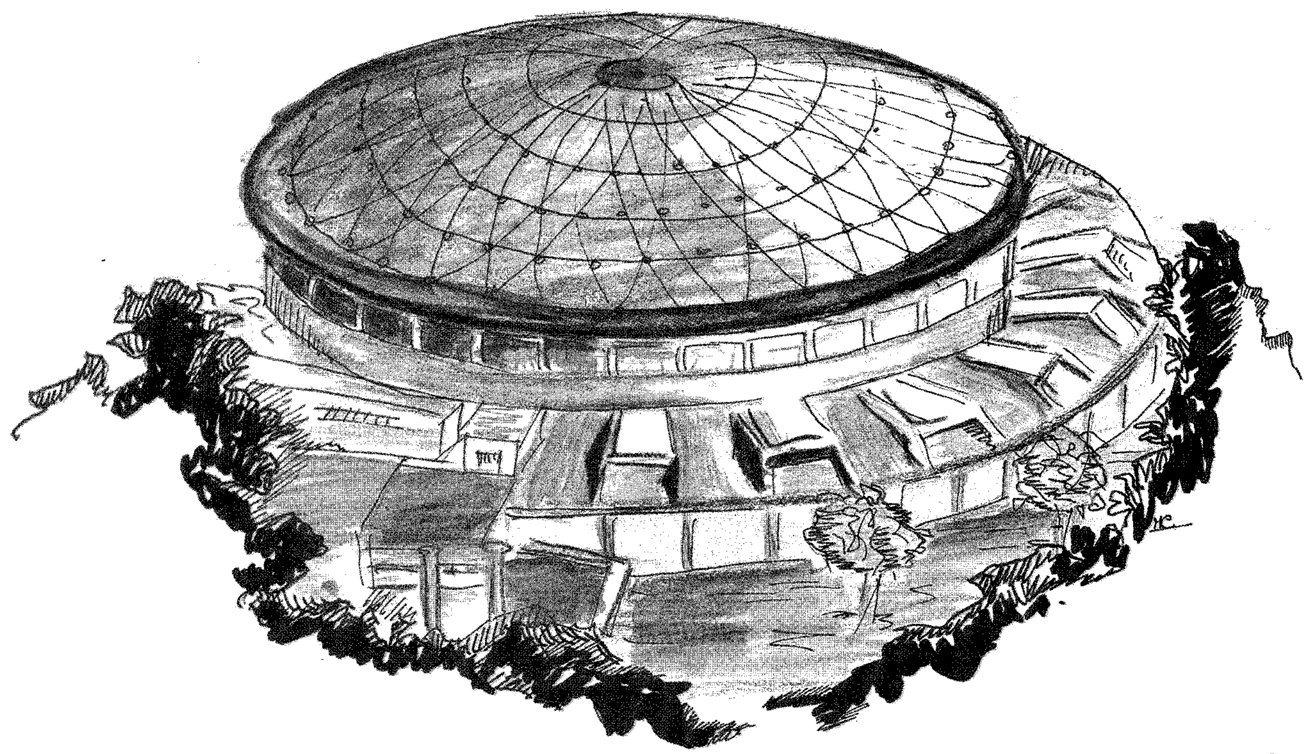
Laboratori Nazionali di Frascati

LNF-90/015(PT)
16 Marzo 1990

S. Bellucci:

ALL YOU NEED TO KNOW ABOUT CHIRAL BOSONS

Talk invited at the
Lake Louise Winter Institute
"The Standard Model and Beyond"
18-24 February 1990 Chateau Lake Louise Alberta, Canada



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ALL YOU NEED TO KNOW ABOUT CHIRAL BOSONS

S. Bellucci*
INFN-Laboratori Nazionali di Frascati, C.P. 13, I-00044 Frascati

ABSTRACT

We discuss the quantization of chiral bosons and the application to the lagrangian formulation of $D < 10$ heterotic strings.

*BITNET: BELLUCCI at IRMLNF

Over the past few years, the study of chiral bosons [1] has blossomed, in part as a consequence of progress in superstring compactification. Bosonization of a two-dimensional Dirac fermion involves a real scalar field [2]. In superstring theory it is important to apply this process also to matter fermions. In the formulation of the heterotic string the internal coordinates can be either chiral fermions or their bosonized form. The bosonization of a Weyl fermion is realized in terms of a left-moving boson, i.e. of a self-dual boson that propagates in one direction only, according to the equation of motion

$$\gamma^{\mu}\partial_{\mu}\psi = 0 \quad . \tag{1}$$

This can be rewritten as

$$(\partial_t - \partial_x)\psi = 0 \quad , \tag{2}$$

where $\psi = \frac{1}{2}(1-\gamma_5)\psi$ and we used the explicit expression of the γ-matrices $\gamma_5 = \sigma_3, \gamma^0 = \sigma_1, \gamma^1 = -i\sigma_2)$ in terms of Pauli matrices. The solution to the equation of motion (2) is then

$$\beta = \beta(x+t) \quad , \tag{3}$$

with the definition $\psi = \binom{0}{\beta}$.

Chiral bosons are building blocks for string models. Fermionization of chiral bosons makes it easy to write actions. Advantages of keeping chiral bosons, rather than fermionizing them, include:

i) the treatment of loop corrections (it is sufficient to consider lower loop graphs on the worldsheet);

ii) symmetries (e.g. $E_8 \times E_8$) are manifest only <u>*before*</u> fermionization;

iii) worldsheet supersymmetry in fermionized theories is due to an anomaly (quantum supersymmetry) [3]; thus, supermodular invariance is non trivial for free-fermion models of $D = 4$ heterotic strings (i.e. quantum supersymmetry must be realized at each loop); in the lagrangian formulation of the theory in terms of chiral bosons, supersymmetry is manifest and it is realized linearly.

The question of coupling background spin-zero fields to $D < 10$ heterotic strings (and/or superstrings) has been considered recently [4]. A solution is provided by (1,0) supersymmetric Thirring models. This solution has the following features:

– it allows the consistent introduction of *independent* right and left gauge groups,

– it possesses manifest $D = 2$ Lorentz invariance, and

– it may be interpreted as the general vertex operator for the emission or absorption of massless spin-zero states in strings.

To accomplish these goals, it has been necessary to construct a new type of two-dimensional, scale-invariant field theory: the lefton-righton Thirring model. The existence of such a model was suggested by a number of well-known facts. The usual Thirring model [5] is very familiar in terms of fermionic fields. Free fermions can be subjected to either abelian [6] or non-abelian [7] bosonization, which at the level of lagrangians leads to abelian [1,8] or non-abelian [9] chiral boson models. For constant values of the Thirring coupling [10], the Thirring model is equivalent to a set of free fermions. We will discuss here the solution to the problem of bosonization within the context of (1,0) supersymmetric theories, since our main interest is heterotic strings.

The solution we discuss in the following lends further evidence to the conjecture that two dimensional lagrangians can be constructed which contain β-function coupling constants (equivalent to vertex operators) for *every* massless mode of string theories. We emphasize that the lefton-righton Thirring model (LRTM) is not just a convenient way to demonstrate explicit spin-0 couplings to the heterotic string. If we demand manifest world-sheet supersymmetry (both classically and quantum mechanically) and complete generality, the LRTM is the only way to couple massless spin-0 background fields. Furthermore, in the case of spinning superstrings, demanding the same conditions imply spin-0 coupling can *only* be written with the use of the LRTM.

From previous studies [8,9] we recall that the (1,0) supersymmetric lefton L in the presence of a specified background field Γ_{--} is described by

$$\begin{aligned} S^*_L(L,\Lambda_+{}^{--},\Gamma_{--}) = \; & i\tfrac{1}{2}\int d^2\sigma\, d\varsigma^- \Big[(D_+L)(\partial_{--}L+2\Gamma_{--}) \\ & - \Lambda_+{}^{--}(\partial_{--}L+\Gamma_{--})^2\Big] \; . \end{aligned} \tag{4}$$

Similarly, the righton R in the presence of a background field Γ_+ is described by

$$\begin{aligned} S^*_R(R,\Lambda_{--}{}^{++},\Gamma_+) = \; & i\tfrac{1}{2}\int d^2\sigma\, d\varsigma^- \Big[(D_+R+2\Gamma_+)(\partial_{--}R) \\ & - i\Lambda_{--}{}^{++}(D_+R+\Gamma_+)D_+(D_+R+\Gamma_+)\Big] \; . \end{aligned} \tag{5}$$

To find the (1,0) supersymmetric Thirring model, we start with $\tilde{S}_{tot}$ defined by adding to the sum of the free actions an interaction term (in the presence of (1,0) supergravity [11])

$$\begin{aligned} \tilde{S}_{tot} = \; & S^*_L(L,\Lambda_+{}^{--},\Gamma_{--}) + S^*_R(R,\Lambda_{--}{}^{++},\Gamma_+) \\ & + S_{LR}(L,R,\Lambda_+{}^{--},\Lambda_{--}{}^{++}) \; . \end{aligned} \tag{6}$$

We obtained the solution in ref. [4]

$$\begin{aligned} S_{LR}(L,R,\Lambda_+{}^{--},\Lambda_{--}{}^{++}) &= i\tfrac{1}{2}\int d^2\sigma\, d\varsigma^- E^{-1}\mathcal{L}_- \;, \\ \mathcal{L}_- = \Big\{ & (\mathbf{D}_+L)\,(\nabla_{--}L+\Gamma_{--}) \; + (\nabla_+L)\,\Gamma_{--} \\ & + (\mathbf{D}_{--}R)\,(\nabla_+R+\Gamma_+) + (\nabla_{--}R)\,\Gamma_+ \\ & - 4S\,\mathbf{D}_{--}R\;\;\mathbf{D}_+L\; - 4\,i\,\Lambda_{--}{}^{++}\,(S\,\mathbf{D}_+L)\;\nabla_+(S\,\mathbf{D}_+L) \\ & - 4S^2\,\frac{\Lambda_+{}^{--}}{1-4iS^2\Lambda^2}\,\Sigma^2_{--}\Big\} \; . \end{aligned} \tag{7}$$

where

$$\begin{aligned}
\mathbf{D}_+ L &= \nabla_+ L - \Lambda_+{}^{--}(\nabla_{--}L + \Gamma_{--}) \ , \\
\mathbf{D}_{--}R &= \nabla_{--}R - i[\Lambda_{--}{}^{++}\nabla_+(\nabla_+ R + \Gamma_+) + \tfrac{1}{2}(\nabla_+\Lambda_{--}{}^{++})(\nabla_+ R + \Gamma_+)] \ , \\
\Sigma_{--} &= \mathbf{D}_{--}R + 2i[\Lambda_{--}{}^{++}\nabla_+(S\mathbf{D}_+ L) + \tfrac{1}{2}(\nabla_+\Lambda_{--}{}^{++})S\mathbf{D}_+ L] \ , \\
\Lambda^2 &= \Lambda_{--}{}^{++}\ \nabla_+\Lambda_+{}^{--} \ .
\end{aligned} \tag{8}$$

As for the bosonic counterpart, this action is invariant under two independent Siegel symmetries. Their transformation laws are given in ref. [4]. We see immediately that in comparison with the bosonic case, this action is highly asymmetrical with respect to $+ \leftrightarrow -$. This is due to the unidexterity of (1,0) superspace. A point worthy of notice is the consistent appearance of the $(\Lambda_{--}{}^{++}\nabla_+ + \frac{1}{2}(\nabla_+\Lambda_{--}{}^{++}))$ operator, which corresponds to the operator $\lambda_{--}{}^{++}\partial_{++}$ of the bosonic model. We found it useful to define this operator, within the context of chiral boson coupled to supergravity theory, already in ref. [9]. A way to understand the structure of this operator is to look at superscale invariance [4]. Note that the action is scale invariant, which is required together with supercoordinate invariance in order to have superconformal invariance. Also note that extension of these results to (1,1) superspace is a simple exercise.

It can be checked that this (1,0) supersymmetric extension contains the previous purely bosonic results [12] with no supersymmetry

$$\begin{aligned}
\mathcal{L}_B = -\tfrac{1}{2}\Big\{ & (\mathbf{D}_{++}B)(D_{--}B) + A_{--}\partial_{++}B \\
& + (\mathbf{D}_{--}C)(D_{--}C) + A_{++}\partial_{--}C \\
& - 4S\ \frac{1}{1-4S^2\lambda^2}\Big[(\mathbf{D}_{++}B)(\mathbf{D}_{--}C)\Big] \\
& + 4S^2\frac{1}{1-4S^2\lambda^2}\Big[\lambda_{--}{}^{++}(\mathbf{D}_{++}B)^2 + \lambda_{++}{}^{--}(\mathbf{D}_{--}C)^2\Big]\Big\} \ ,
\end{aligned} \tag{9}$$

with $\lambda^2 \equiv \lambda_{++}{}^{--}\lambda_{--}{}^{++}$, $\mathbf{D}_{++}B \equiv \partial_{++}B + \lambda_{++}{}^{--}D_{--}B$, $\mathbf{D}_{--}C \equiv \partial_{--}C + \lambda_{--}{}^{++}D_{++}C$, $D_\mu\phi = \partial_\mu\phi + A_\mu$.

Siegel action for chiral bosons has been analyzed from the point of view of constrained hamiltonian systems [13]. It was shown that the chiral constraints, when treated as second class, result in a classical theory which no longer possesses Siegel symmetry. From the discussion in ref. [13] it follows that $\mathcal{L}$ with the particular choice for $\lambda_{++}{}^{--}$ which is forced by the presence of the second class constraints is the correct lagrangian on which to base the quantum theory. Setting $\lambda_{++}{}^{--} = \lambda_{--}{}^{++} = -1$ in (9) we obtain (in terms of time- and space-derivatives)

$$\mathcal{L}_B = -\tfrac{1}{2}\Big\{\dot{B}B' - \dot{C}C' + \frac{1}{1-4S^2}\left[8SB'C' - (1+4S^2)(B'^2 + C'^2)\right]\Big\} . \tag{10}$$

In the particular case of constant S, introducing the field redefinitions

$$B = \alpha\tilde{B} + \beta\tilde{C} \ , \quad C = \gamma\tilde{B} + \delta\tilde{C} \ , \tag{11}$$

with α, δ nonvanishing and $\beta = f\delta$, $\gamma = f\alpha$, where f is defined as either $\frac{1}{2S}$ or $2S$, we can rewrite (10) in the form (dropping for convenience the $\tilde{}$ symbol on the B and C fields)

$$\mathcal{L}_B = -\tfrac{1}{2}(1-4S^2)\left[\alpha^2(\dot{B}B' - B'^2) - \delta^2(\dot{C}C' + C'^2)\right] \tag{12}$$

for $f = 2S$ and, respectively

$$\mathcal{L}_B = \frac{1}{8S^2}(1-4S^2)\left[\alpha^2(\dot{B}B' + B'^2) - \delta^2(\dot{C}C' - C'^2)\right] , \tag{13}$$

for $f = \frac{1}{2S}$. The lagrangian appears as the linear combination of two free lagrangians for chiral bosons with opposite chirality, and so it corresponds to a trivial theory in two dimensions. This is in agreement with the result of ref. [10].

We now turn to the non-abelian case. The starting point is ref. [9]. We restrict the leftons and rightons to be group manifold coordinates; defining the non-abelian lefton by L and the non-abelian righton by R, we may decompose the Lie-algebra valued (1,0) superform components $(\nabla_m L)L^{-1}$ and $(\nabla_m R)R^{-1}$ by $(m = +, --)$:

$$(\nabla_m L)L^{-1} = iL_m^{\hat{\alpha}} t^{\hat{\alpha}} \quad , \quad (\nabla_m R)R^{-1} = iR_m^{\hat{I}} t^{\hat{I}} \ . \tag{14}$$

Here $t^{\hat{\alpha}}$ ($t^{\hat{I}}$) is the generator of the left-(right-) symmetry group. The two group are, of course, not generally the same. For these generators, we choose

$$\begin{aligned} [t^{\hat{\alpha}}, t^{\hat{\beta}}] &= i f^{\hat{\alpha}\hat{\beta}\hat{\gamma}} t^{\hat{\gamma}} \quad , \quad [t^{\hat{I}}, t^{\hat{J}}] = i f^{\hat{I}\hat{J}\hat{K}} t^{\hat{K}} \ , \\ tr(t^{\hat{\alpha}} t^{\hat{\beta}}) &= k_L \delta^{\hat{\alpha}\hat{\beta}} \quad , \quad tr(t^{\hat{I}} t^{\hat{J}}) = k_R \delta^{\hat{I}\hat{J}} \ . \end{aligned} \tag{15}$$

We next take the free non-abelian lagrangians of [9] (lefton and righton), and put in arbitrary Siegel-gauge invariant background fields $\Gamma_{--}^{\hat{I}}$ and $\Gamma_+^{\hat{\alpha}}$. The way to put them in is unique under the requirement that both the lefton and righton actions each possess a Siegel symmetry. One finds, after setting $n_L = -1$ and $n_R = 1$ in [9] and putting the gauge invariant functions in, the lagrangian

$$\begin{aligned} \mathbf{L}_-^0 = & -k_L(L_{--}^{\hat{\alpha}} + \Gamma_{--}^{\hat{\alpha}})(L_+^{\hat{\alpha}} - \Lambda_+{}^{--}(L_{--}^{\hat{\alpha}} + \Gamma_{--}^{\hat{\alpha}})) - k_L L_+^{\hat{\alpha}} \Gamma_{--}^{\hat{\alpha}} \\ & -k_R(R_+^{\hat{I}} + 2\Gamma_+^{\hat{I}})R_{--}^{\hat{I}} + ik_R \Lambda_{--}{}^{++}(R_+^{\hat{I}} + \Gamma_+^{\hat{I}})\nabla_+(R_+^{\hat{I}} + \Gamma_+^{\hat{I}}) \\ & \qquad + \frac{2}{3} ik_R f^{\hat{I}\hat{J}\hat{K}} \Lambda_{--}{}^{++}(R_+^{\hat{I}} + \Gamma_+^{\hat{I}})(R_+^{\hat{J}} + \Gamma_+^{\hat{J}})(R_+^{\hat{K}} - \tfrac{1}{2}\Gamma_+^{\hat{K}}) \\ & - \int_0^1 dy [(\frac{d\tilde{L}}{dy}\tilde{L}^{-1})][(\nabla_{--}(\nabla_+ \tilde{L}\tilde{L}^{-1}) - (\nabla_+(\nabla_{--}\tilde{L}\tilde{L}^{-1})]] \\ & + \int_0^1 dy [(\frac{d\tilde{R}}{dy}\tilde{R}^{-1})][(\nabla_{--}(\nabla_+ \tilde{R}\tilde{R}^{-1}) - (\nabla_+(\nabla_{--}\tilde{R}\tilde{R}^{-1})]] \ . \end{aligned} \tag{16}$$

The final two terms in this action are seen to be the usual extension of the WNZW terms including the addition of the redundant coordinate y required to write such terms in closed form.

To obtain the coupled model, we add an interaction term to (16) and we find in ref. [4] (with $k^2 \equiv k_L k_R$)

$$\begin{aligned}\mathbf{L}_-^{NLRTM} = \mathbf{L}_-^0 \; &-4k^2 S^{\alpha\hat{I}}\mathbf{R}^{\hat{I}}_{--}\mathbf{L}_+^{\hat{\alpha}} + 4k^2 k_R \Lambda_+{}^{--}(M^{-1})^{\hat{I}\hat{K}} S^{\hat{\alpha}\hat{I}} S^{\hat{\alpha}\hat{J}} \Sigma^{\hat{J}}_{--}\Sigma^{\hat{K}}_{--} \\ &+4ik^2 k_L \Lambda_{--}{}^{++} S^{\hat{\alpha}\hat{I}}\mathbf{L}_+^{\hat{\alpha}}\nabla_+(S^{\hat{\beta}\hat{I}}\mathbf{L}_+^{\hat{\beta}}) \\ &-4ik^2 k_L \Lambda_{--}{}^{++} f^{\hat{I}\hat{J}\hat{K}}(\Gamma_+^{\hat{I}} + \tfrac{2}{3}k_L S^{\hat{\alpha}\hat{I}}\mathbf{L}_+^{\hat{\alpha}}) S^{\hat{\beta}\hat{J}}\mathbf{L}_+^{\hat{\beta}} S^{\hat{\gamma}\hat{K}}\mathbf{L}_+^{\hat{\gamma}} \quad ,\end{aligned} \tag{17}$$

with notations

$$\begin{aligned}\mathbf{L}_+^{\hat{\alpha}} &= L_+^{\hat{\alpha}} - \Lambda_+{}^{--}(L^{\hat{\alpha}}_{--} + \Gamma^{\hat{\alpha}}_{--}) \quad , \\ \mathbf{R}^{\hat{I}}_{--} &= R^{\hat{I}}_{--} - i[\Lambda_{--}{}^{++}\nabla_+(R_+^{\hat{I}} + \Gamma_+^{\hat{I}}) + \tfrac{1}{2}(\nabla_+\Lambda_{--}{}^{++})(R_+^{\hat{I}} + \Gamma_+^{\hat{I}}) \\ &\qquad + \tfrac{1}{2}\Lambda_{--}{}^{++} f^{\hat{I}\hat{J}\hat{K}}(R_+^{\hat{J}} + \Gamma_+^{\hat{J}})(R_+^{\hat{K}} - \Gamma_+^{\hat{K}})] \quad , \\ \Sigma^{\hat{I}}_{--} &= \mathbf{R}^{\hat{I}}_{--} - 2ik_L[\Lambda_{--}{}^{++}\nabla_+(S^{\hat{\beta}\hat{I}}\mathbf{L}_+^{\hat{\beta}}) + \tfrac{1}{2}(\nabla_+\Lambda_{--}{}^{++})S^{\hat{\beta}\hat{I}}\mathbf{L}_+^{\hat{\beta}} \\ &\qquad - \Lambda_{--}{}^{++} f^{\hat{I}\hat{J}\hat{K}} S^{\hat{\beta}\hat{J}}\mathbf{L}_+^{\hat{\beta}}(\Gamma_+^{\hat{K}} + k_L S^{\hat{\gamma}\hat{K}}\mathbf{L}_+^{\hat{\gamma}})] \quad , \\ (M)^{\hat{I}\hat{J}} &= \delta^{\hat{I}\hat{J}} - 4ik^2\Lambda^2 S^{\hat{\alpha}\hat{I}} S^{\hat{\alpha}\hat{J}} \quad ,\end{aligned} \tag{18}$$

The general coupling of background spin-zero fields to $D = 4$ compactified heterotic string σ-models follows from a specialization of the non-abelian lefton-righton Thirring model. We may write the the complete action in two pieces $S_{SM} + S_{FB}$. The space-time manifold action [9] is the familiar

$$S_{SM} = \frac{1}{2\pi\alpha'}\int d^2\sigma d\varsigma^- E^{-1}[i\tfrac{1}{2}(\eta_{\underline{ab}} + b_{\underline{ab}}(X))\Pi_+{}^{\underline{a}}\Pi_{--}{}^{\underline{b}} \; + \; \alpha'\Phi(X)\Sigma^+ \,] \quad ,$$

$$\Pi_+{}^{\underline{a}} \equiv (\nabla_+ X^{\underline{m}})e_{\underline{m}}{}^{\underline{a}}(X) \quad , \quad \Pi_{--}{}^{\underline{a}} \equiv (\nabla_{--}X^{\underline{m}})e_{\underline{m}}{}^{\underline{a}}(X) \quad , \tag{19}$$

where ∇_A and Σ^+ are the usual (1,0) supergravity derivative and supergravity field strength [11]. The fiber bundle action is obtained from our non-abelian

Thirring model subject to certain identifications:

$$
\begin{aligned}
&S_{FB} = i\tfrac{1}{2}\int d^2\sigma d\varsigma^- E^{-1}\mathbf{L}_-^{(NLRTM)}(\Gamma_{--}{}^{*\hat{\alpha}}, \Gamma_+{}^{*\hat{I}}, S_{\hat{\alpha}\hat{I}}{}^*) \ , \\
&\Gamma_+{}^{*\hat{I}} \equiv \frac{1}{\sqrt{2\pi\alpha'}}(\nabla_+ X^{\underline{m}})A_{\underline{m}}{}^{\hat{I}}(X) \ , \ \Gamma_{--}{}^{*\hat{\alpha}} \equiv \frac{1}{\sqrt{2\pi\alpha'}}(\nabla_{--} X^{\underline{m}})A_{\underline{m}}{}^{\hat{\alpha}}(X) \ , \\
&S_{\hat{\alpha}\hat{I}}{}^* \equiv \Phi_{\hat{\alpha}\hat{I}}(X) \ .
\end{aligned}
\tag{20}
$$

Here $A_{\underline{m}}{}^{\hat{\alpha}}(X)$ are the "gravi-photons" of extended supergravity theories, $A_{\underline{m}}{}^{\hat{I}}(X)$ are the "matter gauge fields" and $\Phi_{\hat{\alpha}\hat{I}}(X)$ are the "matter scalar fields." For definiteness, we may pick the $N = 4$, $SO(44)$ or $E_8 \times E_8 \times E_6$ models. We need only set $G_L = U(1)^6$ and $G_R = SO(44)$ or $E_8 \times E_8 \times E_6$.

The quantization of chiral bosons has an extremely confusing history [14] (which extends even to the present [15]). The existence of a "reparametrization anomaly" in this theory was pointed out in [14]. We offered a solution for the (1,0) supersymmetric chiral boson multiplets in refs. [8,9]. Chiral boson actions were constructed from truncated D=2 supergravity theories. We adopted a stringy prescription for curing anomalies, i.e. the anomaly was removed in the critical dimension. Coupling leftons and rightons to a curved background allowed us to reproduce the critical dimension formulas for $D < 10$ dimensional superstrings. But there appear at least two more ways ([16] and [13,17]) to quantize chiral boson theories. One of these, used in conjunction with path-integral quantization, is the introduction of Hull's no-movers [16]. At least on genus zero surfaces, this keeps world-sheet Lorentz invariance manifest. Thus, the consistent cancellation of the "Siegel anomaly" [14] requires, according to Hull, the addition of a no-mover (noton) sector. We may introduce $N_{\tilde{\eta}}$ no-movers minus spinors $\tilde{\eta}_-$, $N_{\hat{B}}$ pair superfields $\hat{B}_+^{(s',r')}-\hat{C}^{(s',r')}$ and $N_{\tilde{B}}$ pairs $\tilde{B}_{--}^{(r,s)}-\tilde{C}^{(s,r)}$. Our (1,0) supersymmetric action is

$$S_N = i\tfrac{1}{2}\int d^2\sigma d\varsigma^- E^{-1}\Big\{ \tilde{\eta}_-(\nabla_+ - \Lambda_+{}^{--}\nabla_{--})\tilde{\eta}_- +$$
$$\tilde{B}^{(r,s)}_{--}(\nabla_+ - \Lambda_+{}^{--}\nabla_{--})\tilde{C}^{(s,r)}$$
$$+ \hat{B}^{(r',s')}_+[\nabla_{--} + \Lambda_{--}{}^{++}\nabla_{++} - i\tfrac{1}{2}(\nabla_+\Lambda_{--}{}^{++})\nabla_+]\hat{C}^{(s',r')}$$
$$- \tfrac{1}{2}[(s'-r')(\nabla_{++}\Lambda_{--}{}^{++})\hat{B}_+{}^{(r',s')}\hat{C}^{(s',r')}+$$
$$(s-r)\tilde{B}^{(r,s)}_{--}(\nabla_{--}\Lambda_+{}^{--})\tilde{C}^{(s,r)}]\Big\} ,\tag{21}$$

which possesses the Siegel symmetries

$$\begin{aligned}
\delta\tilde{\eta}_- &= \xi^{--}\nabla_{--}\tilde{\eta}_- + \tfrac{1}{2}(\nabla_{--}\xi^{--})\tilde{\eta}_- \ , \\
\delta\hat{B}^{(r',s')}_+ &= [\xi^{++}\nabla_{++} - i\tfrac{1}{2}(\nabla_+\xi^{++})\nabla_+]\hat{B}^{(r',s')}_+ \\
&\quad + \tfrac{1}{2}(s'-r'+1)(\nabla_{++}\xi^{++})\hat{B}^{(r',s')}_+ \ , \\
\delta\hat{C}^{(s',r')} &= [\xi^{++}\nabla_{++} - i\tfrac{1}{2}(\nabla_+\xi^{++})\nabla_+]\hat{C}^{(s',r')} \\
&\quad - \tfrac{1}{2}(s'-r')(\nabla_{++}\xi^{++})\hat{C}^{(s',r')} \ , \\
\delta\tilde{B}^{(r,s)}_{--} &= \xi^{--}(\nabla_{--}\tilde{B}^{(r,s)}_{--}) - \tfrac{1}{2}(s-r-2)(\nabla_{--}\xi^{--})\tilde{B}^{(r,s)}_{--} \ , \\
\delta\tilde{C}^{(s,r)} &= \xi^{--}(\nabla_{--}\tilde{C}^{(s,r)}) + \tfrac{1}{2}(s-r)(\nabla_{--}\xi^{--})\tilde{C}^{(s,r)} \ ,
\end{aligned}\tag{22}$$

with $\delta\Lambda_+{}^{--}$ and $\delta\Lambda_{--}{}^{++}$ as usual [8]. The notation $\tilde{C}^{(s,r)}$ implies that $\tilde{C}^{(s,r)}$ possesses s upper $(+)$ indices and r upper $(-)$ indices and similarly for the other superfields. By choosing $N_{\tilde{\eta}}$, $N_{\tilde{B}}$, $N_{\hat{B}}$, s, r, s' and r' appropriately, cancellation of the "Siegel anomaly" is achieved.

It is worthwhile at this point to make an observation in relation to string field theory. It should be possible to write two dimensional lagrangians which provide *manifest* realization of all local symmetries of the heterotic string. We have shown by a lefton-righton Thirring model that such lagrangians naturally provide non-linear σ-model "coupling constants" which may be interpreted as the massless backgrounds of the string. But there is another reason why such lagrangians are important. In string fields theory, the linearized equation of motion of the string functional Φ is known to be $Q\Phi = 0$ where Q is the BRST operator of the corresponding first quantized string theory. The simplest way to derive this operator which includes the oscillators for _all_ of the states is from a lagrangian which manifest _all_ of the symmetries.

It was recently pointed out that, due to their noncovariance, the diagrammatic techniques used to derive the critical parameters of superstrings in $D < 10$ may not be suitable to deal with σ-models on D=2 riemannian surfaces [18]. It has been suggested [18] to use a (1,0) generalization of the proper-time technique, instead. A more fundamental problem, though, arises at this stage. This has to do with the question of the intercept, which must vanish in order for the no-movers to really not move. The basic question is to ensure that the theory is modular invariant. Naive Euclideanization does not preserve the self (or anti-self) duality condition of real chiral bosons. At the same time, one has no longer non-moving notons. The requirement of invariance under large Siegel transformation must be imposed when quantizing chiral bosons on higher genus surfaces. The cancellation of the local Siegel anomaly, achieved by Hull's trick, is not sufficient for consistency.

There is a consistent method of quantization of Siegel action for chiral bosons that circumvents the problems of Hull's trick on higher genus surfaces. We have shown [13] that the action for chiral bosons, when treated according to Dirac quantization procedure for hamiltonian systems in the presence of constraints, leads to a theory that has no Siegel anomaly, and thus leads to a consistent bosonization of chiral fermions coupled to gravity. Concentrating on the theory of $O(N)$ non-abelian chiral bosons coupled to gauge fields, we proved in ref. [17] that the chiral constraints allow coupling to only $O(N)_L$ or $O(N)_R$ gauge fields but not both, contrary to the results found in a previous attempt to incorporate gauge fields in the classical theory [19]. The anomaly structure of the theory is just what one would expect from a theory of left-handed or right-handed fermions coupled to gauge fields. In ref. [17], we treated the quantum theory by first deriving the quantum hamiltonian explicitly, and then proving that in the absence of gauge fields the physical currents of the theory satisfy the same Kac-Moody algebra as those for the corresponding free fermion theory. An essential aspect of this result is that when all constraints are resolved (including the first class constraints which vanish on physical states of the theory), the resulting hamiltonian can be derived from the action

$$\begin{aligned} S &= \int d^2x\ \mathcal{L} \\ \mathcal{L} &= -\frac{1}{8\pi}\mathrm{tr}[\partial_- g g^{-1}\partial_+ g g^{-1} - \partial_- g g^{-1}\partial_- g g^{-1}] \\ &+ \frac{1}{8\pi}\int_0^1 dy \mathrm{tr}[\partial_y \overline{g} \overline{g}^{-1}(\partial_- \overline{g}\overline{g}^{-1}\partial_+\overline{g}\overline{g}^{-1} - \partial_+\overline{g}\overline{g}^{-1}\partial_-\overline{g}\overline{g}^{-1})] \\ &- \frac{1}{4\pi}\mathrm{tr}[L_-(\partial_+ g g^{-1} - \partial_- g g^{-1})] - \frac{1}{8\pi}\mathrm{tr}[L_-(L_+ - L_-)] \\ &+ \frac{1}{8e^2}\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}] - \frac{1}{2}\frac{m^2}{e^2}\mathrm{tr}[L_+L_-]\ , \end{aligned} \tag{23}$$

where we introduced a mass term for the gauge fields. The field $g(x)$ is an element of $O(N)$. This action has no Siegel symmetry, so it is not necessary to require cancellation of a "Siegel anomaly", either at the local or at the global level. This means that we do not expect to find inconsistencies in applying our quantization procedure to chiral bosons on the circle. As opposed to Hull's case, we do have only one copy, rather than two, of the Virasoro algebra, and only one modular invariance, rather than two, to be enforced. Thus, presumably, we will be able to construct $D < 10$ heterotic string models beyond the tree-level string approximation, by combining Dirac quantization with the proper time technique of ref. [18].

The action (23) is precisely that which would follow from the gauged Siegel action if we set $\lambda = -1$. This value of λ can be found by requiring terms quadratic in the time derivative of g to be absent. The same holds for the case of chiral bosons coupled to gravity, where the value obtained for λ is more complicated. This lends some further insight to the quantization of chiral bosons. The fact that a boson is chiral implies that it has only half the degrees of freedom of an ordinary boson field. Thus, for any lagrangian describing a chiral boson of the Siegel type ϕ, a constraint $\pi_\phi = f(\phi, \phi')$ is needed, with the function f being independent from $\dot{\phi}$. In order to find the values of λ that give the lagrangian for the quantization of chiral boson theories, we require the vanishing of the coefficient of the term $\dot{\phi}^2$ in the lagrangian of the Siegel approach. We used this observation for the Thirring model with constant S above, where finding the hamiltonian in the presence of λ looks algebraically complicated. This is justified, as long as the actual details of the procedure for eliminating the extra half degrees of freedom do not really matter. In this way, we obtained the simple quadratic lagrangian (10), describing one ordinary real boson. The correspondence with the massless (and thus free) Thirring model emerges after diagonalizing the lagrangian by field redefinitions.

The classical action for D bosons $\{X^m\}$ and M lefthanded chiral bosons $\{\phi^i\}$ coupled minimally to gravity can be written

$$\begin{aligned} S &= \int d^2x\, e^{-1} \left[e_+^\mu \partial_\mu X^m e_-^\nu \partial_\nu X^n \eta_{mn} + e_+^\mu \partial_\mu \phi^i e_-^\nu \partial_\nu \phi^i + \lambda^{--} (e_-^\mu \partial_\mu \phi^i)^2 \right] \\ &= \int d^2x\, \mathcal{L} \quad , \end{aligned} \tag{24}$$

where $e_\pm^\mu$ are components of the inverse zweibein, $e = \det(e_a^\mu)$, λ^{--} is an auxiliary field and η_{mn} is a D–dimensional Minkowski metric:

$$\eta_{mn} = diag(-1, 1, \ldots, 1). \tag{25}$$

The action is invariant under the transformations

$$\begin{aligned} \delta X^m &= \xi^\mu \partial_\mu X^m, \\ \delta e_\pm^\mu &= \xi^\nu \partial_\nu e_\pm^\mu - e_\pm^\nu \partial_\nu \xi^\mu + (\xi_W \pm \xi_L) e_\pm^\mu \quad , \\ \delta \phi^i &= \xi^\mu \partial_\mu \phi^i + \epsilon^- e_-^\mu \partial_\mu \phi^i \quad , \\ \delta \lambda^{--} &= \xi^\mu \partial_\mu \lambda^{--} + 2\xi_L \lambda^{--} - e_+^\mu D_\mu \epsilon^- + \epsilon^- e_-^\mu D_\mu \lambda^{--} - \lambda^{--} e_-^\mu D_\mu \epsilon^- \quad , \end{aligned} \tag{26}$$

where ξ^μ, ξ_L, ξ_W and ϵ^- are the transformation parameters for general coordinate, local Lorentz, Weyl and Siegel transformations respectively, and D_μ denotes the appropriate covariant derivative for the Lorentz group:

$$\begin{aligned} D_\mu \lambda^{--} &= (\partial_\mu - 2\omega_\mu)\lambda^{--} \quad , \\ D_\mu \epsilon^- &= (\partial_\mu - \omega_\mu)\epsilon^- \quad . \end{aligned} \tag{27}$$

In (27) ω_μ is the spin connection explicitly given in terms of the zweibeins by

$$\begin{aligned} \omega_+ &= \quad e_-^-\partial_-(e^{-1}e_+^-) + e_-^-\partial_+(e^{-1}e_+^+) + e_+^-\partial_-(e^{-1}e_-^-) + e_+^-\partial_+(e^{-1}e_-^+) \ , \\ \omega_- &= -\left[e_-^+\partial_-(e^{-1}e_+^-) + e_-^+\partial_+(e^{-1}e_+^+) + e_+^+\partial_-(e^{-1}e_-^-) + e_+^+\partial_+(e^{-1}e_-^+)\right] \ , \end{aligned} \tag{28}$$

and the determinant e is given explicitly by $e = e_+^+ e_-^- - e_+^- e_-^+$.

Using the definition $\partial_\pm = \frac{1}{\sqrt{2}}(\partial_t \pm \partial_x)$ and demanding that the coefficient of the $\dot{\phi}^2$ term in (24) vanishes, we see that the correct lagrangian on which to base the quantum theory is given precisely by the original lagrangian with a special choice of λ^{--}:

$$\widetilde{\mathcal{L}} = \mathcal{L}(\lambda^{--} = \lambda_0^{--}) \ , \tag{29}$$

where

$$\lambda_0^{--} = -\frac{e_+^+ + e_+^-}{e_-^+ + e_-^-} \ . \tag{30}$$

In view of the condition (30) and the elimination of the field λ as a degree of freedom from the lagrangian, the new lagrangian $\widetilde{\mathcal{L}}$ no longer possesses Siegel symmetry, nor is it invariant under the standard general coordinate, Lorentz and Weyl transformations obtained by setting $\epsilon^- = 0$ in (26). Although there is no longer a Siegel invariance, the Siegel transformation is still needed to compensate for the effect of the other transformations in maintaining the condition (30). Thus to achieve full covariance of the action the field ϕ^i must have a rather unconventional transformation law, being a combination of the ordinary transformation and a compensating Siegel transformation. The criterion of the invariance of the action determines what the parameter ϵ^- of the compensating Siegel transformation must be in terms of the zweibein and the other transformation parameters. In what follows, we indicate how the parameter for the compensating transformation of Siegel symmetry $\epsilon^-(\xi^\mu, e^\mu_+, e^\mu_-)$ necessary to keep the lagrangian (29) invariant can be determined.

Using the expressions for the spin connections given in (28) and the transformation laws (26) it is easy to show that the transformation of λ_0^{--} (see (30)) is

$$\delta\left[-\frac{e^+_+ + e^-_+}{e^+_- + e^-_-}\right] = (\xi^\mu\partial_\mu + 2\xi_L)\left[-\frac{e^+_+ + e^-_+}{e^+_- + e^-_-}\right] - \frac{e}{(e^-_- + e^+_-)^2}(\partial_- - \partial_+)(\xi^- + \xi^+) \quad . \tag{31}$$

The first term on the right hand side of (31) contains the standard general coordinate and Lorentz transformations of λ^{--} according to (26), so the second term must be interpreted as a compensating Siegel transformation. Thus comparing (26) with (31) we are able to identify what equation the parameter ϵ^- must satisfy in order to keep the action invariant under general coordinate transformations (Lorentz and Weyl transformations are not affected). In general ϵ^- will have a complicated functional dependence on e^μ_a and ξ^μ and so it was computed in the conformal gauge in ref. [13]. Next, we find the functional dependence in a general gauge.

From (26) and (31) we get

$$X \;=\; -e(e^-_- + e^+_-)^{-2}(\partial_- - \partial_+)(\xi^+ + \xi^-) \quad , \tag{32}$$

where

$$\begin{aligned} X \;\equiv\; & -e^+_+\partial_+\epsilon^- - e^-_+\partial_-\epsilon^- + \epsilon^- e^+_-\partial_+\lambda_0^{--} + \epsilon^- e^-_-\partial_-\lambda_0^{--} - \lambda_0^{--} e^+_-\partial_+\epsilon^- \\ & - \lambda_0^{--} e^-_-\partial_-\epsilon^- + e^+_+\omega_+\epsilon^- + e^-_+\omega_-\epsilon^- - 2\epsilon^- e^+_-\omega_+\lambda_0^{--} \\ & - 2\epsilon^- e^-_-\omega_-\lambda_0^{--} + \lambda_0^{--} e^+_-\omega_+\epsilon^- + \lambda_0^{--} e^-_-\omega_-\epsilon^- \quad . \end{aligned} \tag{33}$$

Using (30) it is easy to see that

$$-e_+^+\partial_+\epsilon^- - e_+^-\partial_-\epsilon^- - \lambda_0^{--}e_-^+\partial_+\epsilon^- - \lambda_0^{--}e_-^-\partial_-\epsilon^- \\ = e(e_-^+ + e_-^-)^{-1}(\partial_- - \partial_+)\epsilon^- \quad , \tag{34}$$

and that

$$\begin{aligned} & e_+^+\omega_+\epsilon^- + e_+^-\omega_-\epsilon^- - 2\epsilon^- e_-^+\omega_+\lambda_0^{--} - 2\epsilon^- e_-^-\omega_-\lambda_0^{--} + \lambda_0^{--}e_-^+\omega_+\epsilon^- \\ & \qquad + \lambda_0^{--}e_-^-\omega_-\epsilon^- = A + B \quad , \\ A &= \epsilon^-[\partial_- e_+^- + \partial_+ e_+^+ - (e_+^+ + e_+^-)(e_-^+ + e_-^-)^{-1}(\partial_- e_-^- + \partial_+ e_-^+)] \quad , \\ B &= \epsilon^-(e_-^+ + e_-^-)^{-1}(\partial_- - \partial_+)\epsilon \quad . \end{aligned} \tag{35}$$

With A,B defined in (35), we have

$$\begin{aligned} & \epsilon^- e_-^+\partial_+\lambda_0^{--} + \epsilon^- e_-^-\partial_-\lambda_0^{--} + A = C + D \quad , \\ & C = \epsilon^-(e_-^+ + e_-^-)^{-1}[e_-^+(\partial_- - \partial_+)e_+^- - e_-^-(\partial_- - \partial_+)e_+^+] \quad , \\ & D = \epsilon^-(e_+^+ + e_+^-)(e_-^+ + e_-^-)^{-2}[e_-^-(\partial_- - \partial_+)e_-^+ - e_-^+(\partial_- - \partial_+)e_-^-] \quad . \end{aligned} \tag{36}$$

Recalling (35) we get

$$\begin{aligned} & B + C = \epsilon^-(e_-^+ + e_-^-)^{-1}[-e_+^-(\partial_- - \partial_+)e_-^+ + e_+^+(\partial_- - \partial_+)e_-^-] \quad , \\ & B + C + D = \epsilon^- e(e_-^+ + e_-^-)^{-2}(\partial_- - \partial_+)(e_-^- + e_-^+) \quad . \end{aligned} \tag{37}$$

We conclude that

$$X = e(e_-^+ + e_-^-)^{-1}(\partial_- - \partial_+)\epsilon^- + \epsilon^- e(e_-^+ + e_-^-)^{-2}(\partial_- - \partial_+)(e_-^- + e_-^+) \quad . \tag{38}$$

Comparing with (32) we obtain

$$\epsilon^- = -(e_-^- + e_-^+)^{-1}(\xi^+ + \xi^-) \quad . \tag{39}$$

This means, introducing

$$\xi^{\pm} = \tfrac{1}{\sqrt{2}}(\xi^0 \pm \xi^1) \ , \tag{40}$$

that

$$\delta\phi \ = \ \Big[\xi^1 - (e_-^+ - e_-^-)(e_-^+ + e_-^-)^{-1}\xi^0\Big]\phi' \ . \tag{41}$$

Equation (41) with the addition of the first two equations in (26) give the general coordinate transformations that leave the lagrangian (29) invariant. We can recast as well (41) in the form

$$\delta\phi \ = \ (e_-^+ + e_-^-)^{-1}(e_-^-\xi^+ - e_-^+\xi^-)(\partial_+ - \partial_-)\phi \ . \tag{42}$$

Note that a chiral boson does not transform like a scalar under general coordinate transformations.

Some new insight may come by considering the problem of first versus second class constraints. If we were to follow the approach of considering the constraint as first class, then we would <u>not</u> normal order the operator $\mathcal{O} = (\pi - \phi')^2$, as its normal ordering would introduce a "Siegel anomaly". This operator is truly given by the sum of terms squared, as opposed to the normal ordered operator which is not a square. As a consequence of <u>not</u> normal ordering the operator $\mathcal{O}$, we will consider, in addition to vanishing matrix elements of $\mathcal{O}$, divergent matrix elements as well. Introducing a cutoff N on the number of oscillator modes of the chiral boson field ϕ we will have a Hilbert space where the nonvanishing matrix elements for $\mathcal{O}$ are proportional to N. On the physical states, i.e. on those states obeying the condition $\mathcal{O}|phys>= 0$, we must have $(\pi - \phi')|phys>= 0$. Thus, the states that give $\mathcal{O}$-matrix elements proportional to N do not belong to the physical sector of the Hilbert space obtained by imposing the first class constraint $\mathcal{O} = 0$. Those unphysical states can be dropped and we can substitute π by ϕ' in any operator, before letting the cutoff go to infinity. Since no need for normal ordering arises, there is no trace of a "Siegel anomaly".

Thus, imposing the first class constraint in the way we described above seems equivalent to implementing the constraint $\pi = \phi'$ as second class, if there are no subtleties related to higher genus Riemann surfaces. From this viewpoint, Hull's trick consists of two compensating mistakes, i.e. normal ordering the operator and then introducing extra-fields (no-movers) [16]. For tree-level strings, following the reasoning above, one may be able to rigorously prove the equivalence of Hull's "solution" with the result of applying hamiltonian quantization to Siegel action [13,17]. When string loop corrections are taken into account some delicate issues may arise, e.g. in the regularization procedure outlined above.

In concluding, we wish to comment on two papers by Harada which appeared recently. Since everything presented there is already worked out in a more detailed and careful way in our own paper [17], we feel compelled to point out that neither of the papers in ref. [20] contains anything new. The introduction of the first paper in [20] contains the incorrect statement that in [17] we "could not get the correct mass for the massive free boson". We solved the abelian theory explicitly by diagonalizing the lagrangian and obtained the same mass for the massive degree of freedom as others, but using a different parameterization. This can be seen by introducing the relation $a = 1 + \frac{m^2}{e^2}$ between Harada's free parameter a and ours m^2. The equivalence of Siegel formulation to the formulation by Floreanini and Jackiw [21], as well as the fact that the covariant features of Siegel approach allow to couple the theory in a more straightforward way to gravity and gauge fields, is worked out in refs. [13,17]. The final result of Harada's paper (equations (12,13)) is explicitly contained in section 5 of [17]. The results derived in the second paper of [20] (where the author does not even refer to our work) were obtained already in our paper. Next, we exhibit a table of correspondence between Harada's main equations and those in our work [17]: eq. (9) corresponds to (3.8), eq. (10) to eqs. (3.7,25), eq. (11) is discussed just after (3.26), eq. (12) to eq. (3.6) 3rd line, eq. (17) to (3.31), eq. (21) to a combination of eqs. (3.8,9), eq. (26) to a combination of eqs. (3.7-9,25,26), eq. (28) to (3.34) and eq. (31) to (2.20,21). This table shows that the work in

[20] is nothing more than a mere plagiarism of our work. We stressed that the advantage in Siegel's approach is that Lorentz covariance is maintained since the beginning, in contrast to the approach by Floreanini and Jackiw, where only a restricted Lorentz invariance exists. We completed recently an investigation of the Lorentz anomaly structure of chiral boson theories quantized canonically and we found results that agree fully with those obtained for Weyl fermions in curved D=2 spacetime [22]. This will be explained in a separate publication.

Acknowledgement

I wish to thank my collaborators for their help in carrying out this research.

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