



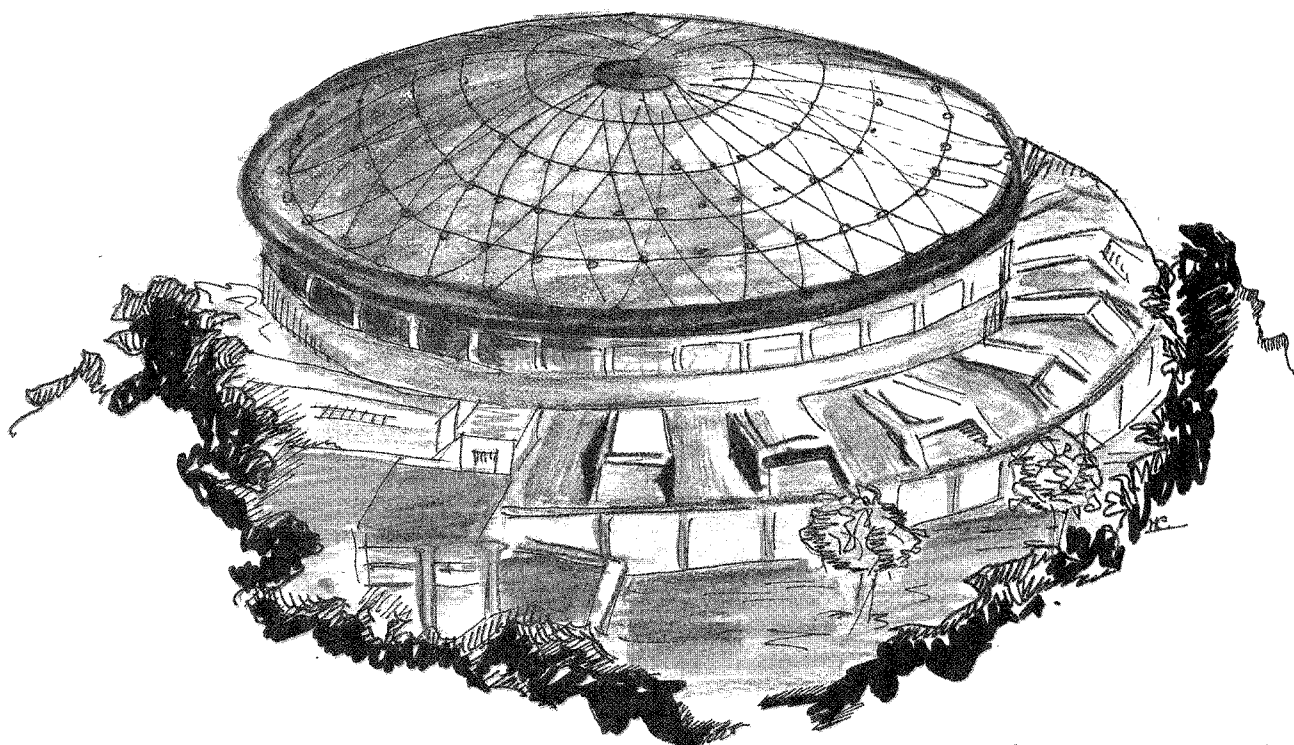
Laboratori Nazionali di Frascati

To be published on IEEE Transaction on Magn.

LNF-90/086(P)
6 Dicembre 1990

F. Celani, R. De Luca, L. Liberatori, S. Pace, A. Saggese:

FLUX CREEP IN JOSEPHSON JUNCTION ARRAYS



Servizio Documentazione
dei Laboratori Nazionali di Frascati
P.O. Box, 13 - 00044 Frascati (Italy)

FLUX CREEP IN JOSEPHSON JUNCTION ARRAYS

F. Celani, L. Liberatori

INFN - Laboratori Nazionali di Frascati, P.O.Box 13, I-00044 Frascati RM (Italy)

S. Pace, A. Saggese, R. De Luca

Dept. of Physics, Univ. of Salerno, 84100 Salerno Italy

ABSTRACT

In order to analyze the diamagnetic properties of weakly coupled structures in high T_c ceramic superconductors, the Josephson junction array model is used. We suppose that the coupling is strong enough to allow magnetic flux trapping inside non superconducting regions surrounded by superconducting loops closed by Josephson junctions. We remark that the presence of currents flowing through the junctions has to be taken explicitly into account in the Hamiltonian. This description leads to a creep model of the Josephson junction array. Indeed we have:

- 1) pinning centers generated by non superconducting regions into the loops,
- 2) pinning potentials determined by fluxon motion barriers due to the Josephson junctions,
- 3) absence of degeneracy of the states corresponding to a different number of fluxons in the loops,
- 4) a reduction of the barrier height due to measuring currents or to diamagnetic shielding currents. The last effect is equivalent to the Lorentz force effect in type-II superconductors. In this way we believe that this picture deeply modifies the usual superconducting glass model.

INTRODUCTION

Just after the discovery of high T_c superconductors two approaches arose to explain the unusual magnetic behaviour of these materials. The first approach¹ aims at describing these systems according to the glassy state, the second² uses flux creep and flux flow phenomena. These two approaches still coexist^{3,4} and the existence of a connection between them is not evident. In particular, great attention has been devoted to the transition between reversible and irreversible magnetization as the temperature or the magnetic field increases⁵. However, it

should be proved that this transition is equivalent either to the disappearance of a long range phase coherence or to the resistive transition. Of course the presence of a reversible regime, for temperatures less but close to T_c , only involves the existence of relaxation times shorter than measurement times. On our knowledge there are no measurements which prove the equivalence of the previously mentioned transitions. Further problems arise from the granular structure found mainly in sintered samples and also in single crystals and thin films. Several authors describe the magnetic behaviour using a spin glass Hamiltonian derived from a Josephson junction array model for this class of samples⁶. Instead, other authors use flux creep and flux flow theories⁷, so that also in this case the relationship between the two approaches is not clear. Furthermore, the analysis of the low field (10-100 Gauss) experimental magnetic behaviour of granular systems is performed⁸ by using critical state models⁹ where the validity of these models for a Josephson junction array is still to be proved.

In this work the analysis is limited to the description of the magnetic behaviour of granular systems with a grain coupling strong enough to generate complete shielding at low magnetic fields.

Let us consider a slab of the sample placed orthogonally to the applied magnetic field and schematized as a square lattice of identical spherical grains surrounding identical intergranular regions. Grains are weakly coupled by Josephson junctions. In this way the sample can be described by Figs. 1a,b or by the equivalent circuit of Fig. 1c.

As it has been recently pointed out¹⁰, the analysis of the Josephson junction network must take explicitly into account the shielding currents crossing the junctions. These currents reduce the potential barrier for a 2π phase flip of the superconducting phase of the junctions. This effect is well known for a single loop closed by a junction described by the potential :

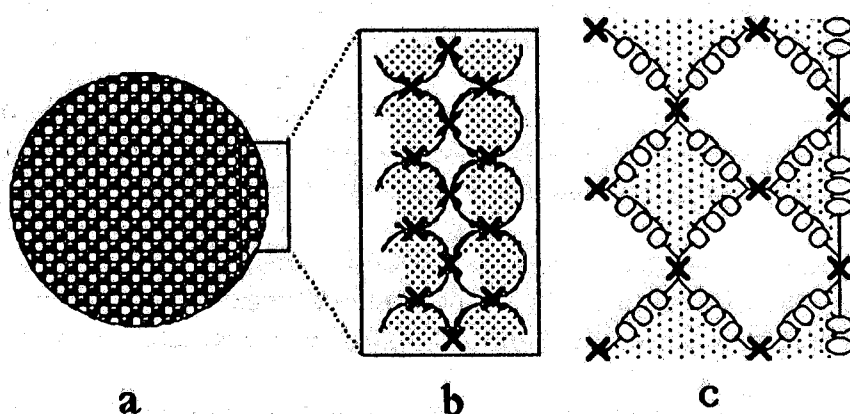


FIG. 1 - a) granular system, b) expanded portion of Fig. 1a, c) equivalent circuit of the granular system; crosses represent Josephson junctions which couple the grains.

$$G = (\Phi - \Phi_{ex})^2 / 2L + E_0 [1 - \cos(2\pi\Phi / \Phi_0)] \quad (1)$$

where $E_0 = E_0(h, T) = I_j \Phi_0 / 2\pi$, $I_j = I_j(h, T)$ is the maximum Josephson current in the local magnetic field h , Φ and Φ_{ex} are respectively the flux of the actual magnetic field and the flux of the field H generated by the external sources, while L is the loop inductance and Φ_0 is the flux quantum. If $\Phi - \Phi_{ex} \gg \Phi_0$, for small magnetic flux variations $\delta\Phi$, the variation of $(\Phi - \Phi_{ex})^2 / 2L$ is $\delta\Phi(\Phi - \Phi_{ex}) / L$. Since now $\Phi - \Phi_{ex} = LI$, Eq. 1 can be approximated by the potential of a junction fed by an external current source:

$$G = E_0(1 - \cos \varphi - \alpha \varphi) \quad (2)$$

where $\alpha = I/I_j$ and φ is the gauge invariant phase difference of the junction. By Eq. 2, if $LI_j \gg \Phi_0$, the barrier height ΔE between metastable states can be approximated by:

$$\Delta E = E_0 [(1 - \alpha^2)^{1/2} - \alpha \cos^{-1} \alpha] \quad (3)$$

As it can be argued from Figs.2a,b and by Eqs.1,2, this problem is similar to the one of pinning centers in the presence of Lorentz forces in a type-II hard superconductor.

In fact we have:

- the non-superconducting region, surrounded by the superconducting loop closed by the Josephson junction, acts as a pinning center;
- the pinning potential barrier is determined by the potential barrier between metastable states due to the Josephson junction;
- the current flowing in the junction removes the states degeneracy and reduces the pinning barriers.

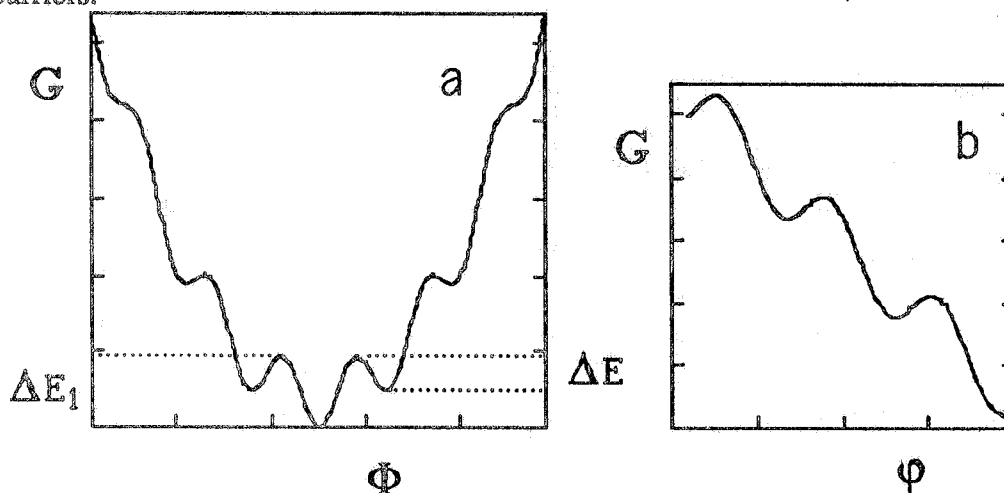


FIG. 2 - a) the potential for a loop in an external field, b) washboard potential for a current biased junction.

In spite of these analogies, we remark some differences:

- in our case the reduction of the barrier is not caused by the Lorentz force, because the magnetic flux is present mainly in the non superconducting region inside the loop, where the currents do not flow, so that the field-current interaction is extremely limited;
- neglecting flux quantization problems, by Eq. 1 the presence of non superconducting regions makes the lower critical field of the granular structure H_{GC1} negligible, namely all the shielding states are always metastable;
- Figs.2a,b show respectively the potential as a function of a flux and a phase, while in the usual pinning problem in the presence of the Lorentz force we have the same picture for the potential as a function of a space coordinate. In spite of this difference, the analogy is not only formal. Indeed any transition between two adjacent minima of G , corresponding to a 2π phase variation and to the entry of a flux quantum into the loop, corresponds to the motion of a flux quantum in real space.

Starting with these considerations, in the second section the single loop results are extended to a network of loops closed by junctions. In this way, a critical state model can be easily derived at $T=0$. In the third section the analysis is extended to finite temperatures leading to a logarithmic time decay of the magnetization. In the last section some comments about the limits of validity of the present description and a comparison with different approaches are presented.

MAGNETIC FLUX PENETRATION AND CRITICAL STATE

The generalization of Eq. 1 for a network of loops closed by Josephson junctions is not trivial, since the presence of currents in every branch induces a magnetic field in the other loops of the network, so that the self and mutual inductance coefficients of all elements should be taken into account. In order to simplify the problem, we limit our discussion to the case where $L I_j \gg \Phi_0$ for any loop and for any junction of the network in the presence of junction currents near the critical values. Under these conditions the effective potential of each junction depends only on the currents flowing through the junction itself. In this way ¹⁰ the potential can be approximated by:

$$G = (\Phi_0/2\pi) \sum_{ik} I_{j_{ik}} \{1 - \cos(\varphi_{ik}) - \alpha_{ik} \varphi_{ik}\} \quad (4)$$

In general, when one analyzes the whole network of junctions, it can be shown that two stationary states obtained by displacing a flux quantum between two adjacent loops are different relatively to the current distribution in the entire network. However, under the condition $L I_j \gg \Phi_0$ for any loop, the displacement of a flux quantum between two adjacent loops changes only the fluxes and the currents of the two adjacent loops, while currents variations in surrounding loops are negligible.

As recently reported^{11,12}, it is possible to predict qualitatively and to determine by numerical simulations the magnetic field penetration dynamics. Summarizing, let's consider first a circular monolayer (with radius R) of identical superconducting spheres (with radius d) arranged into a square lattice and linked, as shown in Fig. 1a, by almost identical Josephson junctions. Fig. 1b shows a magnified detail of Fig. 1a as a set of grain lines interconnected by junctions. We suppose that the critical field H_{c1} and critical currents of these spheres are much higher than all fields and currents considered in this work.

After a zero field cooling condition, all the Josephson junctions of the slice are closed, i.e. a static superconducting current can flow through the links. Since fluxoid quantization must be valid for any superconducting loop of the sample, the application of an uniform weak magnetic field orthogonal to the slice will generate a shielding current I_s which will circulate through the junctions of the most external loop. As for a single loop, this shielding state is stable until the shielding current is lower than the Josephson current $I_{j_{min}}$ of the weakest junction on the most external loop. As soon as I_s reaches $I_{j_{min}}$, the weakest junction will open and flux quanta enter into the sample.

In order to determine the flux trapping, our sample can be described by the equivalent circuit of Fig. 3a, which can be shown to be equivalent to a single loop closed by a junction. By describing the junction with a RSJ model, the number of flux quanta which enter in the small loop depends on both the value of $\gamma = L I_j / \Phi_0$ and the damping coefficient $\beta = L^{1/2} / (RC)^{1/2}$. Blackburn et al. ¹³ have found that there exists a critical value β_c above which only one flux

quantum enters in the loop. Neglecting the $\cos \varphi$ contribution to the junction conductance, in the range $10^2 > \gamma > 10^5$ it holds: $\beta_c = 3\gamma^{1/2}$; in this way the critical condition depends on a sufficiently high value of $1 / (R C^{1/2})$: i.e. : $1 / (R C^{1/2})_{\text{CRIT}} = 3 (I_j / \Phi_0)^{1/2}$.

In this paper, in order to avoid any type of avalanche effect both in penetration and in relaxation of magnetic flux quanta, the analysis is restricted to the high damping case in which any time only one flux quantum jumps inside or outside the loops. This hypothesis is reasonable because the superconducting intergranular coupling is expected to be weak-link like, so that high damping conditions are justified.

Under the conditions $L I_j \gg \Phi_0$ and $\beta > \beta_c$ for any loop and for any junction of the network also in dynamical conditions the motion of a flux quantum between two adjacent loops in general does not change currents and fluxes in surrounding loops unless the junctions in the surrounding loops are very close to the critical values.

Therefore, because of the increase of the magnetic field, a flux quantum enters in the smallest superconducting loop underneath the open junction. In this picture, the lower critical field H_{GC1} , defined as the lowest value which generates flux penetration, is determined by the Josephson current $I_{j\text{min}}$ of the weakest Josephson junction located on the most external loop. During flux penetration the current flowing in the junction decreases while the current flowing in the small internal loop increases. The final current distribution will be the superimposition of both currents ¹².

In an analogous way, if all junctions are almost identical, a further increase of H will lead to progressive temporary opening of all the junctions in the external loop, i.e. on the first line from the right of Fig. 1b, and to the trapping (Fig. 3b) of flux quanta in any loop just underneath the external loop (i.e. in the small loops between the first and the second line). As H increases, first the weakest junction J_{min} and later the other junctions in the external loop will open again, leading to the increasing of both flux quanta trapped and circulating currents in the small loops. The total current distribution is shown in Fig. 3c.

This process continues until the currents reach the Josephson current of the junctions in the second line, which will open and flux quanta will penetrate between the second and the third line.

Following the previously described mechanism a further increase of H will lead to the progressive penetration of flux quanta into inner and inner loops. If all inductances of the smallest intergranular loops are almost identical, the currents flowing through all the junctions in the same line will be almost the same and the current circulating in each internal line will be proportional to the difference between the flux quanta trapped in the two adjacent lines.

The key point of our description is that flux quanta can move and penetrate deeply into the sample only as a critical value I_c of the current of the junctions is reached. At $T=0$ the threshold is determined by I_j . At $T \neq 0$ the threshold is a current value large enough to determine a transition time much shorter than measurements times.

In this way the current flowing on the junctions of all lines will be near the junction critical current except for the deepest line in which some current is flowing ^{11,12}. This line marks the boundary of the magnetic flux penetration. The increase of H and further penetration of flux quanta leads to the increase of the current of the deepest line until the critical current is reached and flux quanta can overcome this line.

If the external magnetic field is now reduced, similar mechanisms determine the flux de-trapping. In fact, the current flowing in the junctions of the most external loop follows the reduction of H until it reaches $-I_{j\text{min}}$, at this point the flux quanta trapped underneath the first

line will get out of the superconductor. Further reduction of H will determine fluxon motion across a line only when the currents flowing in the junctions of the line reach the critical value.

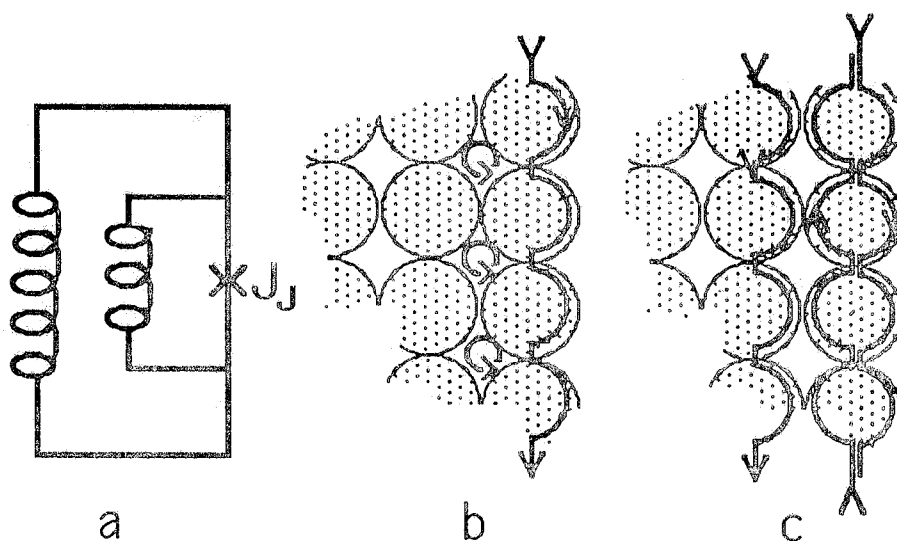


FIG. 3 - a) equivalent circuit for the flux penetration in a loop just underneath the external surface, b) the junctions of the external loop have been temporarily opened and flux quanta has been trapped in each loop underneath the surface, c) equivalent total current distribution.

This description can be easily extended to a cylindrical three dimensional sample in the presence of a magnetic field parallel to the cylinder axis.

It is clear that such description leads to results very close to the critical state Been model. Indeed the existence of a critical state is determined by the condition that flux quantum travel in the sample only after a critical threshold of the current is reached.

MAGNETIC FIELD RELAXATION

In order to analyze the finite temperature effect on the magnetic flux decay in granular systems, we start again from well known results on a single loop closed by a junction. Neglecting quantum tunneling effects, in the analysis of lifetimes of metastable states in a washboard like potential, the overdamped and the underdamped regimes exist^{14,15}, depending on the damping coefficient ϵ . Here $\epsilon = \omega \eta$ with $\eta = (RC)^{-1}$, where R is the normal tunneling resistance of the junction, and C is junction capacity; ω is the frequency of small oscillations around the minimum of the potential of Eq. 1: $\omega^2 = (LC)^{-1} + I_j 2e (1-\alpha^2)^{1/2} / (\hbar C)$.

The escape time τ is given by:

$$1/\tau = 1/\tau_0 \exp(-\Delta E / K_B T) \quad (5)$$

where, in the overdamped case, i.e. $\epsilon \gg 1$, $1/\tau_0 = \omega^2 / (2\pi\eta)$. In the underdamped regime, i.e. $\epsilon \ll 1$, two different cases can be considered¹⁴. For $\epsilon > \epsilon_c = K_B T / 2\pi\Delta E$, we have $1/\tau_0 = \omega / (2\pi)$. For extremely underdamped systems $\epsilon < \epsilon_c$ it has been found $1/\tau_0 = \eta \Delta E / K_B T$.

In the second section we supposed a sufficient high damping to avoid multiple flux quanta input or output from the loop when critical conditions for the current are reached. Here we assume that the multiple flux quanta motion is avoided also for thermal excited jumps.

Moreover we suppose a sufficient low value of R and C to assure the overdamping regime for the time escape.

As shown in previous sections, for $L I_j \gg \Phi_0$ the interactions between non adjacent loops can be neglected, so that for the internal loops the current crossing the junctions depends only on the difference between the fluxes linked with the two adjacent loops.

In the general case the barrier for a flux quantum jump between the two adjacent loops depends not only on the junction common to the two loops but also on the other junctions present in the two loops and on the currents flowing in them. However in a first approximation we neglect the presence of these other junctions so that the barrier will be determined by Eq. 3.

In this way in the junction array in the presence of a magnetic induction $B(r)$ in the direction of the z axis, the decrease of the barrier is only proportional to the circulating shielding currents, i.e. to the gradient of $B(r)$ and the usual dependence of the effective pinning potential on $B(r)$ is not present. In fact, in type-II hard superconductors the dependence on $B(r)$ is due to the existence of both bundles and Lorentz forces. The bundles arise by the existence of the Abrikosov lattice with a strong vortex interaction for distances lower than the London penetration depth.

In the Josephson junction array both the Abrikosov lattice and Lorentz forces do not exist, so that the dependence of the driving force on the number of fluxons (i.e. to $B(r)$) is not present.

The behaviour of the normalized barrier $\Delta E/E_0$ as function of the normalized current I/I_j is reported in Fig. 4. Following P.W. Anderson theory¹⁶, as remarked by Beasley et al.¹⁷, in the critical region of the current, where the probability of decay is maximized, the behaviour of $\Delta E(\alpha)$ can be approximated by a linear dependence.

$$\Delta E / E_0 = E' / E_0 - \zeta \alpha \quad (6)$$

By the shape of the curve $\Delta E(\alpha)$, shown in Fig. 4, both E'/E_0 and ζ will be monotonically decreasing function of α_c . Because α_c depends both on the absolute value of $E_0(h,T)$ and T , then both E'/E_0 and ζ will be functions of h and T . Indeed for a given temperature the increase of the magnetic field reduces the maximum of the Josephson current according to the Fraunhofer pattern¹⁸, reducing both E_0 and consequently α_c . However because the junction area is much smaller than the loops area, for a given external magnetic field, during the decay of the magnetization, it can be reasonable to neglect the dependence of $E_0(h,T)$ on h . In this way α_c and consequently E' and ζ will be independent on h .

To derive the magnetization decay in the easiest way, the Tinkham argument can be followed¹⁹: the logarithmic decay is a consequence of the equation:

$$dB / dt = - A \exp(B / B_0) \quad (7)$$

where A and B_0 are constant. Eq. 7 is correct also in our model because by Eq. 5 the amount of flux quanta coming out from the sample depends exponentially on the barrier ΔE , which by Eq. 6 decreases linearly with the current and of course with the magnetization.

In fact, for simplicity, let us consider a cylindrical annular sample with radius R carried in the critical state by the application and the subsequent removal of the magnetic field. If we suppose negligible the dependence of the Josephson current on the magnetic field, the critical

current density can be considered constant. If the thickness ΔR of the ring is much smaller than R , the flux flow density can be considered independent of the radius.

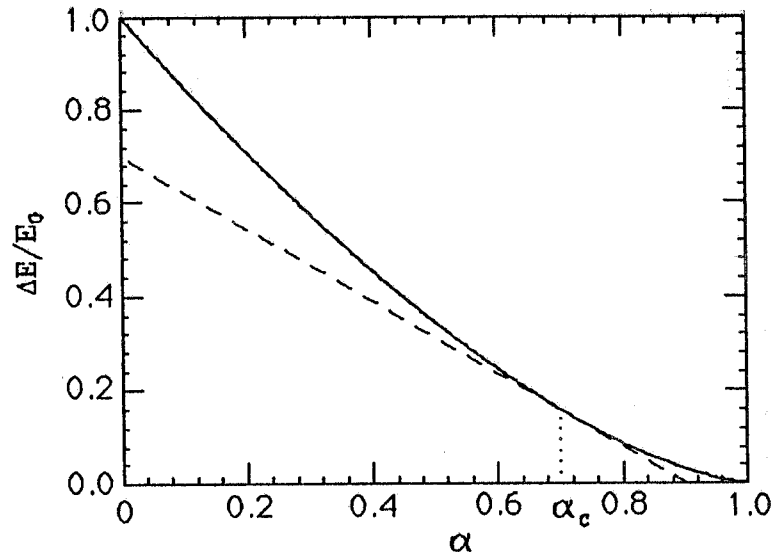


FIG. 4 - Dependence of the normalized barrier versus the normalized bias current.

In these conditions the number $n(t)$ of flux quanta coming out of the sample per unit time will be: $n(t) = N / \tau$, where τ is given by Eq. 5 and N is the number of loops of the external circumference: $N = \pi R / d$ and d is the grain radius. In this way we obtain.

$$dB/dt = -\Phi_0 N / (\pi R^2 \tau) = -\Phi_0 / (\tau_0 R d) \exp(-\Delta E / K_B T) \quad (8)$$

By substituting Eq. 6 into Eq. 8, the time dependence of B is related to the critical current I_c in a single junction, which is proportional to the critical current density $J_c = I_c / d^2$.

By Maxwell equations $B = J_c F$ where F is a geometrical factor: $F = 4\pi/c \Delta R [(R - \Delta R)^2 + R \Delta R] / R^2$, so that the Eq. 7 is obtained with:

$$A = \Phi_0 / (\tau_0 R d) \exp(-E / K_B T) \quad (9)$$

$$B_0 = (K_B T / E_0) F I_j / (\zeta d^2)$$

Since by the previous statements the coefficients A and B_0 are constant, by Eq. 9 the usual logarithmic decay of the magnetization follows.

Of course a more careful analysis should require the solution of the flux flow diffusion equation. However, following the analysis of Beasley et al. ¹⁷, it is clear that the absence of the explicit dependence on B do not modify the time decay but only the space dependence of B .

DISCUSSION AND CONCLUSIONS

In this work, by using a network of superconducting loops closed by Josephson junctions as a model of a superconducting granular system, it is shown that the correct analysis has to consider the shielding currents across the junctions. This is equivalent to consider the effect of

the magnetic field generated by the shielding currents themselves. This analysis conducted to the existence of two parameters which determine the system behaviour:

$$\text{a) } Lj / \Phi_0 = \gamma$$

$$\text{b) } \Delta E / K_B T = \Gamma$$

which are respectively related to the possibility of trapping flux quanta into a single loop of the system, and to the stability of metastable states. The complete analysis should examine the behaviour of the system in the γ versus Γ plane.

This work is limited to the case $\gamma > 1$, $\Gamma > 1$ denominated strong coupling between grains. Under these conditions we have shown that the presence of shielding currents on the junction coupling plays a determinant role and appears to be the main difference among this approach and the spin glass model. At a first sight, the spin glass approach is equivalent to our model only in the limit of negligible currents. This condition is found or by approaching T to the transition temperature or by increasing the magnetic field, since in both cases the irreversible magnetization goes to zero.

However a close view of the potential of Fig. 2a, shows the existence of two barriers: the first ΔE determines the metastable state decays, while the second ΔE_1 is the barrier necessary to generate a 2π phase flip starting from the ground state. The difference $\Delta E_1 - \Delta E \approx \Phi_0^2/2L$ is the energy of a magnetic flux quantum entrapped into the loop corresponding to the 2π phase slip. Only the limit $\Phi_0^2/2L < E_0$, i.e. $Lj \gg \Phi_0$, the magnetic field energy can be neglected for low energy metastable states. Only in this limit the effect of the currents can be neglected.

Out of this limit, ΔE and ΔE_1 may be very different. The energy barrier against the decay of metastable states ΔE may be smaller than $K_B T$, while E_0 remains much larger than $K_B T$, so that a more accurate analysis is necessary. Moreover, as previously stated, in the condition $Lj \leq \Phi_0$, the transition of a single flux quanta changes the electrical parameters not only in the two loops affected by the transition. In this way the energy of the magnetic field introduces a correlation effect among neighboring junctions so that, in the limit of $Lj \ll \Phi_0$, the analysis of a single loop is inadequate and a model similar to the distributed model¹⁸ for long Josephson junctions should be used.

Following the above discussion the spin glass approach appears to correctly describe the irreversible to reversible transition only in the conditions $Lj \gg \Phi_0$ and $E_0 \approx K_B T$. However, in spin glass models the barrier reduction is given by the path integral of the vector potential A associated to the applied magnetic field which reduces the barrier height by means of the frustration. On the contrary, the single loop analysis clearly shows that the barrier undergoes the maximum reduction just in the complete shielding state where the path integral of A on the closed loop is zero.

In conclusion, we have shown that the correct analysis of a granular system described by the Josephson junction array model has to take explicitly into account the effect of the shielding currents on the Josephson barriers. In the limit $Lj \gg \Phi_0$ for any loop of the system, and for $E_0 > K_B T$, the behaviour of a granular system is very similar to a type-II superconductor in the presence of pinning centers. In spite of the absence of the Lorentz forces, the action of the shielding currents flowing in the junctions reduces the pinning potential leading to a critical state and thermally activated creep phenomena. The absence of bundles phenomena implies a non direct dependence of the pinning potential on B ; the absence of B dependence modifies only the space dependence of the magnetization leaving the logarithmic time decay of the magnetization unchanged.

REFERENCES

1. M.P.A.Fisher, "Vortex-Glass Superconductivity: A Possible New Phase in Bulk High-Tc Oxides", *Phys.Rev.Lett.*, 62, 1415-1418, 1989.
2. A.P.Malozemoff, L.Krusin-Elbaum, D.C.Cronemeyer, Y.Yeshurun and F.Holtzberg, "Remanent moment of high-temperature superconductors: implication for flux pinning and glassy models", *Phys.Rev.*, B38, 6490-6499, 1988.
3. M.P.A.Fisher, "Theory of the vortex glass phase in high Tc superconductors", *Physica C* (1991), Proc. of the LT19 Conf.
4. E.H.Brandt, "Fluctuations, melting, depinning, creep & diffusion of the flux line lattice in HTS", *Physica C* (1991), Proc. of the LT19 Conf.
5. K.A.Müller, K.W.Blazey, J.G.Bednorz and M.Takashige, "Superconducting glassy state in high-Tc oxides", *Physica*, 148B, 149-154, 1987.
6. C.Ebner and D.Stroud, "Diamagnetic susceptibility of superconducting clusters: Spin Glass behavior", *Phys.Rev.*, B31, 165-171, 1985.
7. K.H.Müller, "Frequency dependence of A.C. susceptibility in high-temperature superconductors. Flux creep and critical state at grain boundaries", *Physica C*, 168, 585-590, 1990.
8. V.Calzona, M.R.Cimberle, C.Ferdeghini, M.Putti and A.S.Siri, "A.C. Susceptibility and magnetization of high Tc superconductors: Critical state model for the intergranular regions", *Physica C*, 157, 425-430, 1989.
9. C.P.Bean, "Magnetization of high-field superconductors", *Reviews of Modern Physics*, 36, 31-39, 1964.
10. S.Pace, A.Saggese, L.Liberatori, B.Polichetti, and F.Celani, "Shielding currents influence on intergranular coupling in high - Tc superconductors", *Physica C* (1991), Proc. of the LT19 Conf.
11. A.Saggese, S.Pace, B.Polichetti, R.De Luca and C.Naddeo "Magnetic field penetration on superconducting granular systems", to be published on VUOTO.
12. R.De Luca, S.Pace and B.Savo, "Low field magnetic behavior of high Tc superconducting granular systems", submitted to *Physics Letters A*.
13. J.A.Blackburn, H.J.T.Smith and V.Keith, "Probing the phase-dependent conductance and non equilibrium properties of Josephson junctions by means of flux entry into weakly coupled loops", *Phys.Rev.*, B15, 4211-4213, 1977.
14. P.Silvestrini, S.Pagano, R.Cristiano, O.Liengme and K.E. Gray, "Effect of dissipation on thermal activation in an underdamped Josephson junction: first evidence of a transition between different damping regimes", *Phys.Rev.Lett.*, 60, 844-847, 1988.
15. J.Kurkijarvi, "Intrinsic fluctuations in a superconducting ring closed with a Josephson junction", *Phys.Rev.*, B6, 832-836, 1972.
16. P.W.Anderson and Y.B.Kim, "hard superconductivity: Theory of the motion of Abrikosov flux lines", *Reviews of Modern Physics*, 36, 39-43, 1964.
17. M.R.Beasley, R.Labusch and W.W.Webb, "Flux creep in type-II superconductors". *Phys. Rev.*, 181, 682-700, 1969.
18. A.Barone and G.Paterno', "Physics and Applications of the Josephson Effect", Wiley, New York, 1982.
19. M.Tinkham, "Introduction to superconductivity", Chap.5, McGraw Hill, New York, 1975.