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Jet production in hadronic collisions to $O(\alpha_s^3)$

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Abstract. We present the full $O(\alpha_s^3)$ QCD corrections to jet production within a small opening angle δ at large transverse momenta for all partonic subprocesses. Results for CERN and Tevatron colliders are given with particular emphasis on the sensitivity to renormalization and factorization mass scales.

1 Introduction and notation

In a previous paper [1]—hereafter denoted as I—we have presented the results of the full evaluation of all $O(\alpha_s^3)$ cross-sections for parton–parton scattering processes. In particular we have given explicit formulae for one hadron inclusive production and studied the sensitivity of $O(\alpha_s^2)+O(\alpha_s^3)$ cross-sections to the renormalization and factorization scales. We have also given some preliminary phenomenological results for jet production.

In this work we will give the formulae for inclusive jet production within a small cone of semi-aperture δ, as well as more precise predictions for CERN and Fermilab collider experiments.

With the same notations of I, the jet inclusive cross-section in the hadronic reaction $H_1(K_1)+H_2(K_2)\to \text{jet}(P)+X$ is given by

$$E\frac{d\sigma}{d^3P}=\frac{1}{\pi S}\sum_{ij}\int_{VW}^{V}\frac{dv}{1-v}\int_{VW/v}^{1}\frac{dw}{dw}F_{p_i}^{H_1}(x_1,M^2)\cdot F_{p_j}^{H_2}(x_2,M^2)\left[\frac{1}{v}\frac{d\sigma^0_{ij\to\text{jet}}}{dv}(s,v)\delta(1-w)+\frac{\alpha_s(\mu^2)}{2\pi}K_{ij\to\text{jet}}(s,v,w;\mu^2,M^2,\delta)\right] \quad (1)$$

where $S=(K_1+K_2)^2$, $T=(K_1-P)^2$, $U=(K_2-P)^2$, $V=1+(T/S)$, $W=-U/(S+T)$, $x_1=VW/vw$, $x_2=(1-V)/(1-v)$ and $s=x_1x_2S$. Following Furman [2] we define the jet as an arbitrary set of hadrons contained within a small cone of semi-aperture angle δ, and with total momentum P. The smallness of δ implies $P^2\cong 0$ and $E\cong|\mathbf{P}|\cdot F_p{}^H(x,M^2)$ are the structure functions of a parton p inside the hadron H at the factorization scale M^2, $d\sigma^0_{ij}(s,v)$ are the Born partonic cross-sections—which can be found in I—and K_{ij} is the correction term. Furthermore the running coupling constant $\alpha_s(\mu^2)$, evaluated at the renormalization scale μ^2, is given by:

$$\alpha_s(\mu^2)=\frac{2\pi}{\beta_0\ln(\mu^2/\Lambda^2)}\left\{1-\frac{\beta_1\ln[\ln(\mu^2/\Lambda^2)]}{\beta_0{}^2\ln(\mu^2/\Lambda^2)}\right\} \quad (2)$$

with $\beta_0=(\frac{11}{6})N-(\frac{1}{3})N_F$, $\beta_1=(\frac{17}{6})N^2-(\frac{5}{6})NN_F-(\frac{1}{2})C_FN_F$, and $N=3(N_F)$ being the number of colours (flavours) with $C_F=(N^2-1)/2N$.

The calculation of the correction terms K_{ij} in (1) starts from the basic results of Ellis and Sexton [3] for the squared matrix elements of all $(2\to 2)$ and $(2\to 3)$ real and virtual subprocesses to $O(\alpha_s^3)$. Then we perform the phase space integration of the real processes in $n=4-2\varepsilon$ dimensions, cancel the $(1/\varepsilon^2)$ divergences by adding the virtual corrections and absorb the $(1/\varepsilon)$ mass singularities from incoming legs into evoluted structure functions. The collinear divergences associated to final partons are automatically cancelled by adding the contributions of one and two partons in the cone. We will describe the organization of phase space calculation in more details. The basic reaction is $p_1+p_2\to p_3+p_4+p_5$. The jet may consist of one of three final partons (case I) or any pair of them (case II). The configuration consisting of three particles in the cone does not contribute due to the kinematics. For case I we subdivise the phase space region in three parts: (a) where $p_3=P$ is inside the cone and p_4 and p_5 are integrated over full phase space, (b) where $p_3=P$ and p_4 are inside the cone and (c) where $p_3=P$ and p_5 are inside the cone. We have performed the calcula-

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tion in this way since part (a) corresponds to one parton inclusive cross section already evaluated in I. We refer the reader to [2] for more details.

As discussed in detail in I, the finite next-to-leading corrections to structure functions, denoted as $f_{p_ip_j}(x)$ have only been calculated for quarks [4]. As well known they contain terms which become particularly large near the boundary of the phase space. For processes involving gluons, we can similarly incorporate in the f's the relevant kinematical factors by "multiplying" the Altarelli Parisi Kernel $P_{p_ip_j}(x)$ by $\ln[(1-x)/x]$ for $f_{p_ip_j}(x)$. Furthermore imposing energy–momentum sum rules we obtain the following expressions for $f_{p_ip_j}(x)$:

$$f_{gg}(x) = 2N\left\{x\left[\frac{\ln(1-x)}{1-x}\right]_+ - \frac{x\ln x}{1-x} + \left(\frac{5N_F}{24N} - \frac{1}{6}\pi^2 - \frac{1}{2}\right)\delta(1-x)\right\},$$

$$f_{qg}(x) = \tfrac{1}{2}[x^2 + (1-x)^2]\ln\left(\frac{(1-x)}{x}\right),$$

$$f_{gq}(x) = C_F\left[\frac{1+(1-x)^2}{x}\ln\left(\frac{1-x}{x}\right) - \frac{4}{3}\right],$$

$$f_{qq}(x) = C_F\left\{(1+x^2)\left[\frac{\ln(1-x)}{1-x}\right]_+ - \frac{3}{2}\frac{1}{(1-x)_+} - \frac{1+x^2}{1-x}\ln x + 3 + 2x - \left(\frac{9}{2} + \frac{1}{3}\pi^2\right)\delta(1-x)\right\}. \tag{3}$$

In the following we will denote this scheme by $CQ = 1$. To test the sensitivity to the factorization scheme we will consider also the choice $CQ = 0$, which corresponds to $f_{ij} = 0$ except for $i = j = q$.

2 $O(\alpha_s^3)$ jet cross-sections

We present now our results for inclusive cross-section of jets produced at large transverse momentum within a small cone of opening angle δ, in the following processes

$$\begin{aligned} q_j q_k &\to \text{jet} + X \quad (JO1)\\ q_j \bar{q}_k &\to \text{jet} + X \quad (JO2)\\ q_j q_j &\to \text{jet} + X \quad (JO3)\\ q_j \bar{q}_j &\to \text{jet} + X \quad (JO4)\\ q_j g &\to \text{jet} + X \quad (JO5)\\ gg &\to \text{jet} + X \quad (JO6). \end{aligned} \tag{4}$$

With the notation of (1) the Born cross-sections are already identical to those given in I and will not be recalled here. The correction factors $K_{ij\to\text{jet}}$ can be written as:

$$\begin{aligned} K_{ij\to\text{jet}}&(s,v,w;\mu^2,M^2,\delta^2)\\ &= \frac{\pi\alpha_s^2(\mu^2)}{2C_iC_js}\left\{\left[c_1 + \tilde{c}_1\ln\left(\frac{s}{M^2}\right) + \tilde{\tilde{c}}_1\ln\left(\frac{s}{E^2\delta^2}\right) + \hat{c}_1\ln\left(\frac{s}{\mu^2}\right)\right]\delta(1-w) + \left[c_2 + \tilde{c}_2\ln\left(\frac{s}{M^2}\right) + \tilde{\tilde{c}}_2\ln\left(\frac{s}{E^2\delta^2}\right)\right]\frac{1}{(1-w)_+} + c_3\left[\frac{\ln(1-w)}{(1-w)}\right]_+\right\}\\ &\quad + K'_{ij\to\text{jet}}(s,v,w,\delta). \end{aligned} \tag{5}$$

The coefficient C_i is equal to N for quarks and to N^2-1 for gluons.

The explicit expression for the coefficients c_i of the distributions and for the regular terms $K'_{ij\to\text{jet}}$, which are numerically small, is analytically very cumbersome. They will not be given here but could be obtained upon request by bitnet (CHIAPETA@FRCPN11). Two comments are in order here. The correction factor given in (5) contains terms proportional to $\ln\delta$, i.e. sensitive to jet size, which were not present at Born level. Therefore an evaluation at next to leading order is needed to define properly a jet cross section. The regular terms are in principle dependent on the prescription for the average over the spin states of initial gluons. $AL = 1, 0$ corresponds to the average $1/2(1-\varepsilon)$, $1/2$, respectively. The choice $AL = 1$ is mandated by the equivalent choice made in the two loop anomalous dimensions. This ambiguity is numerically irrelevant.

3 Numerical results

We will first give results for proton–antiproton collisions at $\sqrt{S} = 0.63$ and 1.8 TeV, for $\theta_{CM} = 90°$, $\delta = 0.2$ and $N_F = 4$. In order to be consistent with our choices of the factorization scheme we have first modified the Diemoz et al. [5] evolution programme for the structure functions, coherently with our prescriptions $CQ = 0, 1$⋆. The difference with the original Diemoz et al. parametrization affects mainly the large x-range for singlet and glue distributions.

The questions we address mainly are the sensitivity of $O(\alpha_s^2) + O(\alpha_s^3)$ jet cross-sections to the renormalization and factorization scales and the uncertainties due to the factorization scheme. Our results are shown for the scaled jet cross-section $\Sigma \equiv P_T^4 E d\sigma/d^3P$ as a function of $\eta \equiv 2P_T/\sqrt{S}$, where P_T is the transverse momentum of the jet.

We first plot Σ in Fig. 1a (resp. 1b) for $\mu = M = 3P_T/4$ (resp. $\mu = M = 2P_T$), at $\sqrt{S} = 0.63$ TeV for $CQ = 1$. The comparison between the two figures shows that when higher order corrections are included the cross-section is quite stable, unlike the Born cross-section which is varying by a factor of 3. In Fig. 2a–b the same quantities are plotted for $CQ = 0$, showing similar

⋆ This new parametrization of the structure function can be obtained upon request

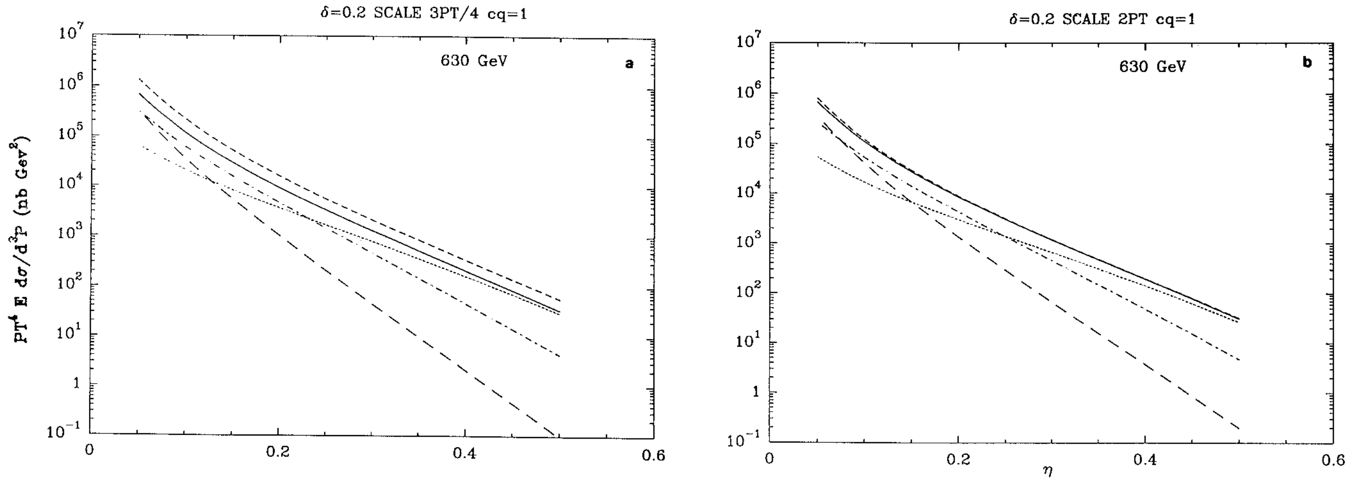

Fig. 1. **a** The scaled jet cross-section $P_T{}^4 E d\sigma/d^3P$ as a function of $\eta = 2P_T/\sqrt{S}$, at $\sqrt{S} = 630\,\text{GeV}$ for $\delta = 0.2$, $CQ = 1$ and $\mu = M = 3P_T/4$. Dashed curve: Born cross-section. Full curve: $O(\alpha_s{}^2) + O(\alpha_s{}^3)$ prediction for all subprocesses. Small dashed curve: $O(\alpha_s{}^2) + O(\alpha_s{}^3)$ prediction for all quark subprocesses. Dot dashed curve: $O(\alpha_s{}^2) + O(\alpha_s{}^3)$ prediction for quark–gluon subprocesses. Long dashed curve: $O(\alpha_s{}^2) + O(\alpha_s{}^3)$ prediction for gluon–gluon subprocesses. **b** Same caption as for **a** for $\mu = M = 2P_T$

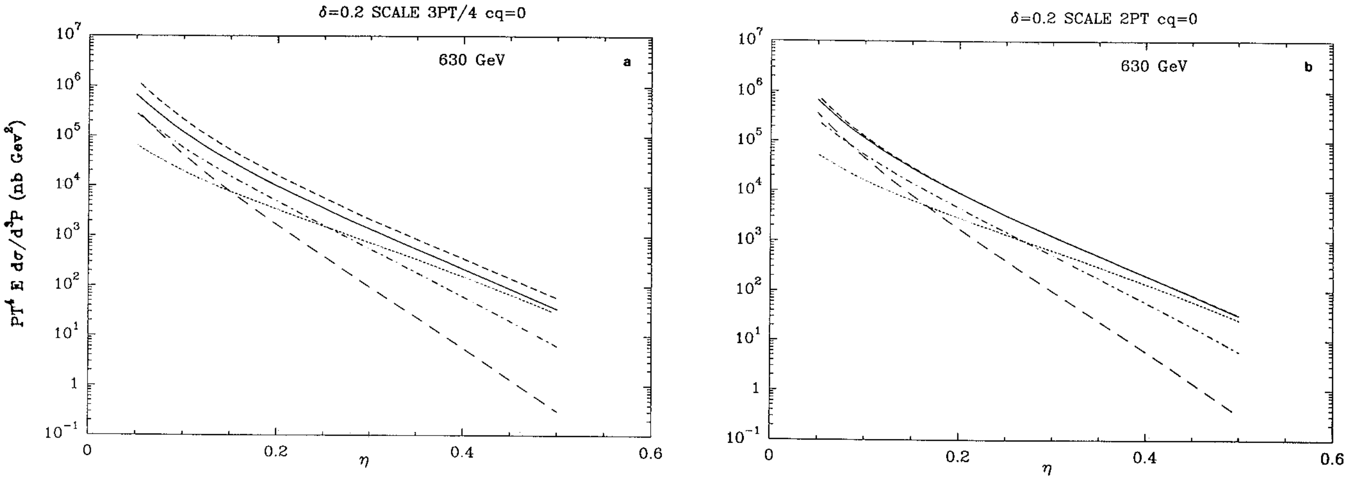

Fig. 2. **a** Same caption as for Fig. 1a for $CQ = 0$. **b** Same caption as for Fig. 1b for $CQ = 0$

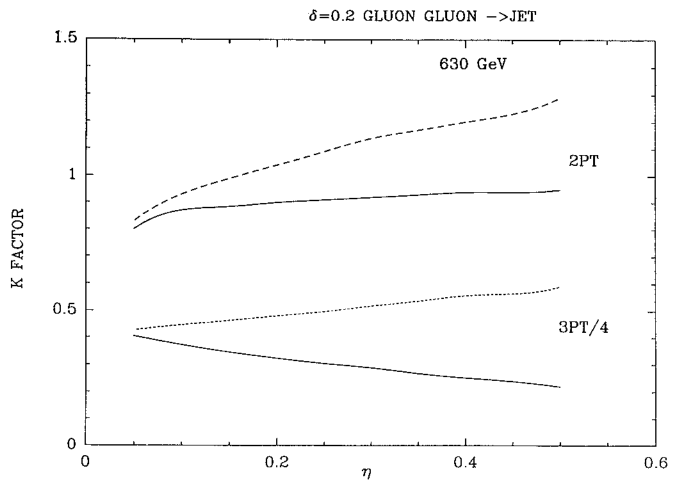

Fig. 3. The K factor for gluon–gluon subprocesses at $\sqrt{S} = 630\,\text{GeV}$ for $\delta = 0.2$. Full curves: $CQ = 1$. Dashed curves: $CQ = 0$. Upper curves: $\mu = M = 2P_T$, Lower curves: $\mu = M = 3P_T/4$

behaviour. The sensitivity to the factorization scheme affects mainly the gg subprocesses at large η—as expected—but does not show up after sum of all contributions. This is shown more in detail in Fig. 3, where the K-factor $\equiv [O(\alpha_s{}^2) + O(\alpha_s{}^3)]/O(\alpha_s{}^2)$—with α_s evaluated to 2-loops (1 loop) in the numerator (denominator)—is plotted for $M = \mu = 3P_T/4, 2P_T$. The scheme corresponding to $CQ = 1$ looks therefore more stable perturbatively. We should emphasize that at the scale $M = \mu = P_T/2$ the gluon–gluon subprocess starts to be more dependent on the factorization scheme.

In Fig. 4a–b we show the same behaviour at $\sqrt{S} = 1.8\,\text{TeV}$. Similar results have been obtained in [6], for the gluon–gluon case, with a different definition of the jet algorithm. Finally in Fig. 5a–b we give Σ at $\sqrt{S} = 40\,\text{TeV}$, for p–p collisions at scale $\mu = M = P_T$, including bottom and top thresholds (with $m_{\text{top}} = 60\,\text{GeV}$). As it could be expected, up to $P_T \sim 2\,\text{TeV}$ g–g and q–g subprocesses dominate over q–q subprocesses.

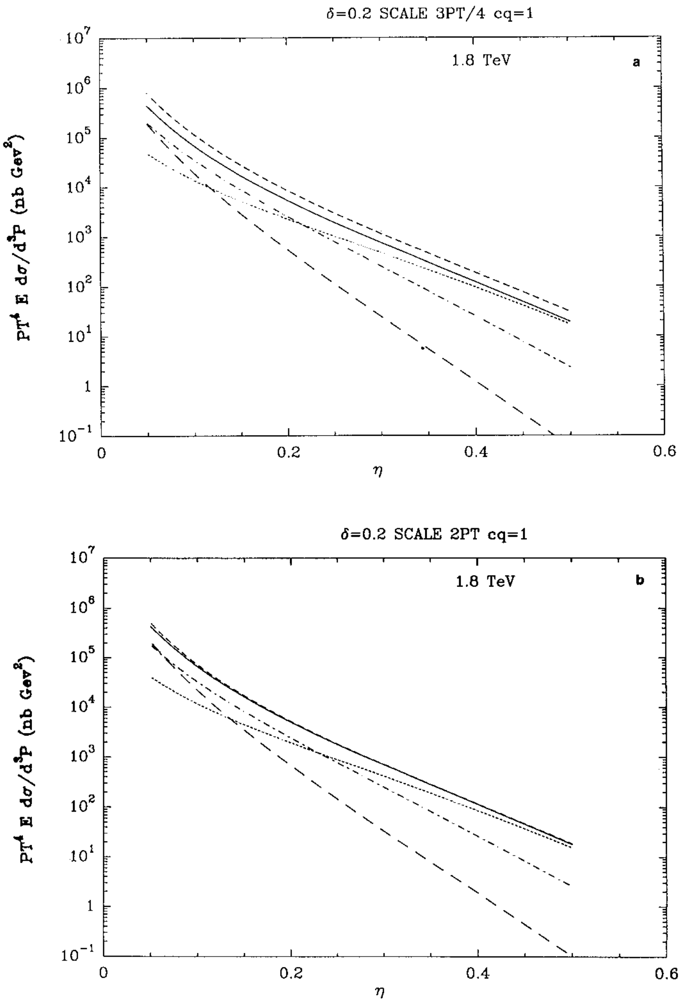

Fig. 4. **a** Same caption as for Fig. 1a at $\sqrt{S} = 1.8\,\text{TeV}$. **b** Same caption as for Fig. 1b at $\sqrt{S} = 1.8\,\text{TeV}$

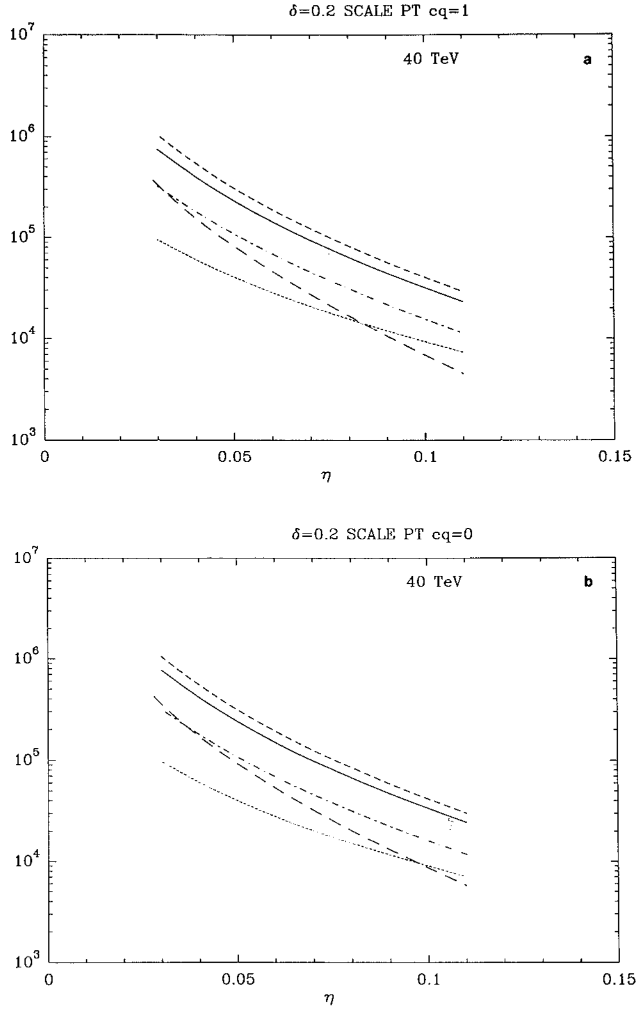

Fig. 5. **a** The scaled jet cross-section $P_T{}^4 Ed\sigma/d^3P$ from p–p collisions at $\sqrt{S} = 40\,\text{TeV}$ as a function of $\eta = 2P_T/\sqrt{s}$ for $\delta = 0.2$, $\mu = M = P_T$ and $CQ = 1$. Same caption as for Fig. 1a. **b** Same caption as for **a** for $CQ = 0$

The total contribution does depend weakly on the factorization scheme.

As a final remark, we are aware that finite corrections of $O(\delta^2)$ could affect our results, and also that present experimental data involve larger cone sizes than included in this analysis. Those problems will be discussed in a future publication, where numerical estimates for larger solid angles will be given.

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