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EFFECTIVE ACTION APPROACH TO THE GREEN-SCHWARZ STRING

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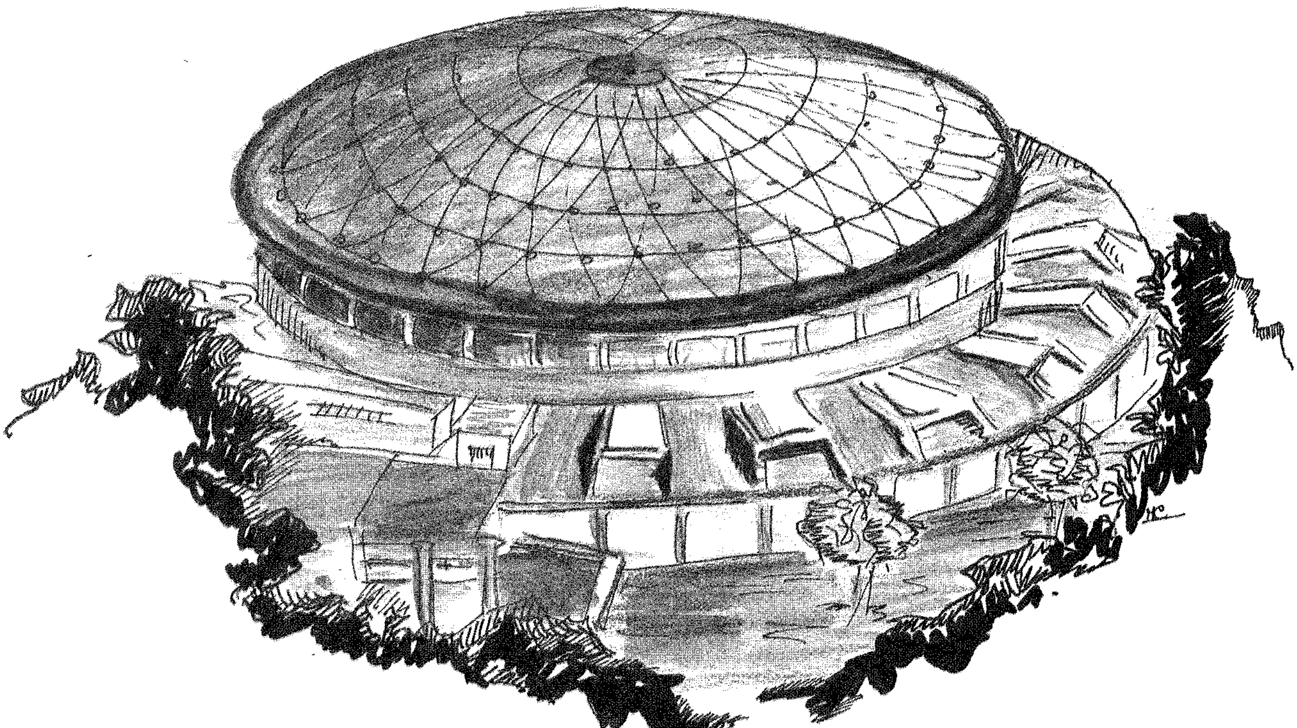

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EFFECTIVE ACTION APPROACH TO THE GREEN-SCHWARZ STRING

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ABSTRACT

We give an introduction to the σ-model for the Green-Schwarz superstring. We analyze the conditions for ultraviolet finiteness comparing them with those needed for conformal invariance. We discuss the Lorentz transformation properties in the quantization by ghost truncation.

1. - INTRODUCTION TO THE GREEN-SCHWARZ σ-MODEL

For the material covered in this section please consult also Ref. [1] and references therein. There exists two formulations of D=10 (heterotic) superstrings: i) the Neveu-Schwarz-Ramond (NSR) action; ii) the GS action. The theory in i) is worldsheet superconformal, and quantization is straightforward. The action ii) possesses D=10 spacetime supersymmetry; there is a difficulty in the quantization. In the light-cone gauge, ii) can be compared with i). For flat backgrounds the two formulations are identical (are related by a triality transformation), at the classical level. For certain nontrivial purely bosonic backgrounds (with no spacetime supergravity fermions and some restrictions on the bosonic fields) the corresponding light-cone σ-models are the same. Even in the presence of fermionic backgrounds it is possible to connect the two formulations of the light-cone string.

The string propagation in bosonic background fields is described by D=2 σ-models. For both the bosonic string and the NSR string the quantum consistency of the σ-model requires the background to satisfy equations of motion identical to those derived from the field theory limit of strings. For a fully supersymmetric background, including spacetime fermions, it is necessary to look at the GS formulation. In fact: 1) coupling to fermionic backgrounds is very natural for i) but difficult for ii); 2) the NSR supermoduli space structure is complicated; 3) there has been some progress in the quantization of ii).

The GS σ-model is obtained coupling the heterotic GS superstring to a D=10 curved superspace. The appropriate background for the GS formulation is the coupled D=10, N=1 supergravity and Yang-Mills system. Consistent coupling is achieved by introducing the Yang-Mills Chern-Simons form (absence of gauge anomalies) and the Lorentz Chern-Simons form [2] (no Lorentz anomaly). For the GS σ-model this is necessary to avoid κ-symmetry anomalies [3,4].

Consistent coupling (invariance of the classical GS action under κ-transformations), requires that the target manifold satisfy the constraints of D=10 supergravity. In the background field method and normal coordinate expansion the supersymmetry of the effective action is preserved but manifest κ-invariance is lost because of nonlinearities in the κ-invariance. The gauge symmetries of the fluctuating part of the supercoordinates may be fixed in a not manifestly covariant way, without affecting the gauge symmetries of the background part [5,6]. This allows to have control on the renormalizability properties of the GS theory. Those of the NSR theory are determined largely by D=2 supersymmetry on the worldsheet.

There is a calculation by Grisaru, Nishino and Zanon which proves the two-loop finiteness of the GS σ-model, restricted to the Yang-Mills-dependent contributions [7]. The ultraviolet finiteness is implied by the classical field equations derived as solutions to the Bianchi identities from the torsion constraints that put the background on-shell. The complete two-loop renormalization is still lacking. This will provide a crucial check of the Lorentz Chern-Simons contribution to the background field equations, whose form has been recently obtained from the geometry of superspace [8].

It has been suggested that adding a D=2 curvature term (or an interaction term proportional to the divergence of the number current of the b,c ghosts obtained from fixing worldsheet reparametrization invariance) is needed for quantum consistency of the σ-model and vanishing of all D=2 (trace and κ-invariance) gauge anomalies.

2. - ULTRAVIOLET FINITENESS VERSUS CONFORMAL INVARIANCE

This section is based on the work contained in Ref. [9]. For the bosonic string σ-model one finds the condition on the Ricci tensor R_{ab} of the manifold, to the one-loop order in α'

$$R_{ab} = D_a V_b + D_b V_a \ , \tag{1}$$

where V represents a generic vector function on the manifold. Cancellation of the on-shell trace anomaly can be achieved by adding a finite local counterterm (the so-called Fradkin-Tseytlin term [10]) in the effective action, and requiring [11]

$$V_a = \frac{1}{2} D_a \Phi \ . \tag{2}$$

Both conditions are needed for a conformal invariant σ-model and a consistent string propagation.

The classical action for the σ-model that describes the heterotic GS string in a supergravity background [3] reads, omitting the heterotic fermions

$$I_{cl} = \int \left(\frac{1}{2} \sqrt{g} g^{ij} V_i{}^a V_{ja} + \varepsilon^{ij} V_i{}^B V_j{}^A B_{AB} \right) d^2x \ , \tag{3}$$

where $V_i{}^A = Z^M{}_{,i} E_M{}^A$, $Z^M = (x^m, \theta^\mu)$, $A = (a, \alpha)$. In writing the two-dimensional action (3) the curved background superspace is described, in terms of the supervielbein $E_A{}^M(x,\theta)$, the torsion superpotential $B(x,\theta)$ and the dilaton superfield $\Phi(x,\theta)$, by the constraints

$$T_{\alpha\beta}{}^c = 2\Gamma^c{}_{\alpha\beta} \ , \qquad T_{\alpha b}{}^c = T_{\alpha b}{}^\gamma = 0 \ ,$$

$$T_{\alpha\beta}{}^\gamma = \left[\delta_{(\alpha}{}^\delta \delta_{\beta)}{}^\gamma - \Gamma^c{}_{\alpha\beta} \Gamma_c{}^{\gamma\delta} \right] \chi_\delta \ ,$$

$$H_{abc} = -\frac{1}{2} T_{abc} \ , \qquad D_\alpha \Phi = \chi_\alpha \ . \tag{4}$$

The classical action (3) possesses a fermionic invariance (κ-symmetry). The one-loop divergent counterterm can be calculated using the propagators and the quantum-background vertices derived from the classical action

$$I_\infty = \frac{1}{2\pi\varepsilon} \int \sqrt{g} \Big[g^{ij} \left(V_i{}^b V_j{}^a D_b D_a \Phi + V_i{}^\beta V_j{}^a D_a \chi_\beta \right) + \varepsilon^{ij} \left(\frac{1}{2} V_i{}^d V_j{}^c T_{cd}{}^b D_b \Phi + (V_i \Gamma^a V_j) D_a \Phi \right) \Big] d^n x \ . \tag{5}$$

Next, we write the change in the quantities of interest to us, under infinitesimal Weyl transformations

$$\delta(\sqrt{g}g^{ij}) = \sqrt{g}g^{ij}\varepsilon\Lambda\ , \tag{6}$$

$$\delta(\sqrt{g}\varepsilon^{ij}) = \sqrt{g}\varepsilon^{ij}\varepsilon\Lambda\ , \tag{7}$$

$$\delta I = \int \sqrt{g}\Lambda T_k{}^k d^n x\ , \tag{8}$$

$$\delta(\sqrt{g}R_{(2)}) = -\,2\sqrt{g}\,\Box\Lambda + O(\varepsilon)\ , \tag{9}$$

$$\Box\Lambda = g^{ij}\Lambda_{;ij} = \frac{1}{\sqrt{g}}(\sqrt{g}g^{ij}\Lambda_{;j})_{;i}\ . \tag{10}$$

The conformal invariance of the classical action does not automatically ensure that the quantum effective action will be so. Let us consider the change in the one-loop counterterm (5)

$$\delta I_\infty = \int \sqrt{g}\Lambda\frac{1}{2\pi}\Big[\ g^{ij}\big(\ V_i{}^b V_j{}^a D_b D_a\Phi + V_i{}^\beta V_j{}^a D_a\chi_\beta\big) + \varepsilon^{ij}\Big(-\ \frac{1}{2}\ V_i{}^d V_j{}^c T_{cd}{}^b D_b\Phi + (V_i\Gamma^a V_j)D_a\Phi\Big)\Big]d^n x\ . \tag{11}$$

Using the classical equation of motion in two dimensions

$$-\ g^{ij}V_j{}^c{}_{;i} + \varepsilon^{ij}\,(-\ \frac{1}{2}\ V_i{}^a V_j{}^b T_{ba}{}^c + V_i{}^\alpha \Gamma^c{}_{\alpha\beta}V_j{}^\beta) = 0 \tag{12}$$

and the constraints (4), we obtain the on-shell value of the trace anomaly

$$T_k{}^k = \frac{1}{2\pi}\ g^{ij}(V_j{}^a D_a\Phi)_{;i}\ . \tag{13}$$

This expression differs from the naive expectation (based on the bosonic result)

$$T_k{}^k = \frac{1}{2\pi}\ g^{ij}\,(V_j{}^A D_A\Phi)_{;i}\ . \tag{14}$$

As we pointed out in Ref. [1], we expect the Fradkin-Tseytlin term to be needed for quantum consistency of the GS σ-model. In fact, the trace anomaly is partially cancelled when such term is introduced. Defining the effective action as the sum

$$I_{eff} = I_{cl} + I_\infty + I_{FT}\ , \tag{15}$$

where

$$I_{FT} = \frac{1}{4\pi} \int \sqrt{g} R_{(2)} \Phi d^2x \,, \tag{16}$$

and recalling (9), we get

$$\delta I_{FT} = -\frac{1}{2\pi} \int \sqrt{g} \Lambda(\Box \Phi) d^2x \,. \tag{17}$$

Thus, considering the change in the effective action under infinitesimal local scale transformations
and using the D=2 equation of motion, we obtain the on-shell trace anomaly

$$T_k{}^k = -\frac{1}{2\pi} g^{ij}(V_j{}^\alpha D_\alpha \Phi)_{;i} \,. \tag{18}$$

One can see that there is no finite counterterm local in both g^{ij} and Z^M that can be added in the effective action to cancel the trace anomaly. This observation is based on the fact that the conformal transformations of the two-dimensional tensors $\sqrt{g} g^{ij}$ and $\sqrt{g} \varepsilon^{ij}$ (6) and (7) are zero in the limit $\varepsilon \to 0$. As a result, one-loop finiteness does not imply invariance of the quantum theory under local scale transformations. The trace anomaly (18) is given by a D=2 covariant derivative. Comparing with (8), we see that there is a residual invariance under the subgroup of dilatations, or rigid scale transformations, which are obtained from the local ones by setting $\Lambda_{,i} = 0$.

The effect of nonvanishing trace anomaly is missed if one makes a redefinition of the string variables in order to eliminate the one-loop divergent counterterm [6]. If one is careful enough to avoid this error, then one discovers that the Fradkin-Tseytlin term is needed for quantum consistency of the σ-model. The inclusion of this term in the effective action is justified in the first place by requiring the equivalence (in the light-cone gauge) of the GS and NSR formulations of the D=10 heterotic string. We have here a clear example of the close connection between conformal invariance and κ-symmetry, since the Fradkin-Tseytlin term introduced in order to cancel part of the conformal anomaly contributes to the κ-symmetry anomaly as well.

The trace anomaly vanishes for flat backgrounds as well as in the limit $\chi_\alpha = 0$. This agrees with some well-known facts, i.e. that the NSR σ-model is superconformal invariant, and also that the free Green-Schwarz formulated heterotic string is consistent at the quantum level. Furthermore, the presence of nonvanishing vacuum expectation values for the D=10 fermions can be associated to the conformal anomaly. In fact, conformally invariant propagation of the GS superstring requires an unbroken N=1 supersymmetry, according to Ref. [12].

For the NSR string the absence of superconformal anomalies defines the background constraints. In the GS formulation a similar role is played by a fermionic gauge symmetry, i.e. κ-symmetry. The requirement of absence of D=2 gauge anomalies should lead to constraints that generalize the coupled D=10, N=1 supergravity and Yang-Mills system, to include the Lorentz Chern-Simons form and the remaining string corrections. Recently the supergravity theory coupled to Yang-Mills fields free from Lorentz and gauge anomalies to the order $O(\alpha')$ has been constructed [2]. There are partial arguments in the literature [4,7] for κ-symmetry and conformal invariance to $O(\alpha')$ by cancellation of a two-loop anomaly. The one-loop conformal anomaly is not in contrast with this result. We wish to remark here that any consistent formulation must include the Lorentz Chern-Simons form to $O(\alpha')$. Therefore, we expect that the cancellation of the worldsheet gauge anomalies to the two-loop order in the GS σ-model will imply the superspace constraints of Ref. [8].

We question the validity of the argument stating that the cancellation of κ-anomaly is necessary and sufficient for the absence of all D=2 anomalies [4]. Any conformal, Lorentz, or other worldsheet gauge anomaly can be related to a κ-symmetry anomaly. According to the conventional wisdom, the absence of κ-anomaly is the only independent requirement. However, the nonvanishing one-loop conformal anomaly implies that the worldsheet anomalies cannot vanish separately. We may conjecture that the sum of the anomalies must vanish, which amounts to introducing a generalized notion of quantum consistency. In order to check this conjecture, we would need to calculate the nonlocal finite one-loop effective action.

All this indicates the inconsistency of noncovariant quantization procedures for κ-symmetry. A light-cone gauge condition has been introduced in Ref. [6], so that the κ-ghosts do not propagate, together with an apparently harmless rescaling of the fermionic normal coordinate which produces the fermion propagator in a simple form. In a covariant quantization of the GS action preserving κ-symmetry, no (potentially anomalous) rescaling of the fermionic normal coordinate will appear. The ghost fields also will contribute to the one-loop order.

The result of this section can be interpreted as a signal that noncovariant quantization of κ-symmetry causes the trace anomaly to be nonvanishing. It is possible that higher-loop β-functions will be affected. The renormalizability of the GS σ-model relies upon the preservation of the worldsheet symmetries at the quantum level. As a consequance, renormalizability itself could be spoiled by noncovariant gauge-fixing. What is wanted is a full covariant quantization of the GS string coupled to D=10 superspace. There exists the possibility that the use of harmonic variables may be helpful.

3. - LORENTZ TRANSFORMATION IN QUANTIZATION BY GHOST TRUNCATION

Let us summarize, at this point. As we have seen, for the bosonic σ-model conformal invariance corresponds to background equations of motion (at 1-loop), and to string corrections (at higher loops). For the NSR string case no fermionic backgrounds have been coupled, which corresponds to having non supersymmetric string corrections. The effective action of the GS σ-model quantized in the conformal gauge by truncating the Batalin-Vilkovisky ghosts (*à la* Kallosh) is renormalizable for on-shell backgrounds. Using the conformal gauge for the $D = 2$ metric obscures the anomaly structure. In Ref. [13] it is shown how terms that are total derivatives in the conformal gauge can be added to the effective action. By performing a loop calculation one can show that, although the Weyl anomaly vanishes, a ten-dimensional Lorentz anomaly arises, owing to arbitrary vector fields in the ghost truncation condition of Ref. [14]. However, no $D = 10$ backgrounds have been included. Also, the θ^μ dependance has not been calculated. The inclusion of fermionic backgrounds is carried out in Ref. [15], which we refer to throughout this section.

Using background-quantum splitting and normal coordinate expansion in superspace leads us to the quadratic action given in [7]

$$L^{(2)} = L_{BB} + L_{FF} + L_{BF} . \tag{19}$$

This lagrangian has Weyl and κ symmetries [1] to be gauge-fixed. We introduce ghosts only for κ-symmetry, and truncate them. We have the gauge fermion

$$\Psi = \hat{C}_\alpha (\Gamma^\ominus \Gamma^\oplus)^\alpha{}_\beta y^M E_M{}^\beta (Z) \tag{20}$$

and the truncation condition

$$(\Gamma^\ominus \Gamma^\oplus)^\beta{}_\alpha C_{+\beta} = C_{+\alpha} , \tag{21}$$

where $C_{+\alpha}$ and $\hat{C}_\alpha$ are the κ ghost and antighost. We introduce the rescaling

$$y^\alpha \to (V_-^\oplus)^{\frac{1}{2}} y^\alpha . \tag{22}$$

We define the two-metric perturbation (weak field approximation) by

$$g_{ij} \equiv \eta_{ij} + h_{ij} \quad , \quad \bar{h}_{ij} \equiv h_{ij} - \frac{1}{2}\eta_{ij} h^k_k \, , \tag{23}$$

Note that $\bar{h}^{+-}$ is of order ϵ. Using the conventions of [7], we have $(\eta^{ij} + \epsilon^{ij}) A_i B_j = A_- B_+$.

The fermion propagator reads

$$\alpha \ \underline{\qquad\quad} \ \beta \quad = \quad i\frac{p_-}{p^2} (\Gamma^{\oplus})^{\alpha\beta} \, , \tag{24}$$

The boson propagator is

$$a \ - - - - - \ b \quad = \quad -\frac{\eta^{ab}}{p^2} \, . \tag{25}$$

The only nonvanishing contribution to the divergent effective action is given by the fermion-boson sector, to linear order in h_{ij}

$$\begin{aligned} \Gamma^{\infty}_{BF} = \frac{1}{\epsilon}\frac{2}{\pi} \int d^2\sigma \, \frac{\bar{h}^{++}}{V^{\oplus}_-} [& 2 H_{abc} V^a_- (V_+ \Gamma^b \Gamma^{\oplus} \Gamma^c V_+) \\ & - V^a_- V^b_+ (V_+ \Gamma^c \Gamma^{\oplus} \Gamma_a T_{bc})] \, . \end{aligned} \tag{26}$$

This counterterm would make the σ-model non-renormalizable. If one makes a light-cone gauge choice for the background fields:

$$(\Gamma^{\oplus} V_+)_\alpha \; = \; 0 \, , \tag{27}$$

then the counterterm (26) vanishes to first order in h_{ij}, by using the two-dimensional equation of motion

$$(\gamma^{ij} + \epsilon^{ij}) V_i^{\ a} V_j^{\ \alpha} (\Gamma_a)_{\alpha\beta} = 0 \, . \tag{28}$$

Thus, we need a finer treatment of the background-quantum splitting outside of the conformal or the light-cone gauge, i.e. for general backgrounds, not restricted by the Lorentz non-covariant condition (27).

In what follows, we set the $D = 10$ background to zero. As a consequence, the σ-model is renormalizable for <u>*general*</u> V_j^α-background. The renormalized effective action

$$\Gamma_{BF}^{non-local} = \frac{4}{\pi}\int d^2\sigma \left[(\Gamma^{\oplus})_{\alpha\beta} \frac{V_+^\alpha}{(V_-^{\oplus})^{\frac{1}{2}}} \left(\frac{\partial_-^3}{\Delta}\right) \frac{V_+^\beta}{(V_-^{\oplus})^{\frac{1}{2}}} + O(h_{ij}) \right] . \tag{29}$$

does not vanish, even on the two-dimensional mass-shell. In Ref. [15] we computed the $O(h_{ij})$ terms. We are omitting those terms in (29), as they are not needed for our considerations here. We should also mention that the $O(h_{ij}^2)$ contribution vanishes, owing to explicit cancellations among various Feynman diagrams. Note that the Jacobian of the transformation (22) does not contribute to $\Gamma^{non-local}$.

Since the gravitational anomaly is non-local, clearly it cannot be cancelled by adding to the effective action a local counterterm (whose variation under $D = 2$ reparametrization is also local). Since

$$\Gamma_{BB}^{non-local} \sim R\Delta^{-1}R \sim O(h_{ij}{}^2) \ ,$$

then the total gravitational anomaly is $\Gamma^{total} = \Gamma_{BF}^{non-local}$, up to $O(h_{ij}^2)$ terms. This shows the lack of Lorentz covariance, outside the light-cone gauge for the background. The anomaly is non-local, so even renormalizability could break down at higher orders in h_{ij}.

One could interpret $\oplus, \ominus$ as harmonic coordinates [16]. In this way, one is replacing the infinite tower of κ-ghosts by a finite set with some symmetry. However, the entire argumentation of the authors of Ref. [16] is wrong. They move $(V_-^{\oplus})^{-\frac{1}{2}}$ (a factor depending on the harmonic variables) through the operator $\frac{\partial_-^3}{\Delta}$, forgetting the pretty well-known fact that integrals do not obey a Leibnitz rule. So, the one-loop counterterm will depend on the harmonic coordinates that one integrates over, giving a violation of $D = 2$ reparametrization invariance.

4. - CONCLUSIONS

We showed that, if the special backgrounds $(\Gamma^{\oplus}V_{+})_{\alpha} = 0$ are set, then there is no conformal anomaly, i.e. $T^k_k \sim g^{ij}(V_j^{\alpha} D_{\alpha}\Phi)_{;i} = 0$. Only for such Lorentz non-covariant backgrounds one has, at present, a consistently quantized sigma-model, but only *after* adding the Fradkin-Tseytlin term.

We should stress that there is a conjecture, underlying the calculation of the trace anomaly. The renormalized action is given by the sum of the regularized action and the divergent counterterm $I_{ren} = I_{reg} - I_{\infty}$, where I_{reg} can be expressed, omitting finite local contributions which are immaterial for our purposes, in terms of a finite non-local contribution

$$I_{reg} = I_{non-local} + I_{\infty} .$$

If I_{reg} is conformal invariant, then

$$\delta_{conf} I_{ren} = - \delta_{conf} I_{\infty}$$

and, in order to find the conformal anomaly, it is sufficient to compute $\delta_{conf} I_{\infty}$. The validity of this conjecture to the one-loop order has been checked for the bosonic case in Ref. [11]. There remains the question whether this is true for the supersymmetric case.

We proved that $\delta_{conf} I_{\infty} = 0$, in non-covariant gauge-fixed backgrounds. It would very interesting to compute the variation $\delta_{conf} I_{non-local}$, to see whether it vanishes or not. This calculation, in addition to the computation of $\delta_{\kappa} I_{FT}$, will determine the κ-anomaly structure. The Wess-Zumino consistency conditions will provide a powerful check on the anomalies. There remains still the open problem of finding the one-loop effective action in Lorentz covariant backgrounds.

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