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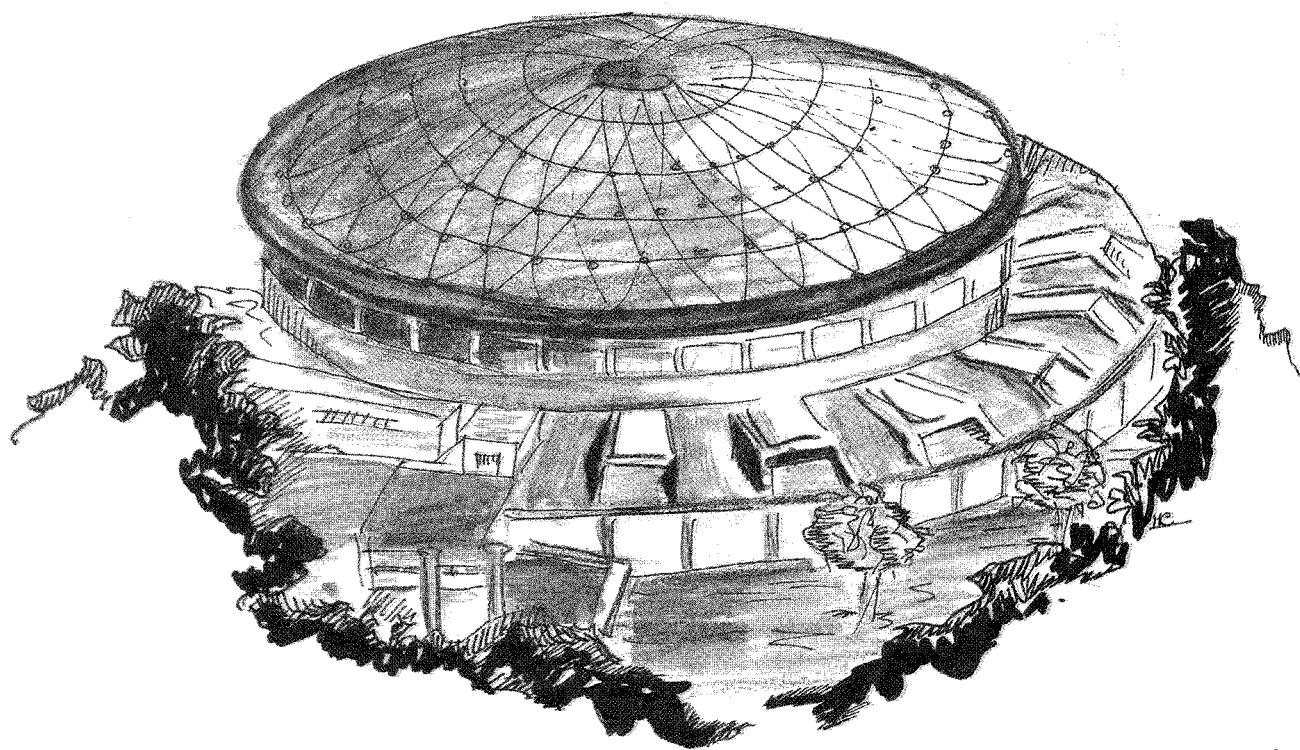
# Laboratori Nazionali di Frascati

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PROCESSES**



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## Contributions of off-shell Top Quarks to Decay Processes

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### Abstract

We quantify the contributions of decay channels proceeding through off-shell top quarks in the decay of the standard model Higgs boson and the Z boson. These contributions turn out to be very small compared to the dominant decay modes except when nearly on-shell.

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## Introduction

Decays into heavy quark pairs are potentially important modes for both the standard model Higgs boson  $H^0$  and the  $Z^0$  boson. It is customary to ignore such channels below the threshold for decay into real quarks; however, the possibility of decay via virtual heavy quarks into light final particles remains open and could in principle be significant. With new limits on the top quark mass<sup>[1]</sup> ( $m_t > 89$  GeV at 95% CL), it appears that  $m_t > M_W$  allowing decay into real W bosons, giving a large width to the top quark and making top decays an important background to W production mechanisms. In the present work we first derive explicit formulae for  $H \rightarrow t^* \bar{t}^* \rightarrow (\text{all})$ , where  $t^*$  denotes a generic top quark that may be either on or off-shell. We compare the partial width for this process with the total width and with other important off-shell decay channels, in particular  $H \rightarrow Z^* Z^* \rightarrow 4\mu$  and  $H \rightarrow W^* W^*$ . We also derive formulae for  $Z \rightarrow t \bar{t}^*$  decay, which is a possible background in the search for  $Z \rightarrow t \bar{c}$ , etc, that could arise in models with enhanced flavour-changing neutral current couplings<sup>[2]</sup>.

## Virtual Contributions to Higgs Decay

### a) $H \rightarrow t \bar{t}^*$ decays

If the Higgs mass exceeds  $m_t + m_b$ ,  $H \rightarrow t^* \bar{t}^*$  decay will be dominated by configurations where either  $t$  or  $\bar{t}$  is on the mass-shell. As a first step, we therefore examine the process

$$H \rightarrow t \bar{t}^* \rightarrow t \bar{b} f \bar{f}'$$

where  $t$  is a real on-shell top quark and  $f, f'$  are light fermions ( $u, d, s, c, e, \mu, \tau, \nu$ ).

The decay rate can be obtained as the product of the following three factors :

- (i) Phase space for the decay  $H \rightarrow t \bar{b} f \bar{f}'$  :

$$dPS(H \rightarrow t \bar{b} f \bar{f}') = dPS(H \rightarrow t \bar{t}^*) \cdot dPS(\bar{t}^* \rightarrow \bar{b} f \bar{f}') \cdot \frac{dQ^2}{2\pi}$$

where  $Q^2$  is the squared invariant mass of  $\bar{t}^*$ .

(ii) Initial state factors, which are given by

$$\frac{1}{2m_H} \text{ for } H \rightarrow t(\bar{t} \rightarrow \text{all}), \quad \frac{1}{2m_{t^*}} \frac{1}{2} = \frac{1}{4Q} \text{ for } \bar{t}^* \rightarrow \bar{b}f\bar{f}'$$

(iii) Squared matrix element for the process  $H \rightarrow t\bar{b}f\bar{f}'$ , which will contain a factor analogous to the squared matrix element for the decay  $H \rightarrow t\bar{t}$ , one for the decay  $\bar{t} \rightarrow \bar{b}f\bar{f}'$  and then extra factors like the top-quark propagator.

The matrix element is written as

$$M = g_t g_W^2 \bar{u}(t) \frac{(\not{t} - m_t)}{(Q^2 - m_t^2) + i\Gamma_{t^*} m_t} \gamma^\mu (1 - \gamma_5) v(\bar{b}) \frac{\bar{u}(f) \gamma_\mu (1 - \gamma_5) v(\bar{f}')}{(Q'^2 - M_W^2) + i\Gamma_W M_W}$$

with  $g_t = \frac{q}{2M_W} m_t$  and  $g_W = \frac{q}{2\sqrt{2}}$ .

The calculation of the squared matrix element together with the phase space factors leads to the expression

$$\Gamma(H \rightarrow t \text{ all (off-shell } \bar{t})) = \int_{m_b^2}^{(m_H - m_t)^2} \frac{Q dQ^2}{\pi} \frac{\Gamma(\bar{t}^* \rightarrow \bar{b}f\bar{f}') \Gamma(H \rightarrow t\bar{t}^*)}{(Q^2 - m_t^2)^2 + (\Gamma_{t^*} m_t)^2} \quad (1)$$

where the width into one real and one virtual quark  $t^*$ , is defined as

$$\begin{aligned} \Gamma(H \rightarrow t\bar{t}^*) &= \frac{3 g_t^2 \lambda^{1/2}(m_H^2, m_t^2, Q^2)}{8\pi m_H^3} \\ &\cdot \left[ (m_H^2 - Q^2 - 3m_t^2) + \frac{m_t^2 - Q^2}{2Q^2} (m_H^2 - m_t^2 - Q^2) \right] \end{aligned} \quad (2)$$

with  $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ .

The decay width of an off-shell top quark of mass  $Q$  has the form

$$\Gamma_Q = 9 \frac{G_F^2 Q^5}{192\pi^3} \text{ for } Q \ll m_W$$

or<sup>[3,4,5]</sup>

$$\Gamma_Q = 9 \frac{G_F^2 m_W^4}{96\pi^3 Q^3} \int dk^2 \frac{\lambda^{1/2}(Q^2, m_b^2, k^2)}{(k^2 - m_W^2)^2 + \Gamma_W^2 m_W^2} [(Q^2 - m_b^2)^2 + k^2(Q^2 + m_b^2) - 2k^4] \quad (3)$$

in the case in which the finite mass of the W-boson compares to that of the quark  $Q$ . Here we assume  $|V_{tb}| = 1$ . The factor 9 comes from summing all  $W^* \rightarrow f\bar{f}'$  channels neglecting final fermion masses and QCD corrections.

Using the narrow width approximation for the quark propagator, one can easily check that the above expression reduces to the known formula for Higgs decay into a fermion-antifermion pair<sup>[6]</sup>. A plot of the Higgs decay width for various top masses , as a function of the Higgs mass, is shown in figure 1. It is interesting to see the opening of the W-threshold, in the  $f\bar{f}'$  channel occurring at  $m_H = m_t + m_b + M_W$ .

A similar formula describes  $H \rightarrow t^*\bar{t}$  decay, with  $\bar{t}$  on-shell and  $t$  off-shell instead; these contributions are not included in Fig.1, which is therefore slightly misleading. Figure 1 and Eq.(1) give the correct on-shell limit, but only half of the off-shell contributions for  $m_H < 2m_t$ , because  $t$  has been artificially constrained to be on-shell throughout. This deficiency is remedied in the next section.

### b) $H \rightarrow t^*\bar{t}^*$ decays

For  $m_H < m_t + m_b$ , neither  $t$  nor  $\bar{t}$  can be on-shell and it is necessary to consider the full decay process

$$H \rightarrow t^*\bar{t}^* \rightarrow (bf_1\bar{f}_2)(\bar{b}f_3\bar{f}_4)$$

as shown in figure 2.

The calculation proceeds in a way rather similar to one outlined above, although the squared matrix element is not immediately factorizable into a product of separate factors as before.

The matrix element now is given by

$$M = g_t g_W^4 \bar{u}(b)\gamma^\alpha(1 - \gamma_5) \frac{(Q_1 + m_t)}{(Q_1^2 - m_t^2) + i\Gamma_{t^*}m_t} \frac{(Q_2 - m_t)}{(Q_2^2 - m_t^2) + i\Gamma_{t^*}m_t} \gamma^\mu(1 - \gamma_5)v(\bar{b}) \cdot$$

$$\cdot \frac{\bar{u}(f_1)\gamma_\alpha(1 - \gamma_5)v(f_2) \bar{u}(f_3)\gamma_\mu(1 - \gamma_5)v(f_4)}{[(K_1^2 - M_W^2) + i\Gamma_W M_W][(K_2^2 - M_W^2) + i\Gamma_W M_W]}$$

After integration over phase space we get the following expression for the Higgs width

$$\Gamma(H \rightarrow t^*\bar{t}^* \rightarrow bf_1\bar{f}_2\bar{b}f_3\bar{f}_4) = \int_{m_b^2}^{(m_H - m_b)^2} \frac{Q_1 dQ_1^2}{\pi} \frac{\Gamma(t^* \rightarrow bf_1\bar{f}_2)}{(Q_1^2 - m_t^2)^2 + (\Gamma_{t^*}m_t)^2} \cdot$$

$$\cdot \int_{m_b^2}^{(m_H - Q_1)^2} \frac{Q_2 dQ_2^2}{\pi} \frac{\Gamma(\bar{t}^* \rightarrow \bar{b}f_3\bar{f}_4)}{(Q_2^2 - m_t^2)^2 + (\Gamma_{t^*}m_t)^2} \Gamma(H \rightarrow t^*\bar{t}^*) \quad (4)$$

where the width into both  $t$  and  $\bar{t}$  off the mass shell is defined as

$$\Gamma(H \rightarrow t^* \bar{t}^*) = 3 \frac{m_t^2 g_t^2}{8\pi m_H^3} \frac{\lambda^{1/2}(m_H^2, Q_1^2, Q_2^2)}{Q_1^2 Q_2^2} \cdot \left[ \frac{1}{2} m_H^2 (Q_1^2 + Q_2^2) - \frac{1}{2} (Q_1^2 + Q_2^2)^2 - 2Q_1^2 Q_2^2 \right] \quad (5)$$

which reduces to eq.(2) when the quark  $t$  is produced on the mass shell ( $Q_1 = m_t$ ).

We show in figure 3 the decay width into two virtual  $t$  and  $\bar{t}$  together with the decay probability into one real  $t$  and one virtual  $\bar{t}^*$ . When  $m_t + m_b < m_H < 2m_t$ , the integrand of Eq.(4) is dominated by configurations where either  $t$  or  $\bar{t}$  is on-shell; in this region the  $t^* \bar{t}^*$  formula correctly gives twice the contribution of the  $t \bar{t}^*$  formula, remedying the deficiency of Fig.1.

### c) $H \rightarrow WW$ or $ZZ$ decays

Above the threshold for production of two Intermediate Vector Bosons (IVB), the search for the Higgs boson relies on the possibility of observing its decay into a pair of  $Z^0$  bosons<sup>[7]</sup>. A much grayer search area is the one below the two  $Z^0$  threshold and in that region, the possibility of using the same signature , i.e.  $H \rightarrow (\mu^+ \mu^-)(\mu^+ \mu^-)$  has been advanced. The decay width for Higgs into a pair of real or virtual  $Z^0$  bosons can be written as<sup>[8]</sup>

$$\Gamma(H \rightarrow \text{all } Z^0 Z^0) = \int_0^{m_H^2} \frac{dQ_1^2 m_Z \Gamma_Z}{\pi \left[ (Q_1^2 - m_Z^2)^2 + (m_Z \Gamma_Z)^2 \right]} + \int_0^{(m_H - Q_1)^2} \frac{dQ_2^2 m_Z \Gamma_Z}{\pi \left[ (Q_2^2 - m_Z^2)^2 + (m_Z \Gamma_Z)^2 \right]} \Gamma(H \rightarrow Z^* Z^*) \quad (6)$$

where  $\Gamma(H \rightarrow Z^* Z^*)$  represents the decay width of a Higgs Boson into a pair of virtual bosons of masses  $Q_1$  and  $Q_2$  and it is given by

$$\Gamma(H \rightarrow Z^* Z^*) = \frac{G_F m_H^3}{16\pi \sqrt{2}} \lambda^{1/2}(1, \lambda_1, \lambda_2) (1 + \lambda_1^2 + \lambda_2^2 + 10\lambda_1\lambda_2 - 2\lambda_1 - 2\lambda_2) \quad (7)$$

with  $\lambda_i = \frac{Q_i^2}{m_H^2}$ . A similar formula holds for the decay  $H \rightarrow W^+W^-$ , with

$$\Gamma(H \rightarrow W^*W^*) = \frac{G_F m_H^3}{8\pi\sqrt{2}} \lambda^{1/2}(1, \lambda_1, \lambda_2) (1 + \lambda_1^2 + \lambda_2^2 + 10\lambda_1\lambda_2 - 2\lambda_1 - 2\lambda_2) \quad (8)$$

As in the top quark case, when the threshold for real production of one intermediate vector boson is reached, the decay into two virtual IVB will be dominated by the production of one real and one virtual IVB<sup>[9,10,11]</sup>. Unlike the virtual top case, the decay of the Higgs boson into a pair of off-shell IVB can be a significant fraction of the total width, at least for Higgs bosons with a mass just below the threshold. This can be seen from fig.4 where we compare the decay width for the three processes

$$H \rightarrow Z^*Z^*$$

$$H \rightarrow (W^+)^*(W^-)^*$$

$$H \rightarrow t^*\bar{t}^*$$

above and below the IVB threshold with the total Higgs Width, for the cases  $m_t = 90$  and  $200$  GeV; the total width shown refers to the case  $m_t = 90$  Gev.

The  $H \rightarrow Z^*Z^*$  and  $H \rightarrow W^*W^*$  contributions do not cut off so sharply below threshold as the  $H \rightarrow t^*\bar{t}^*$  contribution because  $\Gamma_W$  and  $\Gamma_Z \gg \Gamma_t$  (for  $m_t = 90$  Gev). In fact  $H \rightarrow W^*W^*$  remains the dominant decay channel down to about  $m_H = 150$  Gev, where  $H \rightarrow b\bar{b}$  finally takes over.

### **Virtual top Contributions to Z Boson Decay**

#### $Z^0 \rightarrow t\bar{t}^*$ decays

Finally we consider  $Z^0$  decays into one real, one virtual top quark. There is interest in looking for FCNC decays  $Z \rightarrow t\bar{c}$ , etc that would lead to a final state with a real top quark plus very little hadronic activity (assuming  $m_t < M_Z$ , which cannot yet be excluded completely). We note that  $Z \rightarrow t\bar{t}^*$ ,  $t^*\bar{t}$  decays would lead to very similar final states, and must be considered as a potential source of background.

We show in Figure 5 a plot of the  $Z^0$  boson width as a function of the top mass, which is given by

$$\Gamma(Z \rightarrow t \text{ all}(off-shell \bar{t})) = \int_{m_b^2}^{(M_Z - m_t)^2} \frac{Q dQ^2}{\pi} \frac{\Gamma(\bar{t}^* \rightarrow \bar{b} f \bar{f}') \Gamma(Z \rightarrow t \bar{t}^*)}{(Q^2 - m_t^2)^2 + (\Gamma_{t^*} m_t)^2} \quad (9)$$

where the width into one real and one virtual quark  $t^*$ , is defined as

$$\begin{aligned} \Gamma(Z \rightarrow t \bar{t}^*) &= 3 \frac{g_z^2}{12\pi M_Z^3} \left\{ 3m_t^2(g_V^2 - g_A^2) + \frac{1}{Q^2} \left[ 2M_Z^2 - Q^2 - m_t^2 - \frac{(m_t^2 - Q^2)^2}{M_Z^2} \right] \right. \\ &\quad \left. \cdot \left[ \frac{(Q^2 + m_t^2)}{4} (g_V^2 + g_A^2) + \frac{(m_t^2 - Q^2)}{2} g_V g_A \right] \right\} \lambda^{1/2}(M_Z^2, m_t^2, Q^2) \end{aligned} \quad (10)$$

with  $g_z = \frac{g}{\cos\theta_w}$ ,  $g_A = -\frac{1}{4}$  and  $g_V = \frac{1}{4} - \frac{2}{3}\sin^2\theta_w$  to indicate the usual couplings of the  $Z_0$  bosons to the top quark. Fig.5 shows that, with present limits on the top mass, this process has a totally negligible width and cannot be expected to contribute any observable rate of background events to FCNC.

### Conclusions

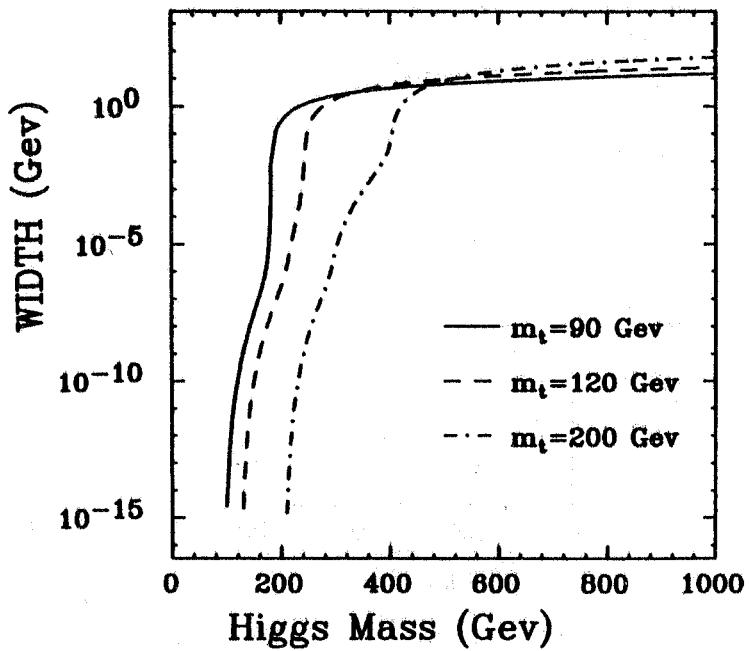
We have shown that for values of the top mass above the present CDF limits, the contribution of virtual top exchanges in the decay of the Higgs boson or of the  $Z_0$  boson is negligible compared to other dominant processes, except when nearly on-shell.

### Acknowledgments

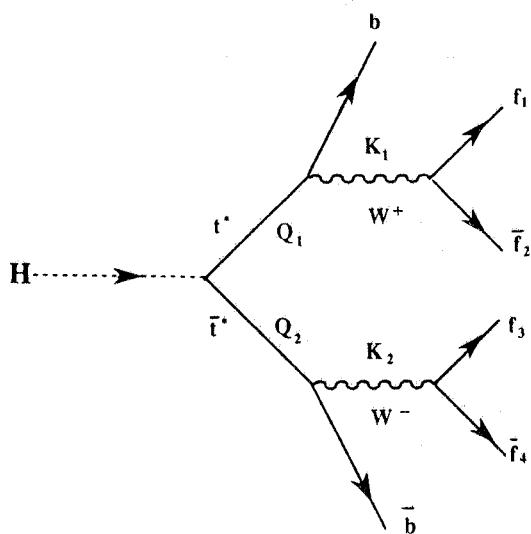
One of us (G.P.) is indebted to C.Rubbia for stimulating our interest in the problem of virtual decay processes of the Higgs boson.

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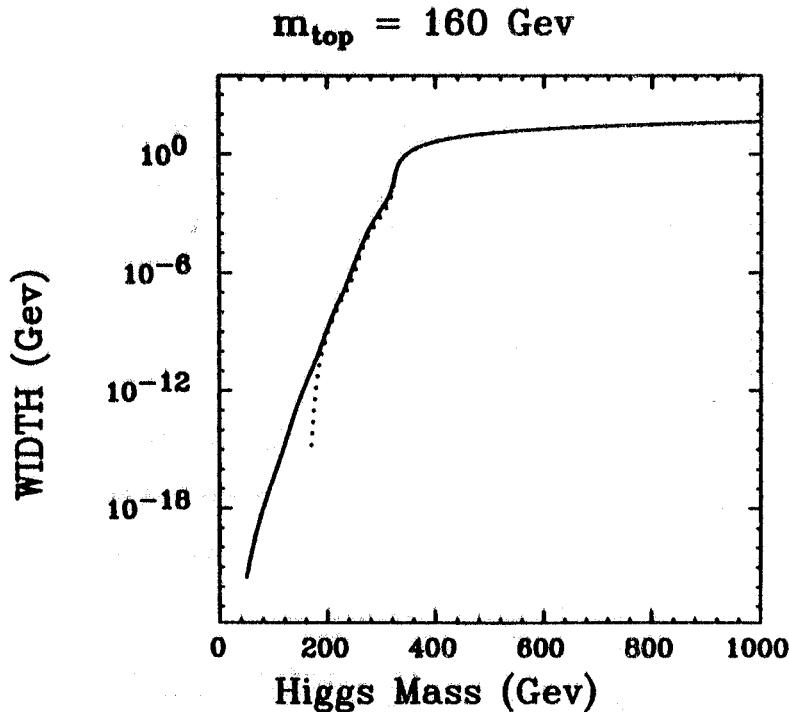
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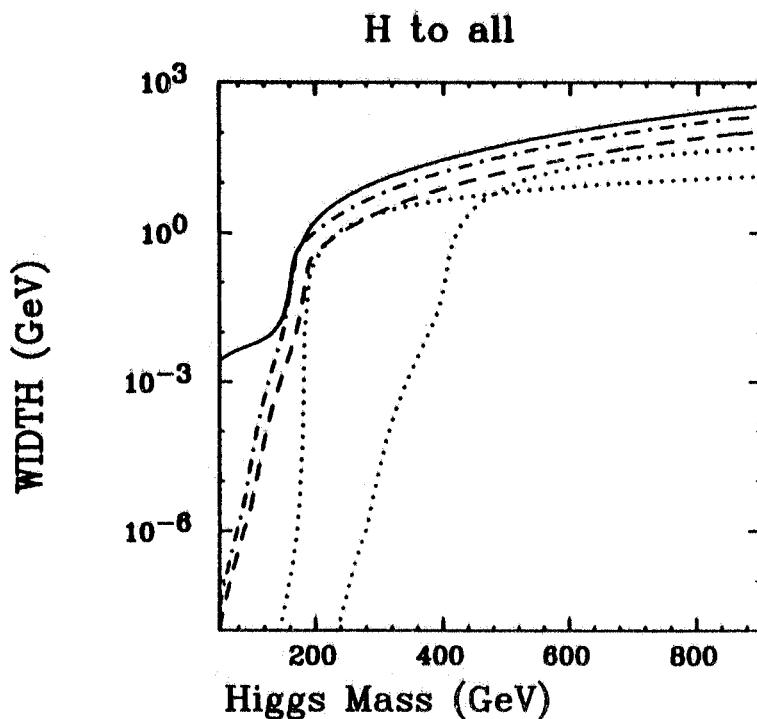
**Fig.1** Dependence of  $\Gamma(H \rightarrow t\bar{t}^*)$ , the decay width of the standard model Higgs boson into one on-shell  $t$  plus one possibly off-shell  $\bar{t}^*$  quark, versus the Higgs mass  $m_H$ . The cases  $m_t = 90, 120, 200$  Gev are illustrated.



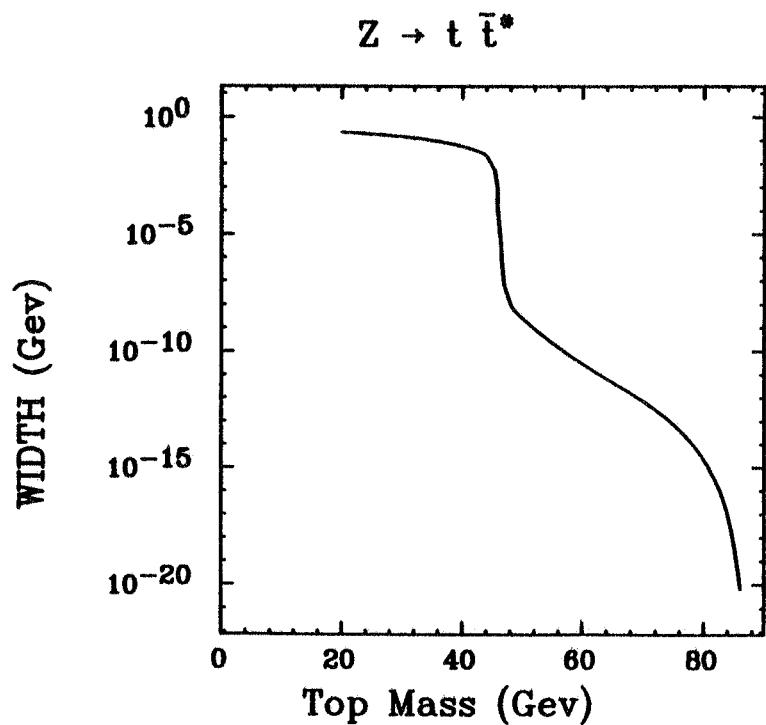
**Fig.2** Feynman diagram describing Higgs boson decay via a pair of real or virtual top quarks.



**Fig.3** The partial width for Higgs boson decay via a pair of real or virtual top quarks; the solid curve shows the complete result from Eq.(4); the dotted curve shows the partial result from Eq.(1), where one final quark is constrained to be on-shell. Results are plotted versus the Higgs boson mass  $m_H$ , for the illustrative case  $m_t = 160$  Gev.



**Fig.4** Decay widths of the standard model Higgs boson into pairs of real or virtual W-bosons, Z-bosons or top quarks. The solid curve denotes the total width (for the case  $m_t = 90$  Gev); the dot-dashed curve denotes  $\Gamma(H \rightarrow W^*W^*)$ ; the dashed curve denotes  $\Gamma(H \rightarrow Z^*Z^*)$ ; the dotted curves show  $\Gamma(H \rightarrow t^*t^*)$  for the cases  $m_t = 90$  and 200 Gev.



**Fig.5** Partial width for  $Z$  decay via a real top quark  $t$  and a virtual (off-shell)  $\bar{t}$  quark, versus top quark mass  $m_t$ .