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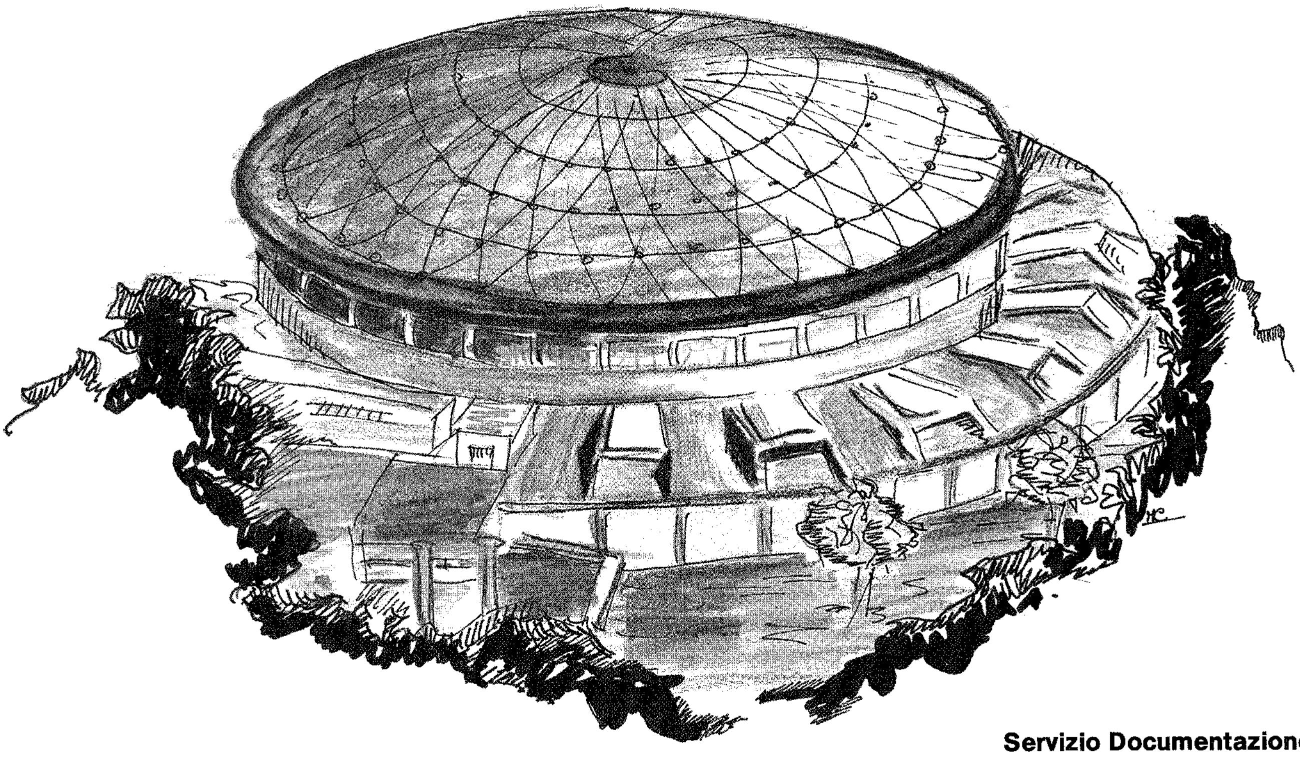

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SIGNAL AND NOISE IN A HOMOGENEOUS, HIGH Z, LIQUID CALORIMETER

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1. Charge and Current Signal from a Drifting Charge

1.1 Introduction

When ionization is produced in "noble" liquids in an electric field ~10 kV/cm, the freed electrons drift in the field at few μs/cm while the positive ions move much more slowly. The external current is given by the charge of the electrons in the gap divided by the time to drift the whole gap.[1] While the initial value of the current is a measure of the total ionization and therefore of the energy deposit in the gap, the integral of the current in the external circuit depends on how the initial charge is distributed along the gap.[2] A measurement of the integrated external current, *i.e.* the charge, increases the sampling fluctuations. Measuring the current for a short time T, just after drift begins, reduces the fluctuations. Since only a fraction of the charge is measured, the electronics noise scales as $1/(T\sqrt{T})$, instead of the usual $1/\sqrt{T}$. In the following we derive expressions for signal and noise.

1.2 Basic Relations

Consider an ion column of length l, ending at a distance d from the positive electrode, with uniform charge density and total charge Q, in a gap of width g with an appropriate field drifting the electrons with velocity v_d, as in figure 1.

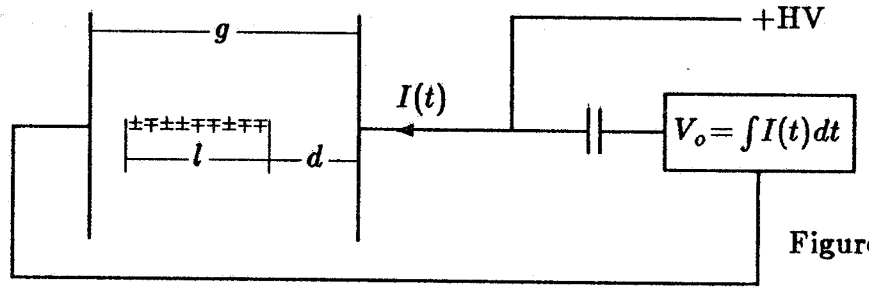

Figure 1. Gap and ion column.

The external current is given by:

$$I_{ext} = \frac{Q}{\tau} \qquad 0 < t < \frac{d}{v_d}$$
$$I_{ext} = \frac{Q}{\tau}\left(1 - \frac{tv_d}{l}\right) \qquad \frac{d}{v_d} < t < \frac{d+l}{v_d}$$

which we write as

$$I(t) = I_0 \left(\theta(t) - \theta(t-t_1)\frac{t-t_1}{\delta} + \theta(t-t_2)\frac{t-t_2}{\delta} \right), \tag{1.1}$$

and is shown in figure 2, where $\theta(x) = 1$ for $x > 0$ and $=0$ otherwise, t_1 is the time for electrons to drift a distance d,

t_2 is the time to drift $d + l$ and $I_0 = Q/\tau$ with $\tau = g/v_d$, the time to drift across the whole gap and $\delta = t_2 - t_1$.

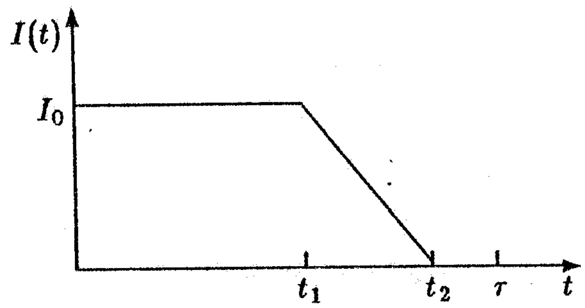

Fig. 2 Current vs time.

While the peak current I_0 is a correct measure of the ionization and therefore the energy deposited in the calorimeter, the charge from integrating the current is not. Depending where in the gap the energy is deposited the external integrated current *i.e.* the charge can vary between 0 and the freed charge Q. One must therefore measure the current, in practice first integrate and then differentiate. Below we derive the most general form of the voltage signal vs time after such manipulations, in order, ultimately, to evaluate the noise. The Laplace transform of the current is:

$$I(s) = \frac{Q}{\tau}\left(\frac{1}{s} - \frac{e^{-st_1}}{s^2\delta} + \frac{e^{-st_2}}{s^2\delta}\right) \tag{1.2}$$

where δ is the time to drift a distance l. If this current is fed to a charge sensitive amplifier, *i.e.* an ideal integrator for which $V_{out}(t) = (1/C)\int I_{in}(t)dt$ or $V_{out}(s) = (1/sC)I_{in}(s)$, where

C is the feedback capacitance, we obtain a voltage signal given by:

$$V(s) = \frac{Q}{s\tau C}\Big(\frac{1}{s} - \frac{e^{-st_1}}{s^2\delta} + \frac{e^{-st_2}}{s^2\delta}\Big). \tag{1.3}$$

To control "parallel" and especially "serias" noise a filter rejecting both low and high frequencies is necessary. Only one differentiation is necessary to remove parallel noise, but a second differentiation is necessary to measure the initial current rather than the integrated charge, which would result in an incorrect measurement of the electromagnetic shower energy. This second differentiation is best done by double sampling (also called *correlated sampling*) and taking the difference of the samples. A typical filter consist of one differentiation and two integrations all with the same cutoff frequency a, $a = 1/rc$. The frequency response, $g(\omega)$, and the transfer function, $g(s)$, of the filter are:[1]

$$|g(\omega)|^2 = |V_{out}(\omega)/V_{in}(\omega)|^2 = \omega^2 a^4/((\omega^2 + a^2)^3), \qquad a = 1/rc \tag{1.4}$$

$$g(s) = V_{out}(s)/V_{in}(s) = sa^2/(s+a)^3 \tag{1.5}$$

and the output signal, for a unit input step, is:

$$V(t) = \frac{(at)^2}{2} e^{-at}, \tag{1.6}$$

If the charge in the gap were collected in a time short with respect to $1/a$, the signal at the output of the filter would be given by equation (1.6). $V(t)$ peaks at $t = 2/a$, with a value $2\exp(-2) = 0.27$ (times the magnitude of input step). For an input current given by equation (1.1), from equations (1.3) and (1.5) we obtain the signal at the filter output as:

$$V_{out}(s) = \frac{Qa^2}{\tau C}\frac{1}{(s+a)^3}\Big(\frac{1}{s} - \frac{e^{-st_1}}{s^2\delta} + \frac{e^{-st_2}}{s^2\delta}\Big)$$

and

$$V_{out}(t) = \frac{Qa^2}{\tau C}\left(f(t) - \frac{g(t-t_1)\theta(t-t_1)}{\delta} + \frac{g(t-t_2)\theta(t-t_2)}{\delta}\right) \tag{1.7}$$

with

$$\begin{aligned} f(t) &= \frac{1}{a^3} - \left(\frac{t^2}{2a} + \frac{t}{a^2} + \frac{1}{a^3}\right)e^{-at} \\ g(t) &= \frac{t}{a^3} - \frac{3}{a^4} + \left(\frac{t^2}{2a^2} + \frac{2t}{a^3} + \frac{3}{a^4}\right)e^{-at} \end{aligned} \tag{1.8}$$

2. Noise

For the filter described above and a charge sensitive preamp with the 2SJ72–2SK147 FET's, the input equivalent noise charge is:[1]

$$Q_{noise} = 2650 \times e \times \frac{C_D}{1\ \mathrm{nF}} \times \sqrt{\frac{1\mu\mathrm{s}}{2/a}}. \tag{2.1}$$

In krypton one electron-ion pair is produced for an energy loss of 24 eV. Thus for a 1 MeV energy loss, $Q_{1-\mathrm{MeV}} = 41700\ e$, where e is the electron charge. If Q is uniformly distributed across the gap, along a line normal to negative and positive electrodes, the measured charge is $Q/2$. If the drift time can be neglected, this is the charge to be compared with the noise in equation (2.1). Thus for $a = 2 \times 10^6\ \mathrm{s}^{-1}$ and C_D=1 nF, the energy noise is (1 MeV)×(2650)/(41700/2)=0.13 MeV. If we were to measure the initial current rather than the charge, we can make a guess about noise as in the following:

1. The signal sampling time should be about one tenth the total drift time
2. The total drift time should be as large as possible, to keep the sampling time long, to keep noise down
3. The filter cutoff frequency a should be chosen to give short shaping times, say equal to the sampling time, to avoid large slopes in the signal.

There is of course a practical constraint, the actual high voltage applied across the calorimeter gap. Few people like to go as high as 10 kV and those who don't are wise. Assuming 10 kV and saturated drift velocity ($E = 1$ kV/mm) gives τ=3 μs, in a gap of 1 cm. Choosing therefore $2/a = 0.1\tau = 0.3\ \mu$s, increases the input equivalent charge noise by $1/\sqrt{.3} \sim 1.8$. In addition since we are integrating the signal for ~1/10 of the time we measure about 1/10 of the charge increasing the noise by another factor 10/2=5. The net result is that the energy noise becomes 0.13×1.8×5=1.17 MeV. This in fact is not so bad, for 1 nF detector capacitance!

Before continuing, one should mention one way to increase the total drift time: operate at non saturated drift velocity. This is however even less wise than going to higher voltage. At low fields electron attachment becomes rapidly a very severe problem. And at contamination levels of 0.01 ppm and non saturated charge collection, it is not even conceivable to attempt to measure the charge loss, since it is going to be different everywhere, in a large calorimeter.

Assuming a 10 mm gap, a preamp with a feedback capacitor C, a filter with two integrations and $a = 3.333 \times 10^6\ \mathrm{s}^{-1}$, we find the signal at 0.6 μs for unit Q/C freed along a line 0.25 mm long, ending at d=0.5, 1.5, ... 9.5 mm from the positive electrode. (If the charge is freed at the positive electrode, the initial current is the same, but since the current lasts for zero time no signal is observed.)

Table I. Signal for 0.6 μs shaping and 3 μs drift times.

d mm	$V(t = 0.6\ \mu s)$ V	$V(t = 0.6\ \mu s)$ normalized
0.5	0.01625	0.503
1.5	0.03161	0.978
2.5	0.03233	1.0
⋮	⋮	⋮
9.5	0.03233	1.0

Note that with the choice of parameters, only 10% of the gap has a severe signal loss. For uniform charge deposition, the normalized response is 0.941. More important is however the fluctuation in the response. For an input (energy) of 1, deposited uniformly and randomly, at a point in the center of a 1 mm layer of liquid, the normalized output is 0.948, with an rms spread of 0.148. This result is not very accurate but not terribly wrong either. A better calculation requires merging EGS with the formulae above.

Finally we can come back to noise. The noise increase, over the value 0.13 MeV above, is $(1/.948)(0.157/0.0323)\sqrt{1/0.6}$=6.61, giving an rms noise of 0.86 MeV. In practical applications it is necessary to measure the base line before the signal of interest appears, the signal, at the appropriate $t = 0.6\ \mu s$ and take the difference of the two measurements. Because of the necessary choice of the filter peaking time for an input step, the two samples are almost completely uncorrelated. The difference therefore has an rms noise of

$\sqrt{2} \times 0.86 = 1.2$ MeV. Scaling of the noise is as $a\sqrt{a}$, because preamp noise grows as $\sqrt{a}$ and the filter output signal decreases as $1/a$, all for constant gap width and drift velocity.

3. A Projective EM Calorimeter

3.1 TOWER PARAMETERS

I have derived the noise for a relative large capacitance, because I wanted to find whether a standard, many gaps, projective structure could deliver reasonable resolution without too much degradation from noise. The read–out planes could be sandwiches of etched aluminized plastic and foam. To properly shield signal lines from pads, four foam layers are necessary and five plastic layers. Two outer ones with charge collecting pads, two ground planes and a center layer with signal traces. For 20 radiation lengths, λ_0, thick calorimeter the sandwiches add up to about $0.05\lambda_0$ of inert material. We assume a structure with sandwiches carrying pads on both sides, alternating with high voltage electrodes, thus each pad views too gaps. For 0.5 mm foam thickness the capacitance to ground of a 3×3 cm^2 double pad is 31.7 pF. Adding a trace, 0.5 mm wide and 50 cm long contributes 8.8 pF, for a total of 40.5 pf per pad. From 1000/40=25, we get that 1 nF corresponds to 2×25 gaps of liquid. For 1 cm gaps and Kr, this corresponds to 10.6 radiation lengths. For projective tower the area increases, approximately doubling. Thus, 3×3 cm^2 at the entrance becomes 4.5×4.5 cm^{2}in average. Four sections with 12×2 cm double gaps each, remain below 1 nF and correspond to 20.4 radiation lengths of krypton. Noise values per tower are $\sqrt{4} = 2$ times larger than for the 1 nF example above. Thus the total noise due to the preamp is 2.4 MeV per tower.

3.2 A CRUDE ESTIMATE OF THE SIGNAL FLUCTUATIONS.

While, as I have mentioned, a proper estimate requires marrying EGS and equation (1.7), I will estimate the fluctuations using the last column in table 1, assuming that energy (charge) is deposited in little 1 MeV clumps, at random, in 1 mm liquid layers. Then, for a 100 MeV shower, I find that one measures in average 94.8 MeV with an rms spread of 1.48 MeV or 1.6%. The average is of course just the sum of the weights in table 1 and is also the measured signal for a uniform column of ionization from one electrode to the other. The estimate of the fluctuation is, I believe, an over-estimate, because at 100 MeV there are a few electrons crossing longer portions of the gap. If the energy clumps in the shower are smaller than I have chosen, the fluctuations are again smaller. Typically 1/2 of a shower energy is in electrons and photons of less than one MeV. Finally the fractional fluctuation scales as $1/\sqrt{E}$, therefore at 20 MeV, the fluctuation is still only 3.3% or 0.7 MeV, quite negligible with respect to noise.

3.3 RADIOACTIVE NOISE

I use the russian[3] quoted value of 300 decays/s/cc. In terms of the signal V(t) produced by a randomly occurring, fixed energy deposit with average frequency f, the mean square fluctuation F^2 of the signal is given by ($\langle V(t)\rangle \equiv 0$):[1]

$$F^2 = \langle (V(t))^2 \rangle = \lim_{T\to\infty} \frac{fT\int_0^T (V(t))^2\,dt}{T} = f\int_0^\infty (V(t))^2\,dt. \tag{3.1}$$

In the equation above one must use $V(t) = V_{out}(t) - V_{out}(t - T_s)$, where T_s is the sampling interval and $V_{out}(t)$ is given in equations (1.7) and (1.8). It's quite straightforward to do a better calculation, by I just assume that all decays happen in the middle of the gap and deposit .33 MeV, one half of the end point of ^{85}Kr β decay spectrum. The thing one must pay attention is the energy to voltage relation. I write:

$$\begin{aligned} \text{Signal from shower} =& K\frac{Q}{C} = K\frac{E\mathcal{R}}{C} \\ \text{Signal noise from radioactivity} =& F\frac{Q}{C} = F\frac{E\mathcal{R}}{C} \end{aligned} \tag{3.2}$$

where F is given in equation (3.1) above, K is the average response to an em shower and $\mathcal{R}$ is the responsivity of the calorimeter, in Coulomb/MeV. E and Q are the energy deposit in the liquid and the charge producing the signal. In this way I find that the total noise from radioactivity for the russian test calorimeter should be 2.6 MeV. They give 0.6 MeV per section (?) and therefore 1.8 to 2.5 for the total, depending how they count sections. Pretty good, since I do not know what filter they use. In table 2, I give values of various parameters, for different shaping times, $2/a$.

Table 2. Signal and noise contribution for a tower, at 100 MeV.

$2/a$ μs	rms sampling fluctuation %	Electronics Noise MeV	Radioactivity Noise MeV	Max signal in units of Q/C	Average sig. in units of Q/C
0.3	0.33	6.9	0.19	0.0162	0.0160
0.6	1.5	2.4	0.26	0.0323	0.0306
0.75	1.88	1.7	0.29	0.0404	0.0376
0.9	2.2	1.3	0.29	0.0485	0.0443
$\gg$ drift time	6	< 0.26	> 0.5	0.270	0.135

4. Conclusions

I have shown that only a moderate amount of fast shaping is necessary, in order to avoid fluctuation in the signal from a homogeneous calorimeter, due to differences in drift paths, thus alleviating problems of cross talk in signal traces. The use of drift fields as low as 100 volts per mm, with the correspondingly severe demand on liquid purity can also be avoided. I have also shown that structures with good shielding do not imply large capacitances and that capacitances in the nF range result in quite acceptable noise levels, better than crystal with silicon diode readouts. I conclude that it is possible to construct a conventional geometry calorimeter with projective tower and adequate resolution and noise, even adding the capacitance of the cables carrying the signals to a feedthrough port. Limitations to the performance of a homogeneous calorimeter using a cryogenic liquid are most likely to come from the wall thickness of the cryostat.

REFERENCES

1. Paolo Franzini, Noise and Pile-up in Liquid Sampling Calorimeters, in Proc. of the 1987 Berkeley Workshop on Experiments, etc. for the Supercollider, R. Donaldson and M. G. D. Gilchriese eds., World Scientific, Singapore, 1988, p. 636.
2. This was well known since the days of ionization chambers. For a cure (impractical in this case) see O. Frisch, British Atomic Energy Report BT–49 (1944), unpublished.
3. G. Godfrey. Private Communication .