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# CONJECTURE ON THE COSMIC RAYS SPECTRUM 

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#### Abstract

The PAMELA experiment has shown that a fraction of cosmic rays could be due to proton fluxes generated by the magnetosphere of the planet Jupiter. Thus, a conjecture that astrophysical objects provided by magnetospheres be sources of the cosmic radiation is put forward. With simple geometric considerations the energy spectrum $E^{-2.5}$ is obtained, independently on the particle species, very close to the experimental CR spectrum, under the hypothesis that particle acceleration mechanisms act uniformly in the magnetospheric region.


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## 1 Introduction

Since the discovery of the cosmic radiation, made simultaneously in 1912 by Domenico Pacini[1] and by Victor Hess [2], the nature of the sources has been debated. After a century mysteries still remain. The suggestion was made, already in 1934 [3], that cosmic rays be generated by the explosion of supernovae, be due to acceleration by shocks from nova and supernova of different types [4], but it is likely that there can be a variety of sources like, i.e. pulsar [5] and, as suggested by Fermi [6], sources of continuous nature distributed in the intergalactic space.

Recently the cosmic rays measured by PAMELA ${ }^{1}$ have been studied in terms of their space-temporal distribution [7, 8]. The result has been that, during a period of eight years, the high energy interplanetary proton fluxes detected by PAMELA are larger, by twelve standard deviations, when the Earth intersects, in its orbit around the Sun, the interplanetary magnetic field (IMF) lines connecting Earth with Jupiter.

Thus: Jupiter emits 1 GeV protons, about $4 \%$ of the cosmic rays detected by PAMELA.
This result draws attention to the idea put forward long ago [10] that magnetospheres of astrophysical objects could contribute to the sources of cosmic rays, that is: higher energy particles leak out from the magnetosphere trapped particle regions and then are ejected into space as cosmic rays.

One important problem for any proposed theory is the derivation of the energy spectrum. In proposing the pulsars as sources of cosmic rays Thomas Gold writes [11] One would like of course that the mechanism proposed should generate the right spectrum; but there we as yet know too little.

In this paper we wish to study the spectrum of the particles escaping from a magnetosphere, according to our conjecture.

## 2 Conjecture on the spectrum of the magnetospheric escaping particles

The basic idea is the following. The source is an astrophysical body surrounded by charged particles in a strong magnetic field (i.e. pulsars, AGN, magnetospheric objects) which we assume to have a dipolar nature. The particles are accelerated by electrical fields acting in the magnetosphere. The accelerated particles follow the magnetic lines of force towards the equator, with a motion that obeys the laws of the adiabatic invariance [12]. At the equator the magnetic field is weakest and if the particles have reached sufficient energy they might leave the magneto-spheric region and go in the outer space,

[^0]contributing to the cosmic rays ${ }^{2}$.
This process goes through the following steps: at the equator the field due to a magnetic dipole $\mathbf{M}$, at distance $r$, is
\[

$$
\begin{equation*}
B=\frac{M}{r^{3}} \tag{1}
\end{equation*}
$$

\]

(For a neutron star with a radius $\sim 10 \mathrm{~km}$ the magnetic moment is of the order of $M=$ $10^{21} T \times m^{3}$.)

A charged particle with electrical charge $q$ and momentum $p$ can be trapped in a magnetic field with Larmor radius

$$
\begin{equation*}
R_{L}=\frac{p}{q B} \tag{2}
\end{equation*}
$$

For a particle to stay trapped it is necessary that the Larmor radius be small enough to satisfy the Alfven condition [14]

$$
\begin{equation*}
\frac{(d B / d r) R_{L}}{B} \leq \xi \tag{3}
\end{equation*}
$$

where the dimensionless quantity $\xi$ is estimated by means of plasma experiments in space and on the Earth and is of the order of a few per cent [15]. Combining the above equations we get

$$
\begin{equation*}
\frac{3 p r^{2}}{q M} \leq \xi \tag{4}
\end{equation*}
$$

An important feature following this conjecture if that the spectrum should terminate at a maximum momentum value (see also [16]). Using Eq. 4 with equal sign we get

$$
\begin{equation*}
p_{\max }=\frac{\xi c q M}{3 R^{2}} \tag{5}
\end{equation*}
$$

In the case of cosmic rays from a neutron star with $R \sim 10 \mathrm{~km}, \mathrm{M}=10^{21} \mathrm{~T} \times \mathrm{m}^{3}$ and with $\xi \sim 0.15$ [17], we calculate

$$
\begin{equation*}
p_{\max }=1.5 \cdot 10^{20} \mathrm{~V} \tag{6}
\end{equation*}
$$

This value ( $1.5 \cdot 10^{20} \mathrm{eV}$ in energy) compares with the highest cosmic ray proton energy detected so far. Higher values can be obtained for larger values of M .

Eq. 4 has to be interpreted in the following way: when an electrically charged particle accelerated by the electrical fields acting in the magnetosphere reaches a momentum $p$ that violate Eq. 4 (or it satisfies with the equal sign) then it escapes from the magnetosphere and goes into the outer space. This is likely to happen at the equator where the magnetic field is the weakest.

We try now to make an estimation of the energy spectrum of the escaping equatorial particles. The equation of a magnetic line of force is

$$
\begin{equation*}
r=L \sin ^{2}(\theta) \tag{7}
\end{equation*}
$$

[^1]where $\theta$ is the co-latitude. The number of particles escaping from the trapping region at the equator at distance L is proportional to the volume contained in the magnetic shell between the lines of force L and $\mathrm{L}+\mathrm{d} \mathrm{L}$ times the density $\rho(L)$.

The volume delimitated by the Earth and the magnetic line of force of Eq. 7 is

$$
\begin{equation*}
V=4 \pi \int_{\theta_{m}}^{90^{\circ}} \sin (\theta) d \theta \int_{R}^{L s \sin ^{2} \theta} r^{2} d r \tag{8}
\end{equation*}
$$

where R is the radius of the solid source and $\theta_{m}=\operatorname{asin}\left(\sqrt{\frac{R}{L}}\right)$ is the angle where the line of force intersect the solid source. Solving the integral we obtain

$$
\begin{equation*}
V=\frac{4 \pi}{3} L^{3}\left\{\cos \theta_{m}-\cos ^{3} \theta_{m}+\frac{3}{5} \cos ^{5} \theta_{m}-\frac{1}{7} \cos ^{7} \theta_{m}-\frac{R^{3}}{L^{3}} \cos \theta_{m}\right\} \tag{9}
\end{equation*}
$$

The volume of the shell included between the lines of force L and $\mathrm{L}+\mathrm{dl}$ can be easily calculated and is

$$
\begin{equation*}
d V=4 \pi L^{2} f\left(\frac{L}{R}\right) d L \tag{10}
\end{equation*}
$$

with the function $f\left(\frac{L}{R}\right)$ that tends to the constant value $\frac{16}{35}$ for $\frac{L}{R}>3$ as shown in fig.1.
Thus the number of particles contained in this volume is

$$
\begin{equation*}
d N=\rho_{o}(L) 4 \pi L^{2} f\left(\frac{L}{R}\right) d L \tag{11}
\end{equation*}
$$

where $\rho_{o}(L)$ is the particle density that, in the general case, depends on L .
The particles at the shell L which violate the Alfvèn breaking condition (Eq. 4 with equal sign) are a fraction $\alpha_{L}(L)$ of $d N$ and have the momentum

$$
\begin{equation*}
p(L)=\frac{\xi c q M}{3 L^{2}}=p_{\max } \frac{R^{2}}{L^{2}} \tag{12}
\end{equation*}
$$

This equation shows a strict relationship between $L$ and $p$,

$$
\begin{equation*}
L(p)=R \sqrt{\frac{p_{\max }}{p}} \tag{13}
\end{equation*}
$$

therefore we also have the inverse function $\alpha_{p}(p)$.
We derive the number of particles which are about to escape from the shell

$$
\begin{equation*}
d n=\alpha_{L}(L) \rho_{o}(L) 4 \pi L^{2} f\left(\frac{L}{R}\right) d L \tag{14}
\end{equation*}
$$

Limiting ourself to the region where $f\left(\frac{L}{R}\right)=\frac{16}{35}$ and using Eq. 12 the above equation becomes

$$
\begin{align*}
d n=\alpha_{p}(p) \rho_{o}[L(p)] 2 \pi & \frac{16}{35} p_{\text {max }}^{3 / 2} R^{3} p^{-1} p^{-3 / 2} d p=  \tag{15}\\
& N_{o} \alpha_{p}(p) \rho_{o}[L(p)] p^{-2.5} d p \tag{16}
\end{align*}
$$

with $N_{o}$ constant and $L(p)$ given by Eq. 13 .


Figure 1: The function $f\left(\frac{L}{R}\right) \times \frac{35}{16}$ calculated versus $\frac{L}{R}$. For $L / r \geq 2 \sim 3$ it takes a constant value equal to $\frac{16}{35}$.

This is the number of particles with momentum between p and $\mathrm{p}+\mathrm{dp}$ which leave, with relativistic velocity, the corresponding shell.

We remark that the spectrum of the escaping particles depends on three processes: a geometrical one expressed by the function $p^{-2.5}$ (the absolute value of the exponent increases for particles leaking out from the region with $L / r \leq 2 \sim 3$, at very high energy), a dynamical one expressed by $\alpha_{p}(p)$ that includes the action of the acceleration mechanisms and, third one, on the density of the plasma distribution in the magnetosphere $\rho_{o}[L(p)]$.

We find impressive that the geometrical process gives a spectrum very close to the experimental one. The dynamical process requires a model for the acceleration mechanisms that bring the particles to the escaping velocity. Such a mechanism might occur trough a diffusion process, studied extensively in [17, 18].

We note that the dependences of the acceleration rate and the density at a certain energy on L are opposite to each other, so that their product shows a much weaker dependence on $\mathbf{L}$. Therefore one can set: $\alpha_{p}(p) \rho_{o}[L(p)] \propto e^{-\beta(p)}$.

Finally we put

$$
\begin{equation*}
d n=N_{o} p^{-\gamma} d p \tag{17}
\end{equation*}
$$

where

$$
\begin{equation*}
\gamma=2.5+\beta(p) \tag{18}
\end{equation*}
$$

## 3 Discussion

At the present stage of our knowledge a complete theory of the cosmic radiation is not possible yet, since many phenomena are still to be explored, in particular the magneto-spheric acceleration mechanisms. Much literature have been published on the pulsar magnetospheres (see in particular [19, 20]), but all works are concerned primarily with the present available observations, in the field of electromagnetic emission.

With the present paper we wish to draw the attention on the new experimental observation: magnetospheres, in addition to emit e.m. radiation, also do emit particles.

In his paper [6] Fermi gives importance to the power law form of his derived spectrum for CR and he states: One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays, in spite of the fact that ad hoc values for the parameters involved in the initial acceleration as well in the particle loss in the interstellar space were necessary.

Following our conjecture the inverse power law spectrum is obtained for a most important part of the spectrum, that concerned with the geometrical process, and this is independent of the particle species comparing favorably with the various measured spectra.

Above the energy of about $10^{14} \mathrm{eV}$ the difference between the $E^{-2.5}$ spectrum and the measured one increases. The experimental logarithmic slope ranges roughly from -2.6 at low energy to -3 at higher energies in regions denominated Knee end Ankle. These regions of the energy spectrum have been studied extensively, i.e [21, 22, 23, 24, 25].

If our conjecture is good, then one expect to find acceleration mechanisms giving values of the $\beta(p)$ ranging, roughly, between 0.1 and 0.5 .

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[^0]:    ${ }^{1}$ PAMELA, launched on 15th June 2006, in a $350-600 \mathrm{~km}$ heigh orbit, is a space-based experiment designed for precise measurements of charged cosmic rays - protons, electrons, their antiparticles and light nuclei in the kinetic energy interval from several tens of MeV up to several hundreds of GeV [9].

[^1]:    ${ }^{2}$ We are aware that we do not propose an explicit acceleration mechanism acting in the magnetosphere. The existence of acceleration mechanisms which lead to a breakdown of the adiabatic conditions has been considered soon after the discovery of the Van Allen belt (see i.e. [13]).

