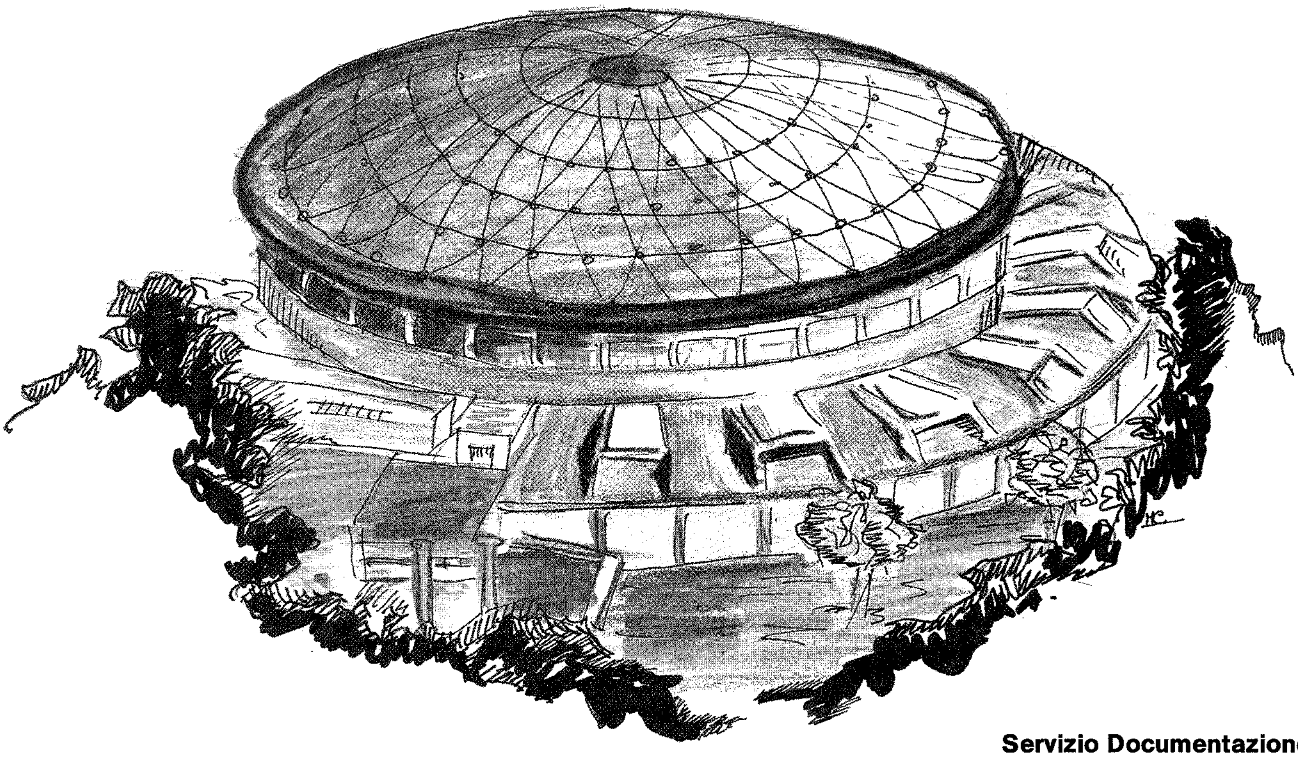
Laboratori Nazionali di Frascati

Submitted to Phys. Letters B

LNF-90/046(PT)
6 Giugno 1990

S. Bellucci, E. Ivanov:

LIOUVILLE REALIZATION OF W_∞-ALGEBRAS



Servizio Documentazione
dei Laboratori Nazionali di Frascati
P.O. Box, 13 - 00044 Frascati (Italy)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-90/046(PT)
6 Giugno 1990

LIOUVILLE REALIZATION OF W_∞-ALGEBRAS

S. Bellucci[1]
INFN - Laboratori Nazionali di Frascati, P.O. Box 13, I-00044 Frascati (Italy)

E. Ivanov
JINR - Laboratory of Theoretical Physics, Head Post Office, P.O. Box 79, 101 000, Moscow,USSR

Abstract

We study the classical field-theoretical realizations of W-algebras in the system of two bosonic fields with the Liouville self-interaction. This system may be important for noncritical W-strings and W-gravity. The combined $W_\infty + w_\infty$ transformations which leave the action invariant are found and their group structure is analyzed. The invariance algebra involves Zamolodchikov's W_2-algebra as a nonlinear subalgebra.

[1] BITNET:BELLUCCI at IRMLNF

The so-called W-algebra has recently become the object of much investigation [1]. This may provide the key to answer many questions concerning two-dimensional (super)gravity and superstring theories. Recently, classical realizations of W-algebras in systems of free bosons were considered [2,3]. It turned out, in particular, that Siegel's chiral bosons [4] naturally appear as a result of gauging the W_∞-algebra. In this work we consider realizations of W-algebras in a simple action for two bosons one of which possesses the Liouville self-interaction. Such a system may be important for noncritical W-strings and, keeping in mind the appearance of the Liouville term in the ordinary two-dimensional Polyakov gravity [5], for the related versions of W-gravities. We find that the adding of the Liouville term reduces a lot of W-type invariances of the free action to a much smaller set.

Let us begin by writing down the free lagrangian

$$\mathcal{L}_0 = -\partial_+\mathcal{U}\partial_-\mathcal{U} - \partial_+\phi\partial_-\phi \equiv -\frac{1}{2}\left[\partial_+\psi\partial_-\overline{\psi} + \partial_+\overline{\psi}\partial_-\psi\right], \tag{1}$$

where we defined $\psi = \mathcal{U} + i\phi$, $\overline{\psi} = \mathcal{U} - i\phi$. The chiral part of the invariances of the corresponding action includes the $U(1)$ Kac-Moody and Virasoro-type transformations (spin 1 and spin 2)

$$\delta_1{}^{(1)}\mathcal{U} = \epsilon_1{}^0 \,,\; \delta_2{}^{(1)}\phi = \epsilon_2{}^0 \tag{2a}$$

$$\delta^{(2)}\mathcal{U} = \epsilon_1{}^-\partial_-\mathcal{U} \,,\; \delta^{(2)}\phi = \epsilon_2{}^-\partial_-\phi \tag{2b}$$

$$\delta^{(2)}\mathcal{U} = \epsilon_3{}^-\partial_-\phi \,,\; \delta^{(2)}\phi = \epsilon_3{}^-\partial_-\mathcal{U} \,, \tag{2c}$$

where all the ϵ parameters depend on x^- but not on x^+. Notice that in (2c) the scalar fields are mixed by the transformation. In addition to the Virasoro sector, there are also w_∞-type transformations [3] that leave the action invariant

$$\delta^{(2)}\mathcal{U} = \epsilon_1{}^{--}(\partial_-\mathcal{U})^2 \ , \ \delta^{(2)}\phi = \epsilon_2{}^{--}(\partial_-\phi)^2 \tag{3a}$$

$$\delta^{(2)}\mathcal{U} = \epsilon_3{}^{--}(\partial_-\mathcal{U})(\partial_-\phi) \ , \ \delta^{(2)}\phi = \frac{1}{2}\epsilon_3{}^{--}(\partial_-\mathcal{U})^2 \tag{3b}$$

$$\delta^{(2)}\mathcal{U} = \frac{1}{2}\epsilon_4{}^{--}(\partial_-\phi)^2 \ , \ \delta^{(2)}\phi = \epsilon_4{}^{--}(\partial_-\mathcal{U})(\partial_-\phi) \ . \tag{3c}$$

The parameters also depend on x^- only. The transformations (3b) mix the two fields $\mathcal{U}$ and ϕ. In checking the Lie bracket structure between mixed transformations, one needs only to commute (3a) with (3b) and (3b) with (3c), owing to a symmetry of exchanging $\mathcal{U}$ and ϕ between (3b) and (3c). Repeated commutators of transformations (3) yield the whole infinite rest of w_∞-transformations.

We complete the list of chiral invariances of the action (1) by writing down the following W_∞-type transformations (corresponding to the conformal spin 3 generators)

$$\begin{aligned} \delta^{(3)}\mathcal{U} &= \epsilon^{--}(\partial_-)^2\phi + \frac{1}{2}(\partial_-\epsilon^{--})\partial_-\phi \ , \\ \delta^{(3)}\phi &= -\,\epsilon^{--}(\partial_-)^2\mathcal{U} - \frac{1}{2}(\partial_-\epsilon^{--})\partial_-\mathcal{U} \ . \end{aligned} \tag{4}$$

Once again, eqs. (4) produce, via commutation, an infinite set of transformations. After setting $\mathcal{U} = e^{\tilde{\mathcal{U}}}$ and $\phi = e^{\tilde{\phi}}$, these transformations are reduced to those of the type given in ref. [3][2]

$$\delta^{(3)}\tilde{\mathcal{U}} = \epsilon^{--}\left[(\partial_-)^2\tilde{\phi} + (\partial_-\tilde{\phi})^2 \right] + \frac{1}{2}(\partial_-\epsilon^{--})\partial_-\tilde{\phi} \ ,$$

2 An essential difference is that eqs. (4) involve two scalar fields while in the realization given in ref. [3] only one field is present. It is an open question how to recover the latter realization within field-theoretical models.

$$\delta^{(3)}\tilde{\phi} = -\epsilon^{--}\left[(\partial_-)^2\tilde{\mathcal{U}} + (\partial_-\tilde{\mathcal{U}})^2\right] - \frac{1}{2}(\partial_-\epsilon^{--})\partial_-\tilde{\mathcal{U}} .$$

The W_∞-type algebra structure constants coincide, at the classical level, with those given in refs. [3,6]. The higher-spin transformations are generated by commuting (4) among themselves. Note that there exist two more sets of W_∞-type invariances which are realized separately on the fields $\mathcal{U}$ and ϕ and involve only the generators with even conformal spins:

$$\delta^{(2k)}\mathcal{U} = (\partial_-)^{k-1}\left[\epsilon_1{}^{-(2k-1)}(\partial_-)^k\mathcal{U}\right] , \; k = 1,2,\dots ,$$

$$\delta^{(2)}\mathcal{U} = \epsilon_1{}^{-}\partial_-\mathcal{U} , \; \delta^{(4)}\mathcal{U} = \partial_-\left[\epsilon_1{}^{---}(\partial_-)^2\mathcal{U}\right] , \; etc. , \tag{5}$$

(and the same for $\phi(x)$). The even spin subalgebra of the W_∞-algebra generated by (4) corresponds to identifying the parameters of the $\mathcal{U}$-transformations (5) with those entering into analogous transformations of $\phi(x)$.

In terms of the complex field $\psi = \mathcal{U} + i\phi$, $\overline{\psi} = \mathcal{U} - i\phi$, we have, for the Virasoro sector

$$\begin{aligned} \delta^{(1)}\psi &= \epsilon_1{}^0 + i\epsilon_2{}^0 \\ \delta_1{}^{(2)}\psi &= \tilde{\epsilon}_1^-\partial_-\psi , \; \delta_2{}^{(2)}\psi = \tilde{\epsilon}_2^-\partial_-\overline{\psi} \\ \delta_3{}^{(2)}\psi &= i\tilde{\epsilon}_3^-\partial_-\overline{\psi} , \end{aligned} \tag{2'}$$

where the parameters $\tilde{\epsilon}_i$, $i = 1,2,3$ are all real. In a similar way one can also rewrite the transformation (3). For the W_∞-type transformation (4) we obtain

$$\delta^{(3)}\psi = -i\epsilon^{--}(\partial_-)^2\psi - i\frac{1}{2}(\partial_-\epsilon^{--})\partial_-\psi . \tag{4'}$$

Our aim is to consider the lagrangian modified by adding a Liouville term[3]

$$\mathcal{L}_{int} = -\partial_+\mathcal{U}\partial_-\mathcal{U} - \partial_+\phi\partial_-\phi + ce^{\mathcal{U}} . \tag{6}$$

3 Any exponential self-interaction $\sim e^{a\mathcal{U}+b\phi}$ is reduced to (6) by a field redefinition.

As we already mentioned, such a lagrangian can play an important role in non-critical W-strings. The chiral invariances of this lagrangian are given by certain combinations of the ones above. We first write the Virasoro-type and $U(1)$ Kac-Moody type symmetries

$$\delta_1{}^{(2)}\mathcal{U} = \epsilon_1{}^-\partial_-\mathcal{U} + \partial_-\epsilon_1{}^- , \ \delta_2{}^{(2)}\phi = \epsilon_2{}^-\partial_-\phi , \ \delta_3{}^{(1)}\phi = \epsilon_3 . \quad (7)$$

In addition, the lagrangian (6) possesses invariances of the type $W + w$

$$\begin{aligned} \delta^{(3)}\mathcal{U} &= (\partial_-\epsilon^{--})\partial_-\phi + \epsilon^{--}\Big[(\partial_-)^2\phi + (\partial_-\mathcal{U})(\partial_-\phi)\Big] , \\ \delta^{(3)}\phi &= \epsilon^{--}\Big[\frac{1}{2}(\partial_-\mathcal{U})(\partial_-\mathcal{U}) - (\partial_-)^2\mathcal{U}\Big] \end{aligned} \quad (8)$$

(the higher-spin transformations follow by commuting (8) with (7) and among themselves). The even spin transformations (5) can also be straightforwardly extended to the invariances of (6):

$$\tilde{\delta}^{(4)}\mathcal{U} = \partial_-\Big[\epsilon^{---}(\partial_-)^2\mathcal{U}\Big] - \frac{1}{2}\epsilon^{---}(\partial_-\mathcal{U})^3 - \frac{1}{2}\partial_-\epsilon^{---}(\partial_-\mathcal{U})^2 , \ etc. \quad (9)$$

(we did not write explicitly the spin > 4 transformations; all these are proper combinations of (5) and of the w_∞ transformations of the type $(3a)$[4]). The action corresponding to (6) is also (trivially) invariant under the full set of $w_{\infty+1}$ symmetries realized on the free field $\phi(x)$. Note that the presence of this field is necessary for invariance under the W transformations with all, odd and even, conformal spins. Without it, we would have only the $W + w$ invariances (9) corresponding to the even spin currents.

4 Note that all the transformations of $\mathcal{U}$ leaving invariant (6) have the form of the Virasoro transformation with the field-dependent parameters

$$\delta\mathcal{U} = A^-\partial_-\mathcal{U} + \partial_-A^- , \ \partial_+A^- \neq 0 .$$

Thus we observe that after including the Liouville term for $\mathcal{U}(x)$, the lot of independent W-type symmetries of the free action is reduced to a more restricted set of invariances. In view of the possible relation to W_∞-gravity and noncritical strings, it seems important to know the structure of the surviving algebra and to gauge it along the lines of refs. [2,3]. Leaving this to future study, here we restrict ourselves to several remarks.

We begin with the issues related to currents and gauging. Determining the currents allows us to construct the gauge invariant action. We are interested in the left currents for the lagrangian (1). Beginning with the Virasoro sector, we have

$$J^{\mathcal{U}}{}_{--} = -\frac{1}{2}(\partial_-\mathcal{U})^2 , \qquad (2a'')$$

$$J^{\phi}{}_{--} = -\frac{1}{2}(\partial_-\phi)^2 , \qquad (2b'')$$

$$J^{\mathcal{U}\times\phi}{}_{--} = -(\partial_-\mathcal{U})(\partial_-\phi) . \qquad (2c'')$$

The properly normalized currents generating, via the Poisson brackets, the w-type transformations read

$$J^{\mathcal{U}}{}_{---} = -\frac{1}{3}(\partial_-\mathcal{U})^3 , \; J^{\phi}{}_{---} = -\frac{1}{3}(\partial_-\phi)^3 , \qquad (3a'')$$

$$J_{---} = \frac{1}{2}(\partial_-\mathcal{U})^2(\partial_-\phi) , \qquad (3b'')$$

$$J_{---} = \frac{1}{2}(\partial_-\phi)^2(\partial_-\mathcal{U}) . \qquad (3c'')$$

The W-type invariance (4) is generated by the current

$$J_{---} = -\frac{1}{2}\left[\partial_-\phi(\partial_-)^2\mathcal{U} - \partial_-\mathcal{U}(\partial_-)^2\phi \right]$$

$$= -i\frac{1}{4}\left[\,\partial_-\overline{\psi}(\partial_-)^2\psi \;-\; \partial_-\psi(\partial_-)^2\overline{\psi}\,\right]. \qquad (4'')$$

We finally give the spin 2, spin 1 and spin 3 currents generating the symmetries (7), (8) of the interacting lagrangian (6)

$$\begin{aligned} \tilde{J}^{(1)}_{--} &= -\frac{1}{2}(\partial_-\mathcal{U})^2 - \frac{1}{2}(\partial_-\phi)^2 + (\partial_-)^2\mathcal{U} \\ &= (\partial_-)^2(Re\,\psi) - \frac{1}{2}\partial_-\psi\partial_-\overline{\psi}\,, \\ \tilde{J}^{(2)}_{--} &= -\frac{1}{2}(\partial_-\phi)^2\,,\quad \tilde{J}^{(3)}_{-} = \partial_-\phi\,, \end{aligned} \qquad (7'')$$

$$\begin{aligned} \tilde{J}_{---} &= \frac{1}{2}(\partial_-\mathcal{U})^2\partial_-\phi - (\partial_-)^2\mathcal{U}(\partial_-\phi) \\ &= \frac{1}{16i}\left\{\left[(\partial_-\psi)^2 - (\partial_-\overline{\psi})^2\right]\left(\partial_-\psi + \partial_-\overline{\psi}\right)\right. \\ &\quad \left. - 4\left[(\partial_-)^2\psi + (\partial_-)^2\overline{\psi}\right]\left(\partial_-\psi - \partial_-\overline{\psi}\right)\right\}, \end{aligned} \qquad (8'')$$

and the spin 4 current corresponding to (9)

$$\begin{aligned} \tilde{J}_{(-4)} &= \frac{1}{2}\left[(\partial_-)^2\mathcal{U}(\partial_-\mathcal{U})^2 - (\partial_-)^2\mathcal{U}(\partial_-)^2\mathcal{U} - \frac{1}{4}(\partial_-\mathcal{U})^4\right] \\ &= -\left[(\partial_-)^2\mathcal{U} - \frac{1}{2}(\partial_-\mathcal{U})^2\right]^2. \end{aligned} \qquad (9'')$$

Note that the expression within the square brackets in (9″) is recognized as the $\mathcal{U}$-part of the full conformal stress-tensor $\tilde{J}^{(1)}_{--}$ (7″). Hence, the infinite-dimensional symmetry (9) in its own right is, to some extent, trivial because all the higher spin currents related to it are expressed as combinations of powers and derivatives of the lowest (spin 2) current generating conformal transformations of the field $\mathcal{U}(x)$ (this property for the currents next to $\tilde{J}_{(-4)}$ (9″) can be checked by direct inspection).

Having found the currents, it is straightforward to gauge the symmetries of the free action (1). We begin with the Virasoro sector and add to the free action the term (for simplicity, we restrict our considerations here to the transformations (2*b*) with identified parameters)

$$+\lambda^{--}\left(\partial_-\mathcal{U}\partial_-\mathcal{U} + \partial_-\phi\partial_-\phi\right). \tag{10}$$

Notice that, simultaneously with the Virasoro transformations, all the diagonal w ones are gauged [3]. Next, we turn to the gauging of the W invariance. We add further to the action the term

$$+\lambda^{---}\left[(\partial_-)^2\mathcal{U}\partial_-\phi - \partial_-\mathcal{U}(\partial_-)^2\phi\right], \tag{11}$$

and check that it compensates, with a proper transformation of λ^{---}, of the form

$$\delta\lambda^{---} = (const)(\partial_+\epsilon^{--}) + (nonlinear\ terms)\ ,$$

noninvariances of the previous terms. The complete gauging of the interacting lagrangian with the Liouville term, which goes beyond the purpose of this work, will be presented in a future publication. We make here only two general comments.

In accordance with the structure of the conformal stress-tensor ($7''$), the minimal gauge coupling (10) will be modified by the term

$$\sim\lambda^{--}(\partial_-)^2\mathcal{U}\ . \tag{12}$$

Also, the independent gauge couplings

$$\sim\lambda_1{}^{--}(\partial_-)^2\phi\ ,\quad \lambda_2{}^{--}(\partial_-)^2\phi \tag{13}$$

may appear, corresponding to the presence of independent $w_{1+\infty}$-algebra transformations of the field $\phi(x)$. As indicated in [7], the adding of terms of the type

(12) to the Siegel's chiral boson lagrangian makes the gauge invariance of the latter hold only quantum-mechanically. We expect the same phenomenon to occur in our case. Furthermore, the spin 3 minimal coupling (11) will be changed to

$$+ \lambda^{---}\left[(\partial_-)^2 \mathcal{U} \partial_- \phi \; - \; \frac{1}{2} (\partial_- \mathcal{U})^2 \partial_- \phi \right] , \tag{11'}$$

Next, we turn to the discussion of some interesting group-theoretical features of our Liouville realization of W-algebras.

Commuting (8) with the conformal transformations (7) one observes

$$[\delta^{(3)}, \delta_c{}^{(2)}] \sim \delta^{(3)} + \delta^{(1)} + \delta_c{}^{(2)} + \delta_2{}^{(2)} , \tag{14}$$

which is different from the analogous Lie bracket of both W_∞ and w_∞, where the spin 3 variation is generated by the current which is primary with respect to the conformal stress-tensor [6] (this corresponds to the absence of variations $\delta^{(1)}$ and $\delta^{(2)}$ in the *r.h.s.* of such a commutator). So, the spin 2 and spin 1 currents $\tilde{J}^{(2)}_{--}$ and $\tilde{J}^{(3)}_{-}$ from the set (7″) essentially enter the group structure we are considering. Commuting $\delta^{(3)}$ with itself and the variations $\delta_2{}^{(2)}$ and $\delta^{(1)}$, one obtains three independent spin 4 variations, generated by the currents (9″) and the following two additional currents

$$\begin{aligned} \tilde{J}^{(1)}_{(-4)} &= \left[\frac{1}{2} (\partial_- \mathcal{U})^2 \; - \; (\partial_-)^2 \mathcal{U} \right] (\partial_- \phi)^2 \\ \tilde{J}^{(2)}_{(-4)} &= \left[\frac{1}{2} (\partial_- \mathcal{U})^2 \; - \; (\partial_-)^2 \mathcal{U} \right] (\partial_-)^2 \phi \, . \end{aligned} \tag{15}$$

There are, of course, the independent spin 4 currents in the ϕ sector, which involve only the field $\phi(x)$.

It is interesting that, combining (8″) with the spin 3 currents of the ϕ sector and derivatives of the spin 2 and spin 1 currents (7″), one may define a new spin

3 current such that it is primary with respect to the modified stress-tensor

$$\begin{aligned}\tilde{\tilde{J}}_{(-2)} &= -\frac{1}{2}(\partial_- \mathcal{U})^2 + (\partial_-)^2\mathcal{U} - \frac{1}{2}(\partial_-\phi)^2 + \alpha(\partial_-)^2\phi \\ &\equiv T_{-2} \,. \end{aligned} \tag{16}$$

This new spin 3 current reads

$$\begin{aligned}\tilde{\tilde{J}}_{(-3)} &= \frac{1}{2}(\partial_-\mathcal{U})^2\partial_-\phi - (\partial_-)^2\mathcal{U}\partial_-\phi + \frac{1}{2}\alpha(\partial_-)^3\mathcal{U} - \frac{1}{2}\alpha(\partial_-)^2\mathcal{U}\partial_-\mathcal{U} \\ &+ \frac{3}{2\alpha}(\partial_-)^2\phi\partial_-\phi - \frac{1}{2\alpha^2}(\partial_-\phi)^3 - \frac{1}{2}(\partial_-)^3\phi \,. \end{aligned} \tag{17}$$

However, for arbitrary α, the corresponding variations do not form a closed subalgebra: the commutator of the modified spin 3 variations contains, besides some new spin 4 variation, also independent spin 1 and spin 2 variations of the field $\phi(x)$. Such unwanted variations drop out only for the choice $\alpha = \pm\sqrt{3}$ in (16), (17). For these values of the α parameter one gets

$$\tilde{\tilde{J}}_{(-3)}(\alpha = \pm\sqrt{3}) = -\frac{1}{2}(T_{-2})(T_{-2}) \,, \tag{18}$$

and the currents (16), (17) generate a closed nonlinear algebra, which is easily recognized as the classical version of Zamolodchikov's W_2 algebra [1,8,2]. This realization of W_2 is different from the one given in [2] and rather coincides with the Toda lattice realization of W_2 [9]. Hence, two W_2's (for $\alpha = \pm\sqrt{3}$) are embedded as nonlinear subalgebras into our Liouville W-algebra. Note that, restricting to these W_2 transformations only, one may find two more invariant self-interactions, namely [8]

$$\mathcal{L}_{s.i.} = e^{\frac{1}{2}(-\mathcal{U}\pm\sqrt{3}\phi)} \,. \tag{19}$$

The sum of (6) and one of the terms (19) (depending on the choice of α) yields just the lagrangian of the Sl_3 Toda lattice. It would be interesting to gauge W_2 in the above realization. Note that the currents (16), (17) at $\alpha = \pm\sqrt{3}$ are reduced, by a field redefinition, to those given in ref. [8] at the quantum level, for the quantum central charge $c = 2 + 12 + 12\alpha^2 = 50$.

We end with several furher remarks. When gauging the action (6), the contribution of the Feigin-Fuchs term [9] $(\partial_-)^2(Re\ \psi)$, contained in the current $\tilde{J}^{(1)}_{(-2)}$ $(7'')$, will reduce to a term $\lambda^{--}(\partial_-)^2(Re\ \psi)$, related to the scalar curvature $(\partial_-)^2\lambda^{--}$ for $SL(2,R)$ (*i.e.* ordinary gravity). Linear terms of this kind appear in the modified Zamolodchikov gauge algebra at the quantum level [8,2]. On the other hand, if we prefer to gauge the spin 3 variations generated by the primary current (17), we should substitute it into eq. $(10')$ instead of the old current $(8'')$. The term $(\partial_-)^3(Im\ \psi)$, contained in the current (17), will be reminiscent of a certain higher curvature term related to W-gravity, as λ^{---} is expected to coincide with some prepotential of W-gravity in a triangular gauge. Thus, acting on this prepotential by $(\partial_-)^3$ may give a curvature term connected to W-gravity.

When going from Minkowski coordinates to $(z,\bar{z})$ variables, by choosing an imaginary coefficient of $(\partial_-)^2(Re\ \psi)$ in the current $\tilde{J}_{--}$ we get minimal models with $c<1$ and a discrete spectrum. If we choose instead a real value of the coefficient, then we obtain a theory with a continous spectrum of states and $c>1$, corresponding to nonminimal models with arbitrary value of the central charge. The structure of the W-algebra should be different for these cases (some structure constants becoming imaginary).

In conclusion, we recall that the standard Siegel action provides a gauge realization of w_∞-algebra. We have introduced a new system for gauging a kind of W_∞-algebra. Adding a Liouville field, the symmetry of the action is restricted and becomes essentially unique (modulo an arbitrariness in the ϕ sector). In the future, the study of supersymmetric extensions of our interacting lagrangian system and its quantum properties will be pursued. Note that the lagrangian (6) is just the bosonic part of the $N=2$ supersymmetric Liouville action [10].

Acknowledgement

It is a pleasure to acknowledge the participation of S. Krivonos at the early stages of this work. We also enjoyed conversations with I. Bakas and E. Sezgin. One of us (E.I.) wishes to thank INFN-Laboratori Nazionali di Frascati for providing hospitality and financial support.

References

[1] A.B. Zamolodchikov, Teor. Mat. Fiz. 65 (1985) 1205; C.N. Pope and X. Shen, Phys. Lett. 236B (1990) 21; C.N. Pope, L.J. Romans and X. Shen, Phys. Lett. 236B (1990) 173.

[2] C.M. Hull, Phys. Lett. 240B (1990) 110.

[3] E. Bergshoeff, C.N. Pope, L.J. Romans, E. Sezgin, X. Shen and K.S. Stelle, Phys. Lett. 243B (1990) 350.

[4] W. Siegel, Nucl. Phys. B238 (1984) 307.

[5] A.M. Polyakov, Mod. Phys. Lett. A2 (1987) 893.

[6] I. Bakas and E. Kiritsis, Nucl. Phys. B343 (1990) 185.

[7] J.M.F. Labastida and M. Pernici, Nucl. Phys. B297 (1988) 557.

[8] V.A. Fateev and A.B. Zamolodchikov, Nucl. Phys. B280 (1987) 644.

[9] A. Bilal and J.-L. Gervais, Phys. Lett. 206B (1988) 412.

[10] B.L. Feigin and D.B. Fuchs, Funkts. Anal. Pril. 16 (1982) 47.

[11] E.A. Ivanov and S.O. Krivonos, Lett. Math. Phys. 7 (1983) 523.