account the fact that the $K^+d \to K^0p(p)$ angular distribution is known from the proceeding experiment (Giacomelli 72b); besides the ambiguous events fall in the forward region of the center of mass angular distribution in the $K^+n(p)$ hypothesis and very forward bins are not included in our angular distribution in order to reduce "interference" effects between $K^+p$ and $K^+n$ amplitudes.

As far as the $K^+n \to K^+n$ total cross-section two different methods have been used; in the first case we used the formula

\[
\sigma(K^+n \to K^+n) = \frac{1}{2}[\sigma_3(K^+n \to K^+n) + \sigma_4(K^+n \to K^+n)] - \sigma(K^+n \to K^0p)
\]

and in the second case the formula

\[
\sigma(K^+n \to K^+n) = \sigma(K^+d \to K^+pn) - \sigma(K^+p \to K^+p)
\]

The results of these two methods are shown in Fig. 15 where the solid curve is a hand-drawn one through the data.

\(^(*)\) Some results of this reaction has also been collected for events in which the proton spectator is not visible, that is for one-prong events (Giacomelli 73).
The $K^+d \rightarrow K^+n(p)$ angular distribution have been obtained with the definition of the proton spectator already used in Sect. 6 but in this case the momentum of the proton was chosen between 100 and 250 MeV/c. Fitting these angular distribution with a sum of Legendre polynomials it has been possible to estimate the angular distribution even in the not included forward bins. In Fig. 16 the $K^+d \rightarrow K^+n(p)$ differential cross-sections are shown, normalized to the hand-drawn curve of Fig. 16.

Also in this case the problem of the normalization will be definitively resolved only with the phase shift analyses. Moreover even in this case these differential cross-sections are not those one would obtain on free neutron. In fact, following the procedure of Alberi (72), we may write:

$$\frac{d\sigma}{d\Omega^\ast} = 2 \left[ \frac{1}{3} g^+ W^- + (|f^+|^2 + \frac{2}{3}|g^+|^2) W^+ + \frac{1}{3} |g^-|^2 W^+ + (|f^-|^2 + \frac{2}{3}|g^-|^2) W^- \right]$$  \hspace{1cm} (20)

where $f^+ = \frac{1}{2}(g^+_0 + f^+_p)$, $f^- = \frac{1}{2}(g^-_0 - f^-_p)$ and analogously for $g^+$ and $g^-$. $f^+_p$, $f^+_0$, $g^+_0$, $g^-_0$, $W^+$ have already been defined in Sect. 6. We can write formula (20) also in the following way, with $I_0 = \frac{1}{2}(W^+ + W^-)$ and $J_0 = \frac{1}{2}(W^+ - W^-)$

$$\frac{d\sigma}{d\Omega^\ast} = I_0(|f^+_p|^2 + |f^+_n|^2 + |g^+_p|^2 + |g^+_n|^2) + 2 J_0 \text{Re}[f^+_p f^-_n + f^+_n g^+_p]$$  \hspace{1cm} (21)

where $f^+_p = f^+_1$, $f^+_n = \frac{1}{2}(f^+_1 + f^+_0)$ and $g^+_p = g^+_1$, $g^+_n = \frac{1}{2}(g^+_1 + g^+_0)$. With our choice of the proton spectator we assume that the simple impulse approximation works and with the cut in the forward bins already discussed we can neglect interference effects between the $K^+p$ and $K^+n$ amplitudes; therefore our measured differential cross-section describes the $K^+n \rightarrow K^+n$ elastic scattering in deuterium when the proton acts as a spectator and we have

$$\frac{d\sigma}{d\Omega^\ast}[K^+n(p) \rightarrow K^+n(p)] = I_0(|f^+_n|^2 + |g^+_n|^2) = I_0 \frac{d\sigma}{d\Omega^\ast}[K^+n \rightarrow K^+n]$$  \hspace{1cm} (22)

where $I_0 \frac{d\sigma}{d\Omega^\ast}[K^+n \rightarrow K^+n]$ is the center of the mass $K^+n \rightarrow K^+n$ differential cross-section on free neutrons. This last formula has in fact been used in the successive phase shift analyses.

In Fig. 17 we show the center of the mass differential cross-sections at $\cos^0 = 1$; the absence of clear structures is evident. The curves of this figure will be discussed latter.

12. - THE $K^+d$ COHERENT REACTIONS

The BGRST collaboration (Giaconelli 74) has also measured the coherent total cross-sections $K^+d \rightarrow K^+d$, $K^+d \rightarrow K^+\sigma^+$ and $K^+d \rightarrow K^+\sigma^+\pi^0$; these cross-sections are shown in Fig. 18 and the differential cross-sections $K^+d \rightarrow K^+d$ are plotted in Fig. 19.
Before entering into the problem of the $I=0$ channel it is necessary to discuss briefly the situation of the $I=1$ phase shift analyses (p.s.a.), already commented in Sect. 5.

In Table 5 we have shortly summarized the conclusions of the $I=1$ p.s.a.. These p.s.a. require in general a very long work and the sets of data used by a group may be different from the set employed by an other group. Nevertheless at this point the overall picture seems to be defined, and we intend in fact to emphasize here this overall picture.

A critical survey of the situation was done by Lea (71) and a review of the various $I=1$ solutions can be found in the Review of Particle Properties by the Particle Data Group (Soding 72).

At this point we believe that the problem of the minimum in the total cross-section at about 700 MeV/c is definitively solved in the sense that there is no minimum at all. Therefore the behaviour of the phase-shifts at low energy is smooth and monotonic with a dominant and negative $S_{1/2}$ phase-shift; this old result of Goldhaber (62) has been confirmed in fact by Adams (73) and Cameron (74). Taking into account this fact all the solutions with a $S_{1/2}$ phase-shift positive below 1 GeV/c must be neglected.

For the remaining solutions the $\chi^2$-test performed by Charles (72a) and by Barber (73) is very instructive. Using the published values for phases and elasticities the $\chi^2$ was calculated for fits to new data, without any reminimization; none of the input solutions gives an acceptable fit. The $\chi^2$ was then reminimized by constraining the phases and elasticities to within 5% of the input values and this reminimization gives acceptable fits practically for all phase-shift solutions. The Albrow (71) $\alpha$, $\beta$, $\gamma$ for Charles (72a) and Albrow (71) $\alpha$, $\gamma$ and Barber (70) $\gamma$ for Barber (73) are the marginally favoured sets of $Kp$ phases. The interesting point in this discussion is that a freedom of only 5% in the old solution is sufficient to achieve new acceptable results; this is very surprising. In Fig. 20 the Argand diagrams for the two solutions Albrow (71) $\alpha$ and Albrow (71) $\gamma$ are shown. These two solutions emerge in some way from the last works; in our opinion these solutions represent an example of a class of solutions that has been practically found in all the p.s.a.. Even in the case of more promising solution (i.e. the $\gamma$ solution) practically all the authors hesitate to clearly state that the loop in the $P_{3/2}$ wave really represents a resonant behaviour.

Up to now few p.s.a. have been carried out in the $I=0$ channel. For the first analysis of the BGRT collaboration (Wilson 72) and the Aaron (73) analysis only part of the data, used in the last analysis by the BGRT collaboration (Giacomelli 74b) was available. The Aaron (73) approach differs from the BGRT one in three respects:
i) Aaron (73) uses the I=1 phase shifts of Rebka (70) (see also Kato 70); BGRT on the contrary uses the I=1 phase shifts of BGRT (Giacomelli 70), Lovelace (71) and Albrow (71);  

ii) Aaron includes the F and G partial waves while BGRT stops at D or at F waves;  

iii) Aaron derives the inelasticities $\eta$ from the calculation of Aaron (71) in all partial waves excepting $S_{1/2}$ and $P_{1/2}$ while BGRT does not use theoretical ingredients for the $\eta$ computations.  

Aaron (73) has performed an energy dependent, modified phase-shift analyses in the region $p_{lab} = 865\pm 1585$ MeV/c with 240 data points; the solutions A and B, shown in Fig. 21, have been found. These solutions contain $D_{3/2}$ and either $S_{1/2}$ (Sol. A) or $P_{1/2}$ (Sol. B) exotic resonances.  

In the comprehensive analysis of the BGRT (Giacomelli 74b) collaboration three different phase shift analyses have been carried out, one energy dependent following the parametrisation of Lea (68) and two energy independent, in which the shortest path method to link the solution at different energies in the Lovelace (71) version was used. Here we do not enter into details of the procedure but we only intend to point out two important points:  

a) Various I=1 amplitudes have been used either as starting points (in the first e.i.p.s.a.) or as fixed parameters (in the e.a.p.s.a. and in the second e.i.p.s.a.).  

b) All the data available up to now in the I=0 channel have been used. The bulk of these data is the $K^+ n \rightarrow K^0 p$, $K^+ n \rightarrow K^+ n$ and the partial inelastic cross sections measured by the BGRT collaboration and illustrated in the preceding sections. If the $K^+ n \rightarrow K^0 p$ polarization data at 600 MeV/c and the counter measurements of the total cross sections in our range are added, we practically obtain all the experimental data used in these p.s.a. The only theoretical ingredients used in these p.s.a. are the real parts of the forward scattering amplitude computed by Martin (68) for I=1 and Carter (67) for I=0.  

The results of these analyses show that it has been possible to obtain satisfactory solutions both on the whole and for each energy or channel; this indicates a reasonable consistency of the different data and also an adequacy of the number of partial waves used.  

Six classes of solutions have been obtained, but if we only limit ourselves to the solutions which have been found at least in two of the three methods only three of them survive; they have been called A, C and D and are plotted in Fig. 22.  

A comparison between the particular features of the data and the different solutions induces in general to prefer the C and D solutions; such a conclusion does not appear from the over-all $\chi^2$. There is in fact only one clear indication in favour of the C and D solutions, that is the polarisation at 600 MeV/c (Fig. 23a); moreover the backward polarisation at $\sim 1.29$ GeV/c seems to be in a good position in order to discriminate between the C and D solutions (Fig. 23c).  

(*) Note that the last counter data of the $K^+ p$ and $K^+ d$ total cross-sections (Carroll 73) have not been used in this analysis.
But it is necessary to point out that this prevision must be taken with a lot of care: when the \( \text{K}^+\text{K}^- \) backward differential cross sections were not yet at our disposal we predicted that a discrimination between the various solutions should be possible if these data were available (see Sect. 10). Now we have the data but the discrimination is always impossible, as it can be seen in Fig. 17 where the different \( L \), \( C \) and \( D \) solutions are plotted.

In Fig. 24 we show four differential elastic \( \text{K}^+\text{d} \) cross-sections. The solid lines represent the fits according to the formula

\[
\frac{d\sigma}{d|t|} = C \left| S \left( \frac{\sqrt{|t|}}{2} \right) \right|^2
\]

where \( S \) is the deuteron form factor, calculated using the S-wave part of the Moravcsik-Garthenhaus wave-function (see for details Giacomelli 74a) and \( C \) is a normalization constant. The dashed lines on the contrary represent the results of the computation with the formula, obtained in the framework of the impulse approximation:

\[
\frac{d\sigma}{d|t|} = \frac{\eta}{p^2} \left\{ \left| f_p + f_n \right|^2 + \frac{2}{3} \left| g_p - g_n \right|^2 \right\} \left| S \left( \frac{\sqrt{|t|}}{2} \right) \right|^2
\]

where \( f_p(n) \) are the non-spin-flip scattering amplitudes for \( \text{K}^+ \) scattering on free protons (neutrons) and \( g_p(n) \) are the corresponding spin-flip amplitudes. \( S \) is the same as in formula (23) and \( p \) is the center of mass momentum in the \( \text{K}^+\text{N} \) system. The \( f \) and \( g \) amplitudes have been determined from the \( D \) solutions of the BERT p.s.a. Computations using other solutions yield values which are different from those plotted in Fig. 24 by at most a few per cent; therefore these \( \text{K}^+\text{d} \rightarrow \text{K}^+\text{d} \) data-not employed in the p.s.a.- seem not to be useful in order to discriminate between the various solutions.

In Fig. 25 we show the computed behaviour of the \( I=0 \) \( \sigma_{\text{tot}} \) for \( C \) and \( D \) solutions; the \( D \) solution is the only one which adequately reproduces the new data (Carroll 73), not included in this analysis.

Then if we trust in this \( D \) solution we show, always in Fig. 25, the elastic cross-section and the contribution of the partial waves to the total cross-section: the structure in the elastic cross-section at \( \sim 0.75 \) GeV/c arises almost entirely from the \( P_{1/2} \) wave, while the structure in the total cross-section at \( 1.15 \) GeV/c arises mainly from the \( D_{3/2} \) wave, a very inelastic wave (Fig. 22) connected in fact with the opening of inelastic channels.

Finally in Fig. 26 we show for the \( I=0 \) \( P_{1/2} \) wave, the \( D \) solutions for the different choice at the \( I=1 \) solution. It may be interesting to point out that, if the \( I=1 \) has a resonant behaviour in the \( P_{1/2} \) wave (Giacomelli (70) and Albrow (71)'s), then also the \( I=0 \) \( P_{1/2} \) shows a resonant behaviour, and the reverse is also true. Thus the behaviour of the \( I=1 \) \( P_{1/2} \) wave and the \( I=0 \) \( P_{1/2} \) wave seems to be linked in some way to one another.

If the \( D \) (BERT) solution is chosen and the Argand diagram is fitted with a Breit-Wigner plus a quadratic background we obtain a resonance with mass 1740 MeV and width \( \sim 300 \) MeV, the elasticity being about 0.85.
15. - WORK IN PROGRESS AND CONCLUSIONS

As far as new experimental data are concerned, we quote the very important measurement of the real part of the forward scattering amplitude $D$ in the $K^p$ elastic scattering (Baillon 74). This measure gives values of $D$ which are somewhat different from the theoretical ones, that have been used in the various phase shift analyses. Unfortunately these $D$ values have only been measured at 3 $K^+$ incident momenta namely: 1.209, 1.798 and 2.606 GeV/c. Therefore for the phase shift analyses in our energy range only one or perhaps two values are available, with a precision of $10\% \pm 5\%$.

The preceding measurement refers obviously to the $I=1$ channel. For the $I=0$ three different experiments are in progress:

i) the $K^0 p \rightarrow K^0 n$ reaction is studied in the 500 - 1500 GeV/c momentum range with electronics technique and very preliminary results have been reported at the Batavia Conference (Armitage 72).

ii) The $K^0 p \rightarrow K^0 n$ reaction is also studied in bubble chamber by a collaboration between Bologna, Edimburg, Glasgow, Pisa and RHEL. (CERN proposal TCC 73 - 18).

iii) The $K^0 S^0 p, K^0 S^0 \rightarrow \pi^+ \pi^-$ reaction is studied in bubble chamber in the 1642 - 1860 MeV/c mass region (CERN proposal TCC 73 - 35 by G.W. London).

In the first two experiments the main purpose is to avoid the problems arising in the study of the inverse reaction $K^0 n(p) \rightarrow K^0 p(p)$, due to the fact that the neutron target is bound in the deuteron. As we have already discussed in Sect. 6, because of the Pauli effect, the $K^0 n \rightarrow K^0 p$ differential cross-section is not directly measurable in the forward direction; therefore this direct measurement can in fact check the computations of the weight factors.

In the third experiment the amplitude for the considered reaction, assuming strangeness and isospin conservation, is

$$T = \frac{1}{4}(Z_0 + 2Z_1 - 2Y_1)$$

where

$Z_0$ is the $I=0$, $KN$ amplitude
$Z_1$ is the $I=1$, $KN$ amplitude
$Y_1$ is the $I=1$, $\bar{K}N$ amplitude

Therefore, if the $I=1$ $KN$ and $\bar{K}N$ amplitudes can be considered to be well known, from the $K^0 p \rightarrow K^0 p$ data we can compute the $I=0$ amplitude. Of course the fact that these $I=1$ amplitudes are necessary, is in principle a difficulty. But the presence in the interested region of a highly elastic and relatively narrow resonance, the $Y_1^+(1765)$, could improve the analysis.

From a pure experimental point of view we note that this last reaction appears cleaner than the $K^0 p \rightarrow K^0 n$ one. In the $K^0 p \rightarrow K^0 p, K^0 S^0 \rightarrow \pi^+ \pi^-$ process the final state is in fact completely measurable while in the $K^0 p \rightarrow K^0 n$ reaction the neutron is not measurable.
After this short review of the work in progress we conclude separately for the 
$I=1$ and for the $I=0$ channel.

$I=1$: the discrimination between the various solutions obtained in the phase shift 
analyses seems not to be possible. Even if we assume for the moment that a solution 
like the $P_3$ of Albrow (71) is the unique one the situation is not better. In fact 
this resonance has not a classical behaviour; the Breit-Wigner parametrisation works 
only partially in this case and we do not observe the expected structure in the speed plot.

$I=0$: In this case the experiments are not as precise as the ones concerning the $I=1$ channel. Therefore we can think that with better precision a discrimination between 
the various $I=0$ solutions should be possible. In this channel, the elastic component 
has a maximum slightly below the opening of the inelastic channels. Moreover all the 
phase shift solutions present a resonant $P_1$ wave, favoured by the existing polarisation data.

Three types of experiments are possible in order to discriminate between the va-
rious solutions: the $K^0p \to K^+n$ charge-exchange, the polarization in the process $K^-p \to K^0n$ 
and the $K^0p \to K^0_{S1}p$ reaction. For the moment, as we have pointed out in the first part 
of this Section, only the $K^0p \to K^+n$ and $K^0_{S1}p \to K^0p$ experiments are in progress while a 
polarisation measurement with high precision seems to be a very remote possibility. An 
improvement of the situation will of course be possible in the near future, when the 
$K^0p \to K^+n$ and $K^0_{S1}p \to K^0p$ results will be available. Nevertheless the precision of these 
experiments is not so high and we cannot exclude a situation similar to that presented 
by the $I=1$ channel.

From all these considerations both in the $I=1$ and $I=0$ channel it is possible that 
in this case we need a different definition of resonance and in fact this has been done. 
It has been shown - in particular in the $I=0$ channel - that the opening up of the inel-
astic channels has repercussion on the elastic channel and we can then observe the 
production of objects that look like resonances (Aaron 73, Amado 73, Moravcsik 73). 
These resonances, driven by virtual effects of the inelastic channels are not pecu-
liar to the $K$-nucleon system, but they can be observed also in the $\pi$-nucleon reac-
tions; in general they are particularly important near threshold. These resonances 
cannot have always a regular behaviour in the Argand diagram and therefore the search 
for a clean and classical resonance will be also in the future elusive, if we limit 
ourselves mainly to the elastic channel. Since these resonances are mainly due to 
the inelastic channels it is obvious that we must analyze these channels and there-
fore we expect a real improvement when new detailed analyses on the $K^0p \to K^0\pi$ and 
perhaps $K^0p \to K^0\pi\pi$ reactions will be available both in the $I=1$ and $I=0$ channels.

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Fig. 1 - $K^+ - p$ total cross-sections versus laboratory incident momentum [● Carroll (73), ○ Cool (70), ■ Goldhaber (62), □ Bowen (73), ◆ Cameron (74), ◆ Bugg (68), ▲ Adams (73), △ Bowen (70)].
Fig. 2 - $K^+$-d total cross-sections versus laboratory incident momentum
[● Carroll (73), ○ Cool (70), □ Bowen (73), ◆ Bugg (68), △ Bowen (70)].
Fig. 3 - $K^+$-nucleon total and inelastic cross-sections in the pure isospin $I=0$ state versus laboratory incident momentum, from Carroll (73) [0 Carroll (73), O Cool (70), □ Bowen (73), △ Bowen (70), ★ Giacomelli (72a)].
Fig. 4 - \( K^+ \)-nucleon total and partial cross-sections in the pure isospin \( I=0 \) and \( I=1 \) states versus laboratory incident momentum: resumé of the situation. The curves are hand-drawn through the relative data.
Fig. 5 - Behaviour of the phase-shifts versus laboratory incident momentum from Cameron (74). The points are the results of an energy independent analysis. The full lines represent the fits of the points to an effective range expansions. The dashed lines represent the results of an energy dependent phase shift analysis.
Fig. 6 - \( K^+ \)-nucleon elastic and inelastic cross-sections in the pure isospin \( I=1 \) state versus laboratory incident momentum. \[ \square \text{Goldhaber (62)}, \triangle \text{Adams (73)}, \text{◆ Cameron (74)}, \bullet \text{Charles (72b)}, \square \text{Giacomelli (70)} \triangle \text{miscellaneous elastic cross-sections, ◇ miscellaneous inelastic cross-sections} \).
Fig. 7 - $K^+p \rightarrow K^0\Delta^{++}_{1236}$ cross-section versus laboratory incident momentum from Griffiths (72). The smooth curve is the fit from the partial wave analysis. [x Bland (70), • Griffiths (72)].
Fig. 8 - The weight factors $W^+$ and $W^-$ at the two laboratory incident momenta of 0.65 and 1.51 GeV/c for the spectator cut $0 < p_{\text{spectator}} < 250$ MeV/c in the reaction $K^+d \rightarrow KNN_{\text{spectator}}$. 
Fig. 9 - The cross-sections for single pion production in deuterium below 1.6 GeV/c versus laboratory incident momentum from Giacomelli (72a). The lines are drawn to guide the eye [Ο Hirata (71b), △ □ ◊ ▲ ● Giacomelli (72a)].
Fig. 10 - The cross-sections for two pion production in deuterium below 1.6 GeV/c versus laboratory incident momentum from Giacomelli (72a). The lines are drawn to guide the eye.
Fig. 11 - The cross-sections $K^+ p \rightarrow K^0 p\pi^+$, $[\bigcirc$ Giacomelli (70) in hydrogen, $\bigotimes$ other authors in hydrogen, $\bullet$ Giacomelli (72)] $K^+ d \rightarrow K^0 p(n)\pi^+$ and $K^+ d \rightarrow K^0(p)n\pi^+$ versus laboratory incident momentum from Giacomelli (72a). The cross-sections for the last two reactions are the raw data, after the cuts on the spectator nucleon.
Fig. 12 - K⁺-nucleon total, inelastic and elastic cross-sections in the pure isospin I=0 state versus laboratory incident momentum: resumé of the situation. The curve for the total and inelastic cross-sections are hand-drawn through the relative data.
Fig. 13 - $K^+d \rightarrow K^0pp$ cross-sections below 1.6 GeV/c versus laboratory incident momentum from Giacomelli (72a). The solid line was drawn by eye. [△Slater (61), □Hirata (71a), ○other authors, ●Giacomelli (72a)].
Fig. 14 - $K^+d \rightarrow K^0pp$ charge-exchange differential cross-sections $d\sigma/d\Omega^*$ from Giacomelli (72b).
Fig. 15 - The integrated $K^+n$ elastic cross-section versus laboratory incident momentum from Giacomelli (73). The solid line is a hand-drawn fit to the data points. The different symbols refer to the two methods quoted in the text. [$\Box: \sigma(K^+pn) - \sigma(K^+p); \bigcirc: \frac{1}{2} (\sigma_0 + \sigma_1) - \sigma_{c.e.}$].
Fig. 16 - $K^+n \rightarrow K^+n$ elastic differential cross-sections $d\sigma/d\Omega^*$ from Giacomelli (73).
Fig. 17 - The center of mass differential cross-sections at $\cos \theta^* = -1$ elastic scattering versus laboratory incident momentum. The curves are the predictions of an energy-dependent phase-shift analysis from Giacomelli (74b): ---- solution A, ——— solution C, ——— solution D.
Fig. 18 - Cross-sections for coherent reactions versus laboratory incident momentum from Giacomelli (74a). [● Giacomelli (74a), □ Hirata (71b)].
Fig. 19 - The differential cross-sections $d\sigma/dt$ for $K^+d \rightarrow K^+d$ elastic scattering from Giacomelli (74a).
Fig. 20 - Argand diagrams for the $\alpha$ and $\gamma$ solutions of Albrow (71).
Fig. 21 - Argand diagrams for the A and B solutions of Aarons (73).
Fig. 22 - Argand diagrams for the A, C and D solutions of Giacomelli (74b).
Fig. 23 - Charge-exchange polarisation data at 0.6 GeV/c (Ray 69) and predictions of the A, C and D solutions of Giacomelli (74b) at a) 0.6, b) 0.98 and c) 1.29 GeV/c.

Fig. 24 - The differential elastic K⁺d cross-sections at four momenta. The solid line represent the fits according to eq. (23), while the dashed line represent the result of the computations according to eq. (24) using the D solution of Giacomelli (74a).
Fig. 25 - $K^+$-nucleon total and partial cross-sections in the pure isospin I=0 state versus laboratory incident momentum. The data are from Carroll (73). The curves on the total cross-sections indicate the C and D solutions of Giacomelli (74b). The I=0 elastic cross-section and the contribution of the partial waves to the total cross-section refer to D solution of Giacomelli (74b).
Fig. 26 - Argand diagram for the $I=0$ $P_{\frac{3}{2}}$-wave of the solutions of D type for different $I=1$ solution:

$I=0$ energy dependent analysis
a) Giacomelli (70),
b) Albrow (71),
c) IA Lovelace (71).

$I=0$ energy independent analysis
d) Albrow (71),
e) IA Lovelace (71).