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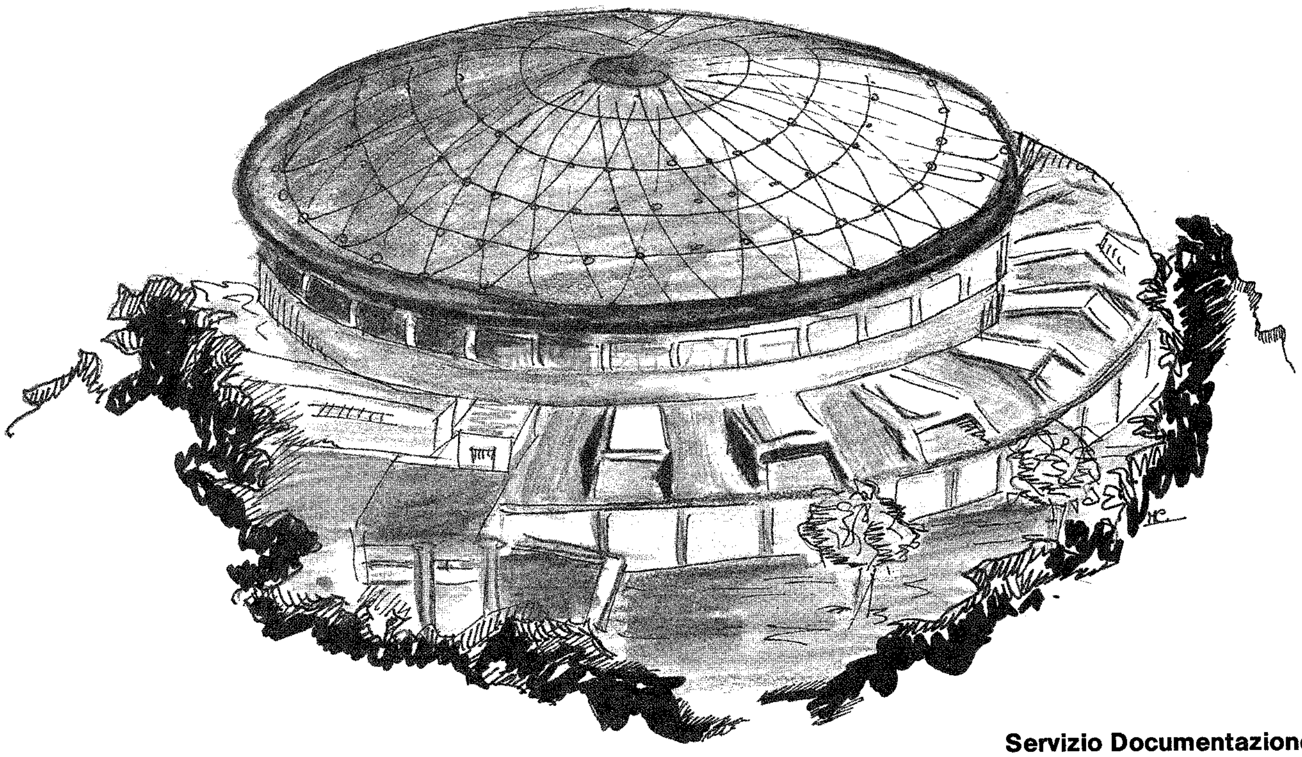

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TWO-LOOP QED CORRECTIONS TO BHABHA SCATTERING NEAR THE Z_0

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ABSTRACT

Leading logarithmic $o(\alpha^2)$ corrections to Bhabha scattering near the Z_0 are presented. These results improve previous analytical formulae for back-to-back $e^+ e^-$ pair production to an accuracy of few ‰ and can be therefore used for precision tests at LEP experiments.

First generation experiments at LEP/SLC have shown [1] so far good agreement with the theoretical expectations of the electroweak standard model. The absolute control of QED radiative corrections to a level of $\lesssim 1\%$ has been decisive to this respect. This result has been achieved by combining soft photon exponentiation [2] with exact calculation up to $o(\alpha^2)$ [3] . In particular analytic [4,5] or semianalytic [6] formulae have been proven to be very useful to extract the electroweak parameters from the data.

The description of QED radiative effects for Bhabha scattering has been limited so far to the one-loop calculation [7,8] , implemented by the resummation to all orders of soft and collinear hard photon effects [7]. The corresponding analytical formulae apply to a kinematical configuration where the electron - positron pair _ eventually detected together with hard collinear photons confined within a small cone δ _ are produced approximately back-to-back. High statistics studies of this channel from actual LEP experiments, which shall obviously add further information in the leptonic sector of the Z decays, clearly require a treatment of QED effects to an accuracy better than 1%, comparable to what has been achieved in the channel $e^+ e^- \to f^+ f^-$ ($f = \mu, \tau, ..$).

To pursue this aim we present in this letter the $o(\alpha^2)$ leading results which improve the previous treatment [7] in two respects. First, following ref.[5], the factorisation properties of the radiative corrections _ the "infrared" factors to all orders and the "finite" ones to $o(\alpha)$ and $o(\alpha^2)$ _ are consistent to the structure observed to two - loop level. This fact has phenomenological implications relevant for the Z_0 line shape in the channel $e^+ e^-$ of the order of few %. Furthermore the leading logarithmic "finite" factors to $o(\alpha^2)$ are explicitly computed for the s and t channels, from the corresponding form factors and bremsstrahlung contributions. The radiative corrected differential cross - section _ with partial inclusion of hard photon effects _ can be then written in the formalism of structure functions [9], after the introduction [10] of K-factors to implement the Born approximation.

We start with the results of ref.[7], where, to one-loop and including infrared factors to all orders, one writes the differential cross section in the form :

$$d\sigma\,(e^+ e^- \to e^+ e^-) = \sum_{i=1}^{10} d\sigma_0\,(i)\,[C^{(i)}_{infra} + C^{(i)}_{F}]\,. \qquad (1)$$

The Born cross sections $d\sigma_0$ (i) refer to all possible γ and Z exchanges in the s and t channels. The infrared factors $C^{(i)}_{infra}$ are obtained after adding virtual and real corrections and fall into three categories corresponding to pure QED, resonant, and QED - resonant interference terms. Then the "finite" $C^{(i)}_{F}$ factors correspond to the left over terms obtained after the exact one - loop calculation. Finally eq.(1) refers to the kinematical configuration of a back - to - back electron positron pair, detected within an acollinearity angle J, which in turn defines the energy resolution $\varepsilon \equiv \frac{\Delta\omega}{E}$ of the experiment, with $E = \sqrt{\frac{s}{2}}$ as the beam energy (ε<<1) .

Hard photon effects are not taken into account in (1). We shall return to this point later.

Following the results of ref.[5], which are based on explicit $o(\alpha^2)$ corrections [3] applied to the reaction $e^+ e^- \to \mu^+ \mu^-$, eq.(1) can be first put in the form :

$$d\sigma\,(e^+ e^- \to e^+ e^-) \equiv \sum_{i=1}^{10} d\sigma(i) = \sum_{i=1}^{10} d\sigma_0\,(i)\,[C^{(i)}_{infra}\,(1+\bar{C}^{(i)}_{F}) + C'^{(i)}_{F}] \quad , (2)$$

where the finite factors $C^{(i)}_{F}$ of eq.(1) are splitted into two parts in eq.(2), according to the factorisation properties explicitly verified to the two loop level. Eqs. (1) and (2) are obviously mutually consistent to $o(\alpha)$, but differ phenomenologically by a few % on the Z resonance. Furthermore the infrared factors are given by the form [2,5,7]

$$C^{(i)}_{infra} = \varepsilon^{2\,\beta_e + 2\,\beta_{int}}\,, \qquad (i= 1,..,6)$$

$$C^{(i)}_{infra} = \frac{\varepsilon^{\beta_e + \beta_{int}}}{\cos \delta_R} \mathrm{Re} \left\{ e^{i \delta_R} \left[\frac{\varepsilon}{1 + \left(\frac{\varepsilon s}{M\Gamma}\right) \sin \delta_R \, e^{i \delta_R}} \right]^{\beta_e} \right.$$

$$\left. \left[\frac{\varepsilon}{\varepsilon + \left(\frac{M\Gamma}{s}\right) \frac{e^{-i \delta_R}}{\sin \delta_R}} \right]^{\beta_{int}} \right\} , \qquad (i= 7,8,9) \qquad (3)$$

$$C^{(10)}_{infra} = \varepsilon^{\beta_e} \left| \frac{\varepsilon}{1 + \left(\frac{\varepsilon s}{M\Gamma}\right) \sin \delta_R \, e^{i \delta_R}} \right|^{\beta_e}$$

$$\left| \frac{\varepsilon}{\varepsilon + \left(\frac{M\Gamma}{s}\right) \frac{e^{-i \delta_R}}{\sin \delta_R}} \right|^{2\beta_{int}} (\cos \beta_e \Phi - \mathrm{ctg}\, \delta_R \sin \beta_e \Phi) ,$$

where, as usual, $\beta_e = \frac{2\alpha}{\pi}(L-1)$, $L= \ln \frac{s}{m_e^2}$, $\beta_{int} = \frac{4\alpha}{\pi} \ln \mathrm{tg} \frac{\theta}{2}$, and the other notations are the same as in ref.[5,7]. The Z width Γ includes the usual dependence $\Gamma = \Gamma(s) = \Gamma(M^2) \frac{s}{M^2}$. The last of eqs. (3) gives a correction of order of $\beta_e{}^2 \sim 0.01$ to the previous [2] resonance tail factor $1+\beta_e \frac{s-M^2}{M\Gamma} \Phi$.

As a next step we shall study to leading $o(\alpha^2)$ the expression of the finite terms in eq.(2). To this aim the strategy to follow is twofold. First, from the explicit expression of the form factors in the s and t channels, up to two loops [11] , and including also the double bremsstrahlung contribution, we shall extend _ to the leading logarithmic accuracy _ the $o(\alpha)$ expression of the factors $C^{(i)}_F$ in ref.[7]. Then, following ref.[5], we also have to add the other $o(\beta^2)$ and $o(\beta\varepsilon)$ terms, corresponding essentially to the hard terms of the initial and final electron radiators in the formalism of the structure functions, taking into account initial and final interference effects, as well. This second task can be achieved by virtue of the explicit correspondence [5] between eq.(2) and the analogous one in the formalism of the structure functions.

In order to discuss the first point, let us consider the expressions of the coefficients $C^{(i)}_F$ in eq.(1). One obtains ($a \equiv \sin \frac{\theta}{2}$, $b \equiv \cos \frac{\theta}{2}$) :

$$C_F^{(i)} = \frac{3}{2}\beta_e + \frac{2\alpha}{\pi}\left[\left(\frac{\pi^2}{3} - \frac{1}{2}\right) + f\right] + \ldots, \qquad (i=1,7,10)$$

$$C_F^{(i)} = \frac{3}{2}\beta_e + \frac{2\alpha}{\pi}\left[\left(\frac{\pi^2}{12} - \frac{1}{2}\right) + \left(\frac{3}{2}\ln a - \ln^2 a\right) + f\right] + \ldots, \qquad (i=2,4,8,9) \qquad (4)$$

$$C_F^{(i)} = \frac{3}{2}\beta_e + \frac{2\alpha}{\pi}\left[-\left(\frac{\pi^2}{6} + \frac{1}{2}\right) + 2\left(\frac{3}{2}\ln a - \ln^2 a\right) + f\right] + \ldots, \qquad (i=3,5,6)$$

where $f \equiv F(a,b)$ defined in ref.[7], and the dots indicate the left over terms also given in [7].

The above results have been obtained from the form factors $F(s) \equiv \delta_v(s)$, $F(t) \equiv \delta_v(t)$ and the bremsstrahlung contribution B to one loop as follows :

$$d\sigma^{(i)} = d\sigma^{(i)}_0 \;|\tilde{F}(s)|^2 \; B^2 \qquad (i=1,7,10)$$

$$d\sigma^{(i)} = d\sigma^{(i)}_0 \;|\tilde{F}(t)|^2 \; B^2 \qquad (i=3,5,6) \qquad (5)$$

$$d\sigma^{(i)} = d\sigma^{(i)}_0 \;\left[\mathrm{Re}\,\tilde{F}(s)\right]\tilde{F}(t)\; B^2, \qquad (i=2,4,8,9)$$

where $\tilde{F}(s) = F^2(s)$, $\tilde{F}(t) = F^2(t)$, and the explicit form of B shall be given below. Then eqs.(5) can be simply generalised to two loops, by taking

$$F(s) = 1 + \frac{\alpha}{\pi}\left[\mathrm{Re}\,F^{(1)}(s) + i\,\mathrm{Im}\,F^{(1)}(s)\right] + \left(\frac{\alpha}{\pi}\right)^2\left[\mathrm{Re}\,F^{(2)}(s) + i\,\mathrm{Im}\,F^{(2)}(s)\right], \quad (6)$$

$$F(t) = 1 + \frac{\alpha}{\pi}F^{(1)}(t) + \left(\frac{\alpha}{\pi}\right)^2 F^{(2)}(t), \qquad (7)$$

$$B = 1 + \frac{\alpha}{\pi}B^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 B^{(2)}. \qquad (8)$$

Then eqs.(4) are simply generalised to $o(\alpha^2)$ using the expression of the form factor of ref.[11]* and the corresponding bremsstrahlung terms to the leading approximation , which we report below for the reader's convenience :

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* In the following lepton loops are not included.

$$\mathrm{Re}\,F^{(1)}(s) = -\frac{1}{4}L^2 + \frac{3}{4}L + \frac{\pi^2}{3} - 1 \tag{9}$$

$$\mathrm{Im}\,F^{(1)}(s) = \pi\left(\frac{L}{2} - \frac{3}{4}\right), \tag{10}$$

$$\mathrm{Re}\,F^{(2)}(s) = \frac{1}{32}L^4 - \frac{3}{16}L^3 + \left(-\frac{5}{4}\frac{\pi^2}{6} + \frac{17}{32}\right)L^2 + \left(\frac{3}{2}\zeta(3) + \frac{\pi^2}{2} - \frac{21}{32}\right)L + \frac{2}{5}\frac{\pi^4}{36} - \frac{9}{4}\zeta(3) - \frac{\pi^2}{2}\ln(2) - \frac{7}{24}\frac{\pi^2}{6} + \frac{45}{24}, \tag{11}$$

$$\mathrm{Im}\,F^{(2)}(s) = \pi\left[-\frac{L^3}{8} + \frac{9}{16}L^2 + \left(\frac{\pi^2}{6} - \frac{17}{16}\right)L - \frac{\pi^2}{8} - \frac{3}{2}\zeta(3) + \frac{21}{32}\right], \tag{12}$$

$$F^{(1)}(t) = -\frac{1}{4}L_t^2 + \frac{3}{4}L_t + \frac{\pi^2}{12} - 1, \tag{13}$$

$$F^{(2)}(t) = \frac{1}{32}L_t^4 - \frac{3}{16}L_t^3 + \left(-\frac{\pi^2}{48} + \frac{17}{32}\right)L_t^2 + \left(\frac{3}{2}\zeta(3) - \frac{\pi^2}{16} - \frac{21}{32}\right)L_t - \frac{59}{40}\frac{\pi^4}{36} - \frac{9}{4}\zeta(3) - \frac{\pi^2}{2}\ln(2) + \frac{139}{48}\frac{\pi^2}{6} + \frac{45}{24}, \tag{14}$$

and

$$B^{(1)} = \frac{1}{2}L^2 - \frac{\pi^2}{3} + f, \tag{15}$$

$$B^{(2)} = \frac{1}{2}\left\{\left[B^{(1)}\right]^2 - \frac{2}{3}\pi^2(L-1)^2\right\} \tag{16}$$

where $L_t \equiv \ln\left(\frac{-t}{m^2}\right) = L + \ln(a^2)$ and ζ is the Riemann function ($\zeta(3) \simeq 1.202$). Notice that we have put the infrared cutoff λ=m in the above equations and also that the usual infrared logarithms ~ln ε do not appear in eqs.(15-16), being already included in the infrared factors $C^{(i)}_{\text{infra}}$ (eqs. (3)) . Furthermore eq.(16) is only expected to give the correct L^2 behaviour.

After substituting in eq.(5) the above expressions and some lengthy algebraic manipulations we finally obtain the vertex and bremsstrahlung contributions $C^{(i)}_1$ to $C^{(i)}_F$ as follows :

$$C_1^{(i)} = \frac{3}{2}\beta_e + \frac{2\alpha}{\pi}\left[\left(\frac{\pi^2}{3} - \frac{1}{2}\right) + f\right] + \left(\frac{\alpha}{\pi}\right)^2 \left\{ \left(\frac{9}{2} - \frac{2}{3}\pi^2 - f\right) L^2 \right.$$
$$+ \left[6\zeta(3) + \frac{17}{6}\pi^2 - \frac{93}{8} + 6f \right] L$$
$$\left. + \left[-9\zeta(3) - 2\pi^2 \ln 2 + \frac{2\pi^4}{45} - \frac{77}{72}\pi^2 + \frac{27}{2} + f^2 + 2\pi^2 f - 8f \right] \right\} ,$$

(i=1,7,10)

$$C_1^{(i)} = \frac{3}{2}\beta_e + \frac{2\alpha}{\pi}\left[-\left(\frac{\pi^2}{6} + \frac{1}{2}\right) + 2\left(\frac{3}{2}\ell - \ell^2\right) + f \right]$$
$$+ \left(\frac{\alpha}{\pi}\right)^2 \left\{ \left[8\ell^2 - 12\ell - \frac{2}{3}\pi^2 + \frac{9}{2} - f \right] L^2 \right.$$
$$+ \left[16\ell^3 - 36\ell^2 + \frac{4}{3}\pi^2\ell + 34\ell + 6\zeta(3) - \frac{\pi^2}{6} - \frac{93}{8} - 8\ell f + 6f \right] L$$
$$+ \left[8\ell^4 - 24\ell^3 + 34\ell^2 + \frac{4}{3}\pi^2\ell^2 + 12\zeta(3)\ell - 3\pi^2\ell - \frac{93}{4}\ell - 9\zeta(3) - 2\pi^2 \ln 2 \right.$$
$$\left.\left. - \frac{11}{90}\pi^4 + \frac{211}{72}\pi^2 + \frac{27}{2} + f^2 - 8\ell^2 f + 12\ell f - 8f \right] \right\}, \qquad (i=3,5,6) \qquad (17)$$

$$C_1^{(i)} = \frac{3}{2}\beta_e + \frac{2\alpha}{\pi}\left[\left(\frac{\pi^2}{12} - \frac{1}{2}\right) + \left(\frac{3}{2}\ell - \ell^2\right) + f\right]$$
$$\left(\frac{\alpha}{\pi}\right)^2 \left\{ \left[2\ell^2 - 6\ell - 7\frac{\pi^2}{6} + \frac{9}{2} - f \right] L^2 \right.$$
$$+ \left[4\ell^3 - 12\ell^2 + 17\ell - \frac{\pi^2}{3}\ell + 6\zeta(3) + \frac{17}{6}\pi^2 - \frac{93}{8} - 4\ell f + 6f \right] L$$
$$+ \left[2\ell^4 - 6\ell^3 - \frac{\pi^2}{3}\ell^2 + \frac{25}{2}\ell^2 + 6\zeta(3)\ell - \frac{93}{8}\ell - 9\zeta(3) - 2\pi^2 \ln 2 \right.$$
$$\left.\left. - \frac{59}{360}\pi^4 - \frac{7}{36}\pi^2 + \frac{27}{2} + f^2 - 4\ell^2 f + 6\ell f - 8f + \pi^2 f \right] \right\} , \qquad (i=2,4,8,9)$$

where $\ell = \ln a$.

The above equations extend, to the two–loop leading logarithmic accuracy, the results of ref.[7]. The splitting of the factors $C_F^{(i)}$ into $\bar{C}_F^{(i)}$ and $C'^{(i)}_F$ in eq.(2) will be discussed later, after considering further terms of $o(\beta^2)$ and $o(\beta\varepsilon)$, which can be obtained by the method of the structure functions [7]. Indeed it has been shown [5] in this framework, for the reaction $e^+ e^- \to \mu^+ \mu^-$, that a representation of the form

$$d\sigma^{(i)} = \int_0^{\varepsilon} dx \, d\sigma^{(i)}{}_0 (s(1-x)) \left[\Delta_e (s) \, \Delta_\mu (s) \, \beta_e \, x^{\beta_e - 1} (\varepsilon - x)^{\beta_\mu} + R(x,..)\right] \quad (i{=}\mathrm{QED,RES,INT}) \tag{18}$$

has an analytic solution of the type (2), with the infrared factors indicated in (3). In the above equation the first term in the r.h.s. takes into account the effect of the soft radiation to all orders, while R(x,..) include the hard one up to $o(\alpha^2)$ and gives further correction terms of $o(\beta^2)$ and $o(\beta\varepsilon)$ in the final cross section.

At the light of this result, eq(2), together with (3) and (15), can be rewritten in a form similar to (18), taking into account two observations. First, the s , t channel structure of the various cross sections $d\sigma^{(i)}$ in eq.(2) does not imply the same s – channel initial and final states radiators for all $d\sigma^{(i)}$, as in the case $e^+ e^- \to \mu^+ \mu^-$. This can be maintained however, as in the case of $o(\alpha^3{}_S)$ quark - antiquark scattering in QCD [12], where the same structure functions are used, of course, for all s, t sub - diagrams, provided appropriate K - factors are introduced for three classes of $d\sigma^{(i)}$ corresponding to eqs.(17). These K - factors essentially take into account the different scale involved and can be easily obtained by subtracting the first from the second and the third of eqs. (17). For the sake of brevity we shall not report explicitly this result here. We also observe that the structure function representation of the differential cross section, as in eq.(18) or in refs.[5,10], has to be limited to the very specific kinematical region considered. The generic case, which includes hard photon production, involves a much more complex analysis, as done for example in ref.[13] for $e^+ e^- \to \mu^+ \mu^-$, which is out of the purpose of this paper.

Having settled the general framework, we are able now to calculate some further corrections, induced by the hard photon term R(x,..) in eq.(18), as obtained in ref.[5]. We then get for the QED, interference and resonant contributions to the coefficient $C^{(i)}_F$, respectively :

$$C_F^{(i)} \to -\beta_e\varepsilon - \frac{\pi^2}{6}\beta_e^{\,2} - \frac{1}{4}\beta_e^{\,2}\ \varepsilon^{1-\beta_e}, \qquad (i=1,..,6) \tag{19}$$

$$C_F^{(i)} \to -\beta_e\varepsilon - \frac{\pi^2}{6}\beta_e^{\,2} - \beta_e \frac{\cos[(1+\beta_e)\Phi] + \mathrm{tg}\,\delta_R \sin[(1+\beta_e)\Phi]}{\cos(\beta_e\Phi) + \mathrm{tg}\,\delta_R \sin(\beta_e\Phi)} \left|\frac{\varepsilon}{1+\varepsilon z}\right|$$

$$-\frac{1}{4}\beta_e^{\,2} \frac{\cos\Phi + \mathrm{tg}\,\delta_R \sin\Phi}{\cos(\beta_e\Phi) + \mathrm{tg}\,\delta_R \sin(\beta_e\Phi)} \left|\frac{\varepsilon}{1+\varepsilon z}\right|^{1-\beta_e}, \qquad (i=7,8,9) \tag{20}$$

$$C_F^{(10)} \to -\beta_e\varepsilon - \frac{\pi^2}{6}\beta_e^{\,2} - \beta_e \frac{\cos[(1+\beta_e)\Phi] - \mathrm{ctg}\,\delta_R \sin[(1+\beta_e)\Phi]}{\cos(\beta_e\Phi) - \mathrm{ctg}\,\delta_R \sin(\beta_e\Phi)} \left|\frac{\varepsilon}{1+\varepsilon z}\right|$$

$$-\frac{1}{4}\beta_e^{\,2} \frac{\cos\Phi - \mathrm{ctg}\,\delta_R \sin\Phi}{\cos(\beta_e\Phi) - \mathrm{ctg}\,\delta_R \sin(\beta_e\Phi)} \left|\frac{\varepsilon}{1+\varepsilon z}\right|^{1-\beta_e}, \tag{21}$$

with the same notation of [5], in particular with $M^2_R = M^2 - i\,\Gamma\,M$, and $z = \frac{s}{M^2_R - s}$.

Notice that the coefficient of the third term in eq.(21) is only a half of the one of ref. [5], because of the s – dependence of the Z – width, introduced here.

So far the β_{int} – dependence has been taken into account in the infrared factors (3) only, which do contain the main effect. Some next – to – leading terms can also be calculated, as done in ref.[10] for the reaction $e^+ e^- \to \mu^+ \mu^-$. Then by collecting all the effects discussed so far, we are finally lead to the following results for the coefficients $\bar{C}_F^{(i)}$ appearing in eq.(2):

$$\bar{C}_F^{(i)} = C_I^{(i)} - \beta_e\varepsilon - \frac{\pi^2}{6}\bar{\beta}_e^{\,2} - \frac{1}{4}\beta_e\bar{\beta}_e\ \varepsilon^{1-\bar{\beta}_e} \qquad (i=1,..,6) \tag{22}$$

$$\bar{C}_F^{(i)} = C_I^{(i)} - \beta_e\varepsilon - \frac{\pi^2}{6}\bar{\beta}_e^{\,2} - \bar{\beta}_e \frac{\cos[(1+\beta_e)\Phi] + \mathrm{tg}\,\delta_R \sin[(1+\beta_e)\Phi]}{\cos(\beta_e\Phi) + \mathrm{tg}\,\delta_R \sin(\beta_e\Phi)} \left|\frac{\varepsilon}{1+\varepsilon z}\right|$$

$$-\frac{1}{4}\beta_e\bar{\beta}_e \frac{\cos\Phi + \mathrm{tg}\,\delta_R \sin\Phi}{\cos(\beta_e\Phi) + \mathrm{tg}\,\delta_R \sin(\beta_e\Phi)} \left|\frac{\varepsilon}{1+\varepsilon z}\right|^{1-\beta_e} \qquad (i=7,8,9) \tag{23}$$

$$\bar{C}^{(10)}_{F} = C^{(10)}_{1} - \beta_e\varepsilon - \frac{\pi^2}{6}\bar{\beta}_e\bar{\beta}_e - \bar{\bar{\beta}}_e \frac{\cos[(1+\beta_e)\Phi] - \mathrm{ctg}\,\delta_R \sin[(1+\beta_e)\Phi]}{\cos(\beta_e\Phi) - \mathrm{ctg}\,\delta_R \sin(\beta_e\Phi)} \left|\frac{\varepsilon}{1+\varepsilon z}\right|$$

$$-\frac{1}{4}\beta_e\bar{\bar{\beta}}_e \frac{\cos\Phi - \mathrm{ctg}\,\delta_R \sin\Phi}{\cos(\beta_e\Phi) - \mathrm{ctg}\,\delta_R \sin(\beta_e\Phi)} \left|\frac{\varepsilon}{1+\varepsilon z}\right|^{1-\beta_e} \tag{24}$$

with $\bar{\beta}_e = \beta_e + \beta_{int}$ and $\bar{\bar{\beta}}_e = \beta_e + 2\,\beta_{int}$.

The expression of the factors $C^{'(i)}_{F}$ to $o(\alpha)$ can be extracted from ref.[7]. Being no large logarithms involved, there is no need of the $o(\alpha^2)$ terms, which would require a complete two – loop calculation. Of course some relevant $o(\alpha^2)$ terms, e.g. from box diagrams, are already included in the exponentiated $C^{(i)}_{infra}$ factors. Then we get for the factors $C^{'(i)}_{F}$, with the same notation of [7] :

$$C^{'(1)}_{F} = \frac{\alpha}{\pi}\left\{ V^{\gamma}_{1f}(s) + \frac{2z}{1+z^2} A^{\gamma}_{1}(s) \right\}$$

$$C^{'(2)}_{F} = \frac{\alpha}{2\pi}\left\{ V^{\gamma}_{1f}(s) + A^{\gamma}_{1}(s) + V^{\gamma}_{1f}(t) + A^{\gamma}_{1}(t) \right\}$$

$$C^{'(3)}_{F} = \frac{\alpha}{\pi}\left\{ V^{\gamma}_{1f}(t) + \frac{b^4-1}{b^4+1} A^{\gamma}_{1}(t) \right\}$$

$$C^{'(4)}_{F} = \frac{\alpha}{2\pi}\left\{ V^{\gamma}_{1f}(s) + A^{\gamma}_{1}(s) + V^{Z}_{1f}(t) + A^{Z}_{1}(t) \right\}$$

$$C^{'(5)}_{F} = \frac{\alpha}{2\pi}\left\{ \left[V^{\gamma}_{1f}(t) + V^{Z}_{1f}(t) \right] + \right.$$

$$\left. \frac{(f_v^2+f_A^2)\,b^4 - (f_v^2-f_A^2)}{(f_v^2+f_A^2)\,b^4 + (f_v^2-f_A^2)} \left[A^{\gamma}_{1}(t) + A^{Z}_{1}(t) \right] \right\} \tag{25}$$

$$C^{'(6)}_{F} = \frac{\alpha}{\pi}\left\{ V^{Z}_{1f}(t) + A^{Z}_{1}(t) \frac{b^4\left[(f_v^2+f_A^2)^2 + 4 f_A^2 f_v^2 \right] - (f_v^2-f_A^2)^2}{b^4\left[(f_v^2+f_A^2)^2 + 4 f_A^2 f_v^2 \right] + (f_v^2-f_A^2)^2} \right\}$$

$$C'^{(7)}_F = \frac{I'(s)}{R'(s)} \delta'_\pi (s) + \frac{\alpha}{2\pi} \left\{ V^\gamma_{1f}(s) + V^Z_{1f}(s) + 2\pi \frac{I'(s)}{R'(s)} \left[V^\gamma_2(s) - V^Z_2(s) \right] \right\} +$$
$$\frac{\alpha}{2\pi} \left\{ \frac{f_A^2(1+z^2) + f_v^2 2z}{f_V^2(1+z^2) + f_A^2 2z} \right\} \left\{ A^\gamma_1(s) + A^Z_1(s) + 2\pi \frac{I'(s)}{R'(s)} \left[A^\gamma_2(s) - A^Z_2(s) \right] \right\}$$

$$C'^{(8)}_F = \frac{\alpha}{2\pi} \left\{ V^\gamma_{1f}(t) + V^Z_{1f}(s) + A^\gamma_1(t) + A^Z_1(s) \right\}$$
$$+ \alpha \frac{I'(s)}{R'(s)} \left\{ V^\gamma_{2f}(t) - V^Z_2(s) + A^\gamma_2(t) - A^Z_2(s) + \frac{3}{2} \right\}$$

$$C'^{(9)}_F = \frac{\alpha}{2\pi} \left\{ V^Z_{1f}(s) + V^Z_{1f}(t) + A^Z_1(s) + A^Z_1(t) \right\}$$
$$+ \alpha \frac{I'(s)}{R'(s)} \left\{ V^Z_{2f}(t) - V^Z_2(s) + A^Z_2(t) - A^Z_2(s) + \frac{3}{2} \right\}$$

$$C'^{(10)}_F = \frac{\alpha}{\pi} \left\{ V^Z_{1f}(s) + \frac{4 f_A^2 f_v^2 (1+z^2) + (f_v^2 + f_A^2)^2 2z}{(f_v^2 + f_A^2)^2 (1+z^2) + 8 f_A^2 f_v^2 z} A^Z_1(s) \right\}$$

Eqs. (22-25) together with (2) and (17) represent our final result. Notice that we have grouped the form factors and the soft bremsstrahlung θ – dependent terms in the $\bar{C}^{(i)}_F$, and the corresponding box contributions in $C'^{(i)}_F$. This separation is indeed somehow arbitrary, in the absence of a complete exact two loop calculation. However, due to the smallness of those terms to $o(\alpha^2)$ a different procedure will lead to irrelevant consequences for LEP experiments. We also observe that we have not included in eqs. (22-25) the vacuum polarization terms $\delta_\pi(s)$ and $\delta_\pi(t)$, unlike in ref.[7]. This, of course, implies a corresponding use of the running coupling constant $e^2(s)$ and $e^2(t)$ in the effective Born amplitudes.

So far we have not yet considered the emission of hard photons collinear to the final particles, which are often detected within a small cone around the electron – positron directions [14]. This effect was considered in ref.[7] to all orders for the exponentiated factors and to $o(\alpha)$ for the rest. The analysis can be simply extended to $o(\alpha^2)$ in the leading logarithmic accuracy, by applying the Kinoshita – Lee – Nauenberg theorem [15] on the mass singularities to eqs.(22-24). Then, following

[5,10], defining δ the half opening angle of the small cone ($\delta << 1$) and $\beta_\delta = \frac{2\alpha}{\pi} L_\delta$ with $L_\delta = \ln \frac{4}{\delta^2}$, we obtain

$$C_{infra} \to \tilde{C}_{infra} = C_{infra}\, \varepsilon^{\beta_\delta - \beta_e} \tag{26}$$

and

$$\bar{C}_F^{(i)} \to \tilde{C}_F^{(i)} = \tilde{C}_1^{(i)} - \beta_\delta \varepsilon - \frac{\pi^2}{6} \bar{\beta}_e (\beta_\delta + \beta_{int}) - \frac{1}{4} \beta_e \bar{\beta}_e\, \varepsilon^{1-\bar{\beta}_e}, \qquad (i=1,..,6)$$

$$\bar{C}_F^{(i)} \to \tilde{C}_F^{(i)} = \tilde{C}_1^{(i)} - \beta_\delta \varepsilon - \frac{\pi^2}{6} \bar{\beta}_e (\beta_\delta + \beta_{int})$$

$$- \bar{\beta}_e \frac{\cos[(1+\beta_e)\Phi] + \mathrm{tg}\, \delta_R \sin[(1+\beta_e)\Phi]}{\cos(\beta_e \Phi) + \mathrm{tg}\, \delta_R \sin(\beta_e \Phi)} \left| \frac{\varepsilon}{1+\varepsilon z} \right|$$

$$- \frac{1}{4} \beta_e \bar{\beta}_e \frac{\cos\Phi + \mathrm{tg}\, \delta_R \sin\Phi}{\cos(\beta_e \Phi) + \mathrm{tg}\, \delta_R \sin(\beta_e \Phi)} \left| \frac{\varepsilon}{1+\varepsilon z} \right|^{1-\bar{\beta}_e}, \qquad (i=7,8,9) \tag{27}$$

$$\bar{C}_F^{(10)} \to \tilde{C}_F^{(10)} = \tilde{C}_1^{(10)} - \beta_\delta \varepsilon -$$

$$\frac{\pi^2}{6} \beta_\delta \bar{\bar{\beta}}_e - \bar{\bar{\beta}}_e \frac{\cos[(1+\beta_e)\Phi] - \mathrm{ctg}\, \delta_R \sin[(1+\beta_e)\Phi]}{\cos(\beta_e \Phi) - \mathrm{ctg}\, \delta_R \sin(\beta_e \Phi)} \left| \frac{\varepsilon}{1+\varepsilon z} \right|$$

$$- \frac{1}{4} \beta_e \bar{\bar{\beta}}_e \frac{\cos\Phi - \mathrm{ctg}\, \delta_R \sin\Phi}{\cos(\beta_e \Phi) - \mathrm{ctg}\, \delta_R \sin(\beta_e \Phi)} \left| \frac{\varepsilon}{1+\varepsilon z} \right|^{1-\bar{\beta}_e},$$

where the factors $\tilde{C}_1^{(i)}$ are finally given by

$$\tilde{C}_1^{(i)} = \frac{3}{4}(\beta_e + \beta_\delta) + \frac{2\alpha}{\pi}(1+f) + \left(\frac{\alpha}{\pi}\right)^2 \left\{ \left(\frac{9}{4} - \frac{2}{3}\pi^2 - f\right) \frac{L^2 + L_\delta^2}{2} \right.$$

$$+ \frac{9}{4} L L_\delta + \left[6\zeta(3) + \frac{17}{6}\pi^2 - \frac{93}{8} + 6f \right] \frac{L + L_\delta}{2}$$

$$+\left[-9\,\zeta(3)-2\,\pi^2\ln 2+\frac{2\pi^4}{45}-\frac{77}{42}\pi^2+\frac{27}{2}+f^2+2\,\pi^2 f-8\,f\right]\Bigg\}\ ,$$

(i=1,7,10)

$$\tilde{C}_1^{(i)}=\frac{3}{4}(\beta_e+\beta_\delta)+\frac{2\alpha}{\pi}\left(1-\frac{\pi^2}{2}-2\,\ell^2+3\,\ell+f\right)$$

$$+\left(\frac{\alpha}{\pi}\right)^2\Bigg\{\left(\frac{9}{4}-\frac{2}{3}\pi^2-f+8\,\ell^2-12\,\ell\right)\frac{L^2+L_\delta^2}{2}$$

$$+\frac{9}{4}L\,L_\delta+\left[16\,\ell^3-36\,\ell^2+\frac{4}{3}\pi^2\ell+34\,\ell+6\,\zeta(3)-\frac{\pi^2}{6}-\frac{93}{8}-8\,\ell\,f+6\,f\right]\frac{L+L_\delta}{2}$$

$$+\left[8\,\ell^4-24\,\ell^3+34\,\ell^2+\frac{4}{3}\pi^2\ell^2+12\,\zeta(3)\,\ell-3\,\pi^2\ell-\frac{93}{4}\ell-9\,\zeta(3)-2\pi^2\ln 2\right.$$

$$\left.-\frac{11}{90}\pi^4+\frac{211}{72}\pi^2+\frac{27}{2}+f^2-8\,\ell^2 f+12\,\ell\,f-8\,f\right]\Bigg\},\qquad (i=3,5,6)\qquad (28)$$

$$\tilde{C}_1^{(i)}=\frac{3}{4}(\beta_e+\beta_\delta)+\frac{2\alpha}{\pi}\left(1-\frac{\pi^2}{4}-\ell^2+\frac{3}{2}\ell+f\right)$$

$$+\left(\frac{\alpha}{\pi}\right)^2\Bigg\{\left(\frac{9}{4}-\frac{7}{6}\pi^2-f+2\,\ell^2-6\,\ell\right)\frac{L^2+L_\delta^2}{2}$$

$$+\frac{9}{4}L\,L_\delta+\left[4\,\ell^3-12\,\ell^2+17\,\ell-\frac{\pi^2}{3}\ell+6\,\zeta(3)+\frac{17}{6}\pi^2-\frac{93}{8}-4\,\ell\,f+6\,f\right]\frac{L+L_\delta}{2}$$

$$+\left[2\,\ell^4-6\,\ell^3-\frac{\pi^2}{3}\ell^2+\frac{25}{2}\ell^2+6\,\zeta(3)\,\ell-\frac{93}{8}\ell-9\,\zeta(3)-2\pi^2\ln 2\right.$$

$$\left.-\frac{59}{360}\pi^4-\frac{7}{36}\pi^2+\frac{27}{2}+f^2-4\,\ell^2 f+6\,\ell\,f-8\,f+\pi^2 f\right]\Bigg\},\qquad (i=2,4,8,9)$$

This concludes the discussion of collinear hard photon effects. We would like to add a few comments on the phenomenological applications of our formulae, in particular for LEP experiments. The kinematical regime envisaged by our formulae, e.g. soft and / or collinear photon emission, is of course oversimplified to allow a simple analytical treatment of the problem. In practical applications one has to correct for the kinematical configurations not included in the formalism but clearly implied by data, as for example, by applying a fixed cut on the acollinearity of the final electrons. This, in turn, implies the necessity of relying on Montecarlo calculations for Bhabha scattering. A treatment beyond one loop corrections [16] is still missing, although some work along this line is actually in progress [17]. We expect, however, that our formulae can give an accurate description of Bhabha events on the Z_0 peak, providing therefore an additional leverage to study the purely leptonic final states, where a combined fit to all channels could provide a better determination of the Z_0 production and decay properties.

To conclude, we have extended a previous analysis of e.m. radiative corrections to Bhabha scattering in the vicinity of the Z_0 to include two - loop effects which are relevant to improve the theoretical accuracy to a few ‰ . Our analytical treatment is similar to what already exists in the $e^+ e^- \to f^+ f^-$ channels and can be therefore used to perform an overall coherent analysis of leptonic final states in LEP experiments.

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REFERENCES

[1] See for example,
ALEPH Collaboration: D. DECAMP et al., Phys. Lett. B235,399 (90) ;
DELPHI Collaboration: P. AARNIO et al., Phys. Lett. B241,425 (90);
L3 Collaboration: B. ADEVA et al., Phys. Lett. B238,122 (90);
0PAL Collaboration : P. L. AKRAWY et al., Phys. Lett. B240,497 (90),
and references therein.

[2] M. Greco, G. Pancheri and Y. Srivastava, Nucl. Phys. B101 (1975) 234 and B171 (1980)118.

[3] F.A. Berends, W.L. Van Neerven and G.J.H. Burgers, Nucl. Phys. B297 (1988) 429;
O. Nicrosini and L. Trentadue, Z. Physic. C39 (1988) 479.

[4] A.Borrelli, M.Consoli, L.Maiani and R.Sisto, Nucl. Phys. B333(1990)357.

[5] F. Aversa and M. Greco, Phys. Lett. B228 (1989) 134. see also M. Greco, LNF-89/069 (1989).

[6] For a review, see for example, Z Physics at LEP, G.Altarelli, R.Kleiss and C. Verzegnassi editors, CERN 89-08(1989)

[7] M. Greco, Phys. Lett. B199 (1986) 97. See also M. Greco, La Rivista del Nuovo Cimento, Vol.11 n.5 (1988).
For the corresponding FORTRAN code see, for example, M. Caffo and E. Remiddi in [6], and A. Pullia and G. Zumerle, DELPHI 89-84 PHYS 56, Nov. 1989.

[8] M.Boehm, A.Denner and W.Hollik, Nucl. Phys. B304(1988)687.

[9] E.A. Kuraeev and V.S. Fadin, Sov. J. Nucl. Phys. 41 (1985) 466 ; G. Altarelli and G. Martinelli, in Physics at LEP, ed. by J. Ellis and R. Peccei, CERN 86-02 (1986); O. Nicrosini and L. Trentadue, Phys. Lett. 196B (1987) 551. For a review see, for example, O. Nicrosini and L. Trentadue, CERN-TH 5437/89 (1989).

[10] M. Greco and O. Nicrosini, Phys. Lett. B240, 219 (90)

[11] R. Barbieri, G.A. Mignaco and E. Remiddi, Nuovo Cimento 11A (1975) 824 and 865; G.J.H. Burgers, Phys. Lett. 164B (1985) 167.

[12] F. Aversa, P. Chiappetta, M. Greco and J. Ph. Guillet, Nucl. Phys. B327(1989)105.

[13] D. Bardin, M. Bilenk, A. Chizhov, A. Sazonov, O. Fedorenko, T. Riemann and M. Sachwitz, Preprint PHE 89-19, March 1990.

[14] M. Caffo, R. Gatto and E. Remiddi, Nucl. Phys. B252 (1985) 378.

[15] T. Kinoshita, J. Math. Phys. 3 (1962) 650 ;
T.D. Lee and M. Nauenberg, Phys. Rev. 133 (1964) 154.

[16] For a review see, for example, R. Kleiss in [6].

[17] G. Bonvicini, private communication.