Laboratori Nazionali di Frascati
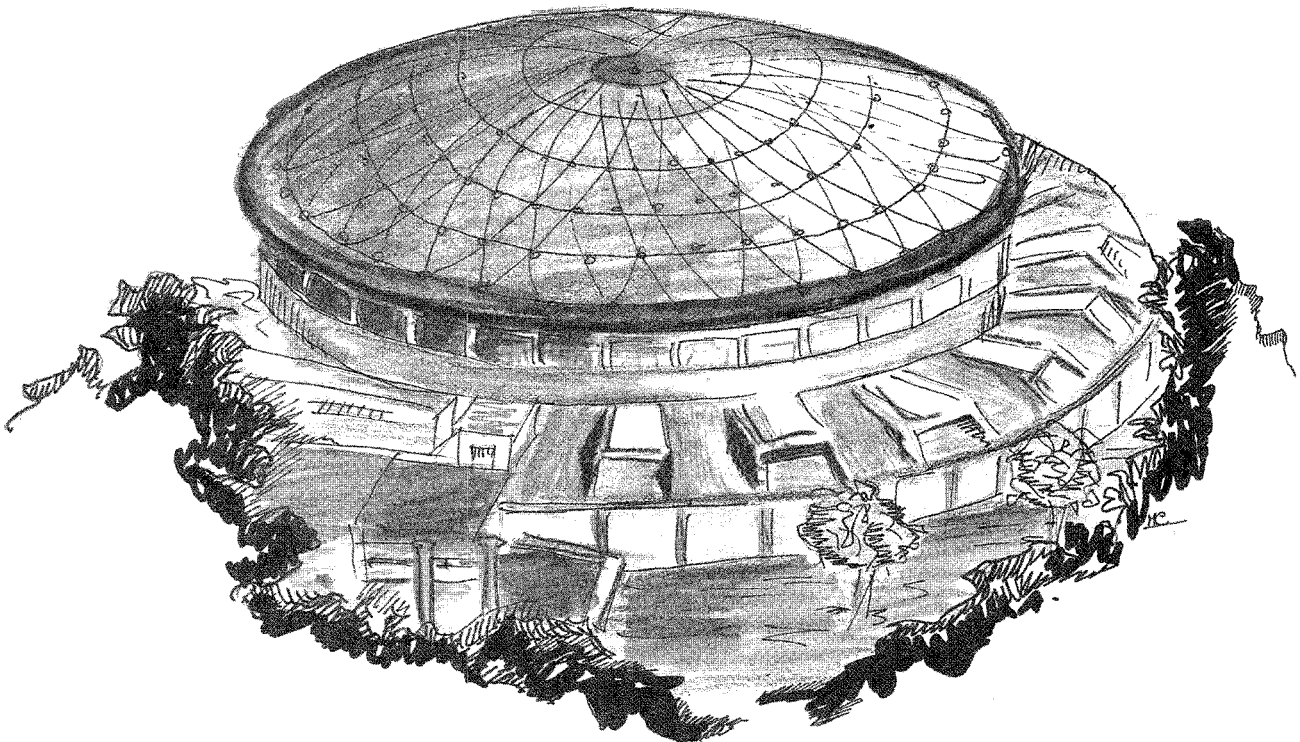
M.E. Biagini, S. Dubnicka, E. Etim, P. Kolàr:

PHENOMENOLOGICAL EVIDENCE FOR A THIRD RADIAL EXCITATION OF ρ(770)

PHENOMENOLOGICAL EVIDENCE FOR A THIRD RADIAL EXCITATION OF ρ(770)

M.E. Biagini, S. Dubnicka*, E. Etim
INFN - Laboratori Nazionali di Frascati, P.O.Box 13, I-00044-Frascati, Italy

and

P. Kolàr
Institute of Physics, Czechoslovak Academy of Sciences, Na Slovance, 18040 Prague 8, Czechoslovakia

ABSTRACT

To bring theory and experiments into more satisfactory agreement in the much studied pion form factor, and also in the kaon and nucleon form factors, especially in the time-like region , we assume the existence of a third radial excitation of ρ(770) at a mass of $m_{\rho'''} = 2150$ MeV and width $\Gamma_{\rho'''} = 320$ MeV. We interprete ρ''' (2150) therefore as one of the structures in the total cross section of e^+e^- annihilation into hadrons around the nucleon-antinucleon threshold. If the collected statistics is high enough, this resonance would be observable in the on-going FENICE experiment at Frascati, designed to investigate baryon form factors. There is thus an additional exciting challenge for FENICE.

* Present address: Joint Institute for Nuclear Research, JINR, Dubna, USSR.

The total cross section of e^+e^- annihilation into multi-pion states reveals interesting narrow and large structures in the nucleon-antinucleon ($N\overline{N}$) threshold region[1]. If some of these structures correspond to meson resonances then they would be directly observable also in the exclusive channels $e^+e^- \rightarrow N\overline{N}$, for N corresponding to the proton (p) and neutron (n). There are new, high statistics, results on $p\overline{p} \rightarrow e^+e^-$ [2] suitable to look for such structures. Much less easily, the same resonances would be observable in the reaction $p\overline{p} \rightarrow$ all[3] and in anti-proton-deuteron scattering [3] (i.e. $\overline{p}$ d $\rightarrow$ all). By far the cleanest and easiest way to look for these resonances is in the e^+e^- annihilation channel. This means, effectively, the study of the time-like behaviour of hadronic electromagnetic form factors.

We propose in this paper to show, from a phenomenological fit to the pion form factor[4], that there is a case for the existence of a third radial excitation of $\rho(770)$ of mass $m_{\rho'''} \simeq 2150$ MeV and width $\Gamma_{\rho'''} \simeq 320$ MeV. The parameters of this resonance point to the same structure identified by Clegg and Donnachie [5] in the process $e^+e^- \rightarrow 6\pi$. The main idea behind the phenomenological analysis is quite simple: one notes that in the fit to the pion time-like form factor with only the contributions of $\rho(770)$, $\rho'(1450)$ and $\rho''(1700)$, there is a net excess of the experimental values over theory for squared momentum transfer s in the integral $4.5 \leq s \leq 10(GeV)^2$. There are six experimental points in this interval: five of these points have relatively large errors[6], but the sixth point, sitting on the $\psi(3.1)$ resonance, has an extremely small error[7] and it is in agreement with the former points. Its position, taken together with those of the other five points in the interval $4.5 \leq s \leq 10(GeV)^2$, indicates an overall trend distinctly above the theoretical fit with the three isovector resonances $\rho(770)$, $\rho(1450)$ and $\rho(1700)$. A third radial excitation of $\rho(770)$ would improve the theoretical fit in the right direction. In more details the first five points are achieved by measuring $\sigma_M(s) = \sigma(e^+e^- \rightarrow \pi^+\pi^-) + \sigma(e^+e^- \rightarrow K^+K^-)$, which is much easier from an experimental point of view, and deducing the pion form factor according to a theoretical estimation of the K/π ratio. However our conclusions are valid also for σ_M. An even more relevant case would arise if the excess in cross section would be due to the kaon form factor. The sixth point is elicited according to the standard hypothesis that the J/ψ coupling with the photon is responsible of the G-parity violation decay $J/\psi \rightarrow \pi^+\pi^-$. Therefore the pion form factor is deduced from the reduction in the amplitude for the J/ψ.

We first present the fits and then describe how they were obtained. In Fig. 1 we plot the pion form factor for both space-like and time-like momentum transfer squared s. The theoretical curve is based on a vector meson dominance model (VMD) with the three isovector mesons $\rho(770)$, $\rho'(1450)$ and $\rho''(1700)$[8]. This curve is distinctly below the eperimental data in the region $4.5 \leq s \leq 10(GeV)^2$. In Fig. 2 we exhibit our improved fit to the same data, based on a VMD model with four isovector mesons, namely the previous three plus an additional one ρ''' at a higher energy. For ρ''' we find

$$m_{\rho'''} = 2169 \pm 48 \text{ MeV}$$

$$\Gamma_{\rho'''} = 320 \pm 147 \text{ MeV} \tag{1}$$

$$\frac{g_{\rho'''\pi\pi}}{f_{\rho'''}} = 0.524 \pm 0.0065$$

where $g_{V\pi\pi}$ ($V \equiv \rho, \rho', \rho'', \rho'''$) is the coupling of the vector meson V to $\pi^+\pi^-$, while m^2_V/f_V is the coupling to the photon, m_V is the vector meson mass. As a check of the ρ''' parameters obtained in this way, we have performed a simultaneous fit to the pion and kaon form factors with four isovector mesons. The ρ''' mass and width come out to be

$$m_{\rho'''} = 2153 \pm 37 \text{ MeV}$$

$$\tag{2}$$

$$\Gamma_{\rho'''} = 389 \pm 79 \text{ MeV}$$

All the quoted errors are the standard MINOS [9] ones.

Predictions for the total cross sections $\sigma(e^+e^- \to \pi^+\pi^-)$ and $\sigma(e^+e^- \to K^+K^-)$ are given in Fig. 3. Their sum $\sigma_M(s)$ too exhibits distinct resonance structures including the one corresponding to $\rho'''(2150)$ and it is within the capabilities of the non-magnetic FENICE detector.

We now turn to the description of the model. The starting point is the VMD model expression

$$F_h(s) = \sum_v \frac{g_{vh\bar{h}}}{f_v} \frac{m^2_v}{m^2_v - s} \tag{3}$$

for the form factor $F_h(s)$ of the hadron h. For $h \equiv$ a meson (i.e. $\pi^\pm$, $K^\pm$, K°, $\bar{K}^\circ$), the asymptotic behaviour of $F_h(s)$ given by eq(3) i.e.

$$F_h(s) \xrightarrow[s \to \infty]{} \frac{1}{s} \tag{4}$$

is consistent with the quark parton model prediction (i.e. quark counting rule)[10]. We limit ourselves to this case (i.e. $h \equiv \pi^\pm, K^\pm, K^\circ, \bar{K}^\circ$). Let s_0 be the threshold for $h\bar{h}$ production and s_1 the threshold for inelastic contributions to $F_h(s)$. As it stands, eq(3) contains no information related to these thresholds. These thresholds characterise, though not completely, the analyticity of the form factor. According to eq(3) on the other hand the only singularities of $F_h(s)$ are

simple poles on the real axis. The VMD formula therefore represents the analyticity behaviour of the form factor rather poorly. The fact of the simple poles being on the real axis can be remedied by including the widths Γ_v of the vector mesons. This 'ad hoc' shift in the positions of the poles has no relationship with the threshold cuts in the structure of the form factor. The complex poles destroy the property of real analyticity of the form factor exhibited by the expression for $F_h(s)$ in eq(3).

Eq(3) can nevertheless be modified firstly to incorporate implicitly the analytic cut structure represented by the thresholds s_0 and s_1. Secondly the modification can be so carried out that its real analytic property is preserved. The mathematical procedure is as follows[8]: one operates the change of variables

$$s = s_0 - 4 \frac{\left(s_1 - s_0\right)}{\left(u - \frac{1}{u}\right)^2} \tag{5}$$

in eq(3). In terms of the new variable u, the form factor becomes

$$F_h(s(u)) = \frac{\left(1 - u^2\right)^2}{\left(1 - u_0^2\right)^2} \sum_v \frac{g_{vh\bar{h}}}{f_v} \frac{\left(u_o - u_v\right)\left(u_o + u_v\right)\left(u_0 - \frac{1}{u_v}\right)\left(u_0 + \frac{1}{u_v}\right)}{\left(u - u_v\right)\left(u + u_v\right)\left(u - \frac{1}{u_v}\right)\left(u + \frac{1}{u_v}\right)} \tag{6}$$

u_0 is the value of u for $s = 0$ and u_v the value of u for $s = m^2_v$. Eq(6) is in a factorised form: the over all factor $(1 - u^2)^2$ outside of the sum over the vector mesons, determines the asymptotic behaviour of $F_h(s(u))$ for $s \to \infty$. This is so because $s \to \infty$ corresponds to $u \to \pm 1$, according to eq(5). The second factor is represented by the remaining V-dependent terms. Obviously, these terms determine the analytic behaviour of $F_h(s(u))$. The factorisation of $F_h(s(u))$ allows thus to separate its dominant asymptotic behaviour from its analytic behaviour in the finite s-plane. Of the two behaviours, the analytic one is the less well known. It is the one which is subjected to approximations. We do this also here. More precisely, to incorporate the asymptotic behaviour

$$F_h(s) \xrightarrow[|s| \to \infty]{} s^{-\left(N_q - 1\right)} \tag{7}$$

of form factors predicted by the quark counting rule[10] for a hadron h containig Nq valence quarks, we replace eq(6) by

$$F_h(s(u)) = \left(\frac{1-u^2}{1-u_0^2}\right)^{2(N_q-1)} \sum_v \frac{g_{vh\bar{h}}}{f_v} \frac{(u_o - u_v)(u_o + u_v)\left(u_0 - \frac{1}{u_v}\right)\left(u_0 + \frac{1}{u_v}\right)}{(u - u_v)(u + u_v)\left(u - \frac{1}{u_v}\right)\left(u + \frac{1}{u_v}\right)} \tag{8}$$

$N_q = 2$ corresponds to mesons. In this case eq(8) reduces to (6).

Now the change of variables in eq(5) is characterised by two symmetries. They are the inversion R: $u \to 1/u$ and the reflection or parity P: $u \to -u$. Both R and P leave the function $s(u)$ invariant. Consequently, for given s the roots u_i ($i = 1,2,3,4$) of the quartic equation

$$\left(u - \frac{1}{u}\right)^2 + 4\frac{(s_1 - s_0)}{(s - s_0)} = 0 \tag{9}$$

are given by

$$u_1 := u$$

$$u_2 := Ru_1 = +\frac{1}{u}$$

$$u_3 := Pu_1 = -u \tag{10}$$

$$u_4 := PRu_1 = RPu_1 = -\frac{1}{u}$$

Explicitly, the roots are

$$u = \left(\frac{s_1 - s_0}{s - s_0}\right)^{\frac{1}{2}}\left[\pm\left(\frac{s - s_1}{s_1 - s_0}\right)^{\frac{1}{2}} \pm i\right] \tag{11}$$

so that for $s < s_1$, $Pu = -u = u^*$ while for $s > s_1$, $Ru = 1/u = u^*$. Making use of these relationships, eq(8) can be rewritten as[8]

$$F_h(s(u)) = \left(\frac{1-u^2}{1-n_0^2}\right)^{2(N_q-1)}\left[\sum_j \frac{g_{jh\bar{h}}}{f_j} \frac{(u_0 - u_j)\left(u_0 - u_j^*\right)\left(u_0 - \frac{1}{u_j}\right)\left(u_0 - \frac{1}{u_j^*}\right)}{(u - u_j)\left(u - u_j^*\right)\left(u - \frac{1}{u_j}\right)\left(u - \frac{1}{u_j^*}\right)} + \right.$$

$$\left. + \sum_k \frac{g_{kh\bar{h}}}{f_h} \frac{(u_0 - u_k)\left(u_0 - u_k^*\right)(u_0 + u_k)\left(u_0 + u_k^*\right)}{(u - u_k)\left(u - u_k^*\right)(u_0 + u_k)\left(u + u_k^*\right)}\right] \tag{12}$$

The merit of eq(12) is that it is explicitly in the form of a real analytic function. In this form it lends itself to an analytically acceptable generalisation which allows to incorporate vector meson resonance widths. This is done through the standard replacement $m^2_v \rightarrow (m_v - i \, \Gamma_v /2)^2$ The complex masses are on unphysical sheets. Eq(5) is thus to be understood more generally as a transformation relating two complexs variables i.e., s and u.

For the pion form factor $F_\pi(s)$, one makes use of eq(12) with Nq = 1 and with four contributing vector mesons $\rho(770)$, $\rho'(1450)$, $\rho''(1700)$ and $\rho'''(2150)$.

The resulting fit to the data is in Fig. (2) and we obtain a marked improvement in the χ from 382 to 343 for 276 degrees of freedom[8]. We interprete this improvement as an argument in favour of the existence of the radial excitation $\rho''(2150)$.

For the kaon form factors $F_{K^+}(s)$ and $F_{K^\circ}(s)$ one first decomposes them into their isoscalar $F^s_K(s)$ and isovector $F^v_K(s)$, parts, that is,

$$F_{K^+}(s) = F^s_K(s) + F^v_K(s)$$

$$F_{K^\circ}(s) = F^s_K(s) - F^v_K(s)$$

(13)

The normalisations of these form factors at s = 0 are

$$F_{K^+}(0) = 2 \, F^s_K(0) = 1$$

(14.a)

$$F_{K^\circ}(0) = F^v_K(0) = 0$$

(14.b)

Eq (12) is then applied to $F^s_K(s)$ and $F^v_K(s)$ with Nq = 1. Thus

$$F^s_K(s(u)) = \left(\frac{1 - u^2}{1 - u^2_0} \right)^2 \left[\sum_{j = \omega, \phi} \frac{g_{jK\bar{K}}}{f_j} \frac{(u_0 - u_j)(u_0 - u^*_j)\left(u_0 - \frac{1}{u_j}\right)\left(u_0 - \frac{1}{u^*_j}\right)}{(u - u_j)(u - u^*_j)\left(u - \frac{1}{u_j}\right)\left(u - \frac{1}{u^*_j}\right)} + \right.$$

(15)

$$\left. + \frac{g_{jK\bar{K}}}{f_{\phi'}} \frac{(u_0 - u_\phi)(u_0 - u^*_\phi)(u_0 + u_\phi)(u_0 + u^*_\phi)}{(u - u_\phi)(u - u^*_\phi)(u - u_\phi)(u - u^*_\phi)} + \right]$$

$$F_K^v(s(u)) = \frac{\left(1 - u^2\right)^2}{\left(1 - u_0^2\right)} \left[\frac{g_{\rho K\bar{K}}}{f_\rho} \frac{\left(u_0 - u_\rho\right)\left(u_0 - u_\rho^*\right)\left(u_0 - \frac{1}{u_\rho}\right)\left(u_0 - \frac{1}{u_\rho^*}\right)}{\left(u - u_\rho\right)\left(u - u_\rho^*\right)\left(u - \frac{1}{u_\rho}\right)\left(u - \frac{1}{u_\rho^*}\right)} + \right.$$

$$\left. + \sum_{j = \rho',\rho'',\rho'''} \frac{g_{j K\bar{K}}}{f_j} \frac{\left(u_0 - u_j\right)\left(u_0 - u_j^*\right)\left(u_0 + u_j\right)\left(u_0 + u_j^*\right)}{\left(u - u_j\right)\left(u - u_j^*\right)\left(u + u_j\right)\left(u + u_j^*\right)} \right]$$

(16)

It turns out, from the fit to the kaon form factors, that

$$\frac{g_{\rho'' K\bar{K}}}{f_{\rho''}} = -0.005 \pm 0.003$$

(17)

$$\frac{g_{\rho''' K\bar{K}}}{f_{\rho'''}} = --0.020 \pm 0.004$$

This means that $\rho'''(2150)$ is more strongly coupled to the $K\bar{K}$ pair than $\rho''(1700)$. In fact a good fit to the kaon form factor is obtained also from the contributions of $\rho(770)$, $\rho'(1450)$ and $\rho'''(2150)$ only. We interprete this again as additional argument in favour of the existence of $\rho'''(2150)$. On the basis of these analyses of the pion and kaon form factors we claim that the $\rho'''(2150)$ resonance structure in the combined cross section $\sigma_M(s)$ is detectable, even in a non-magnetic detector, if the collected statistics is high enough. The same claim applies to the observability of $\rho'''(2150)$ in the threshold region of proton and neutron form factors. The FENICE experiment[11] now running at Frascati, has thus an additional challenge.
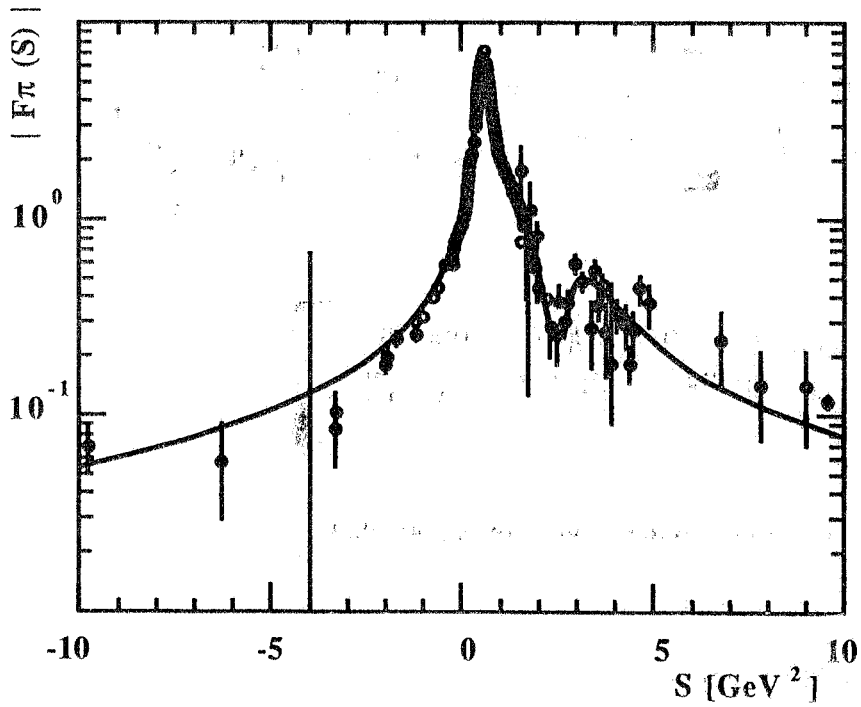
FIG. 1 - Comparison of the VMD model for the modulus of the pion form factor with data. The contributing vector mesons are: ρ(770), ρ'(1450) and ρ"(1700).
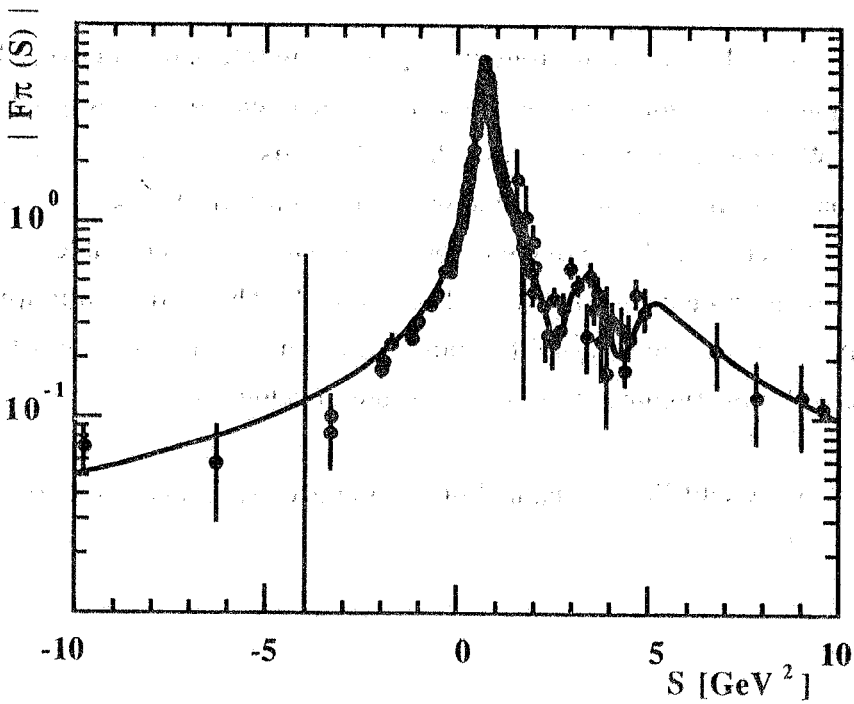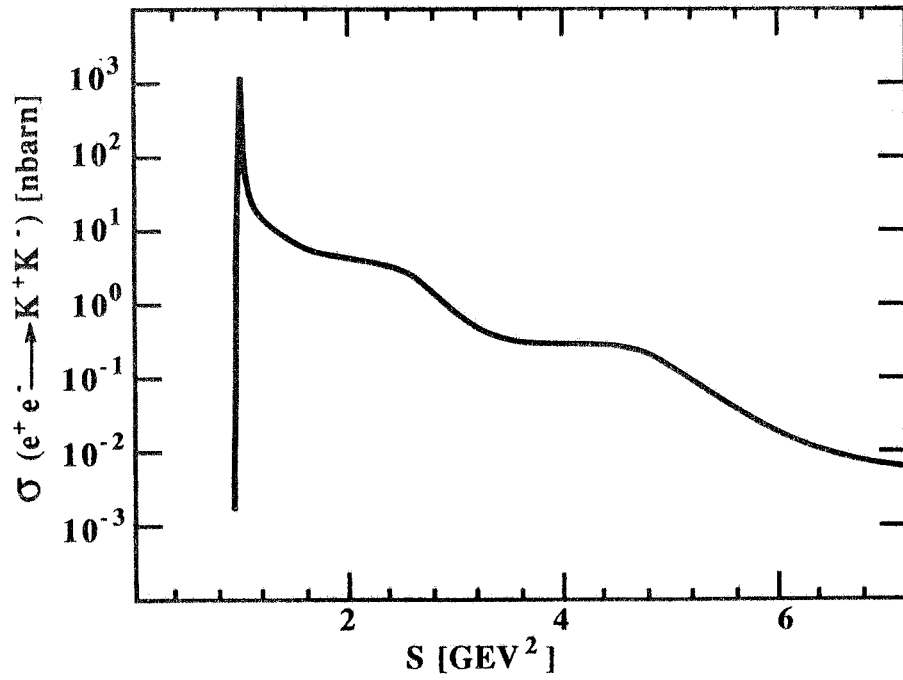

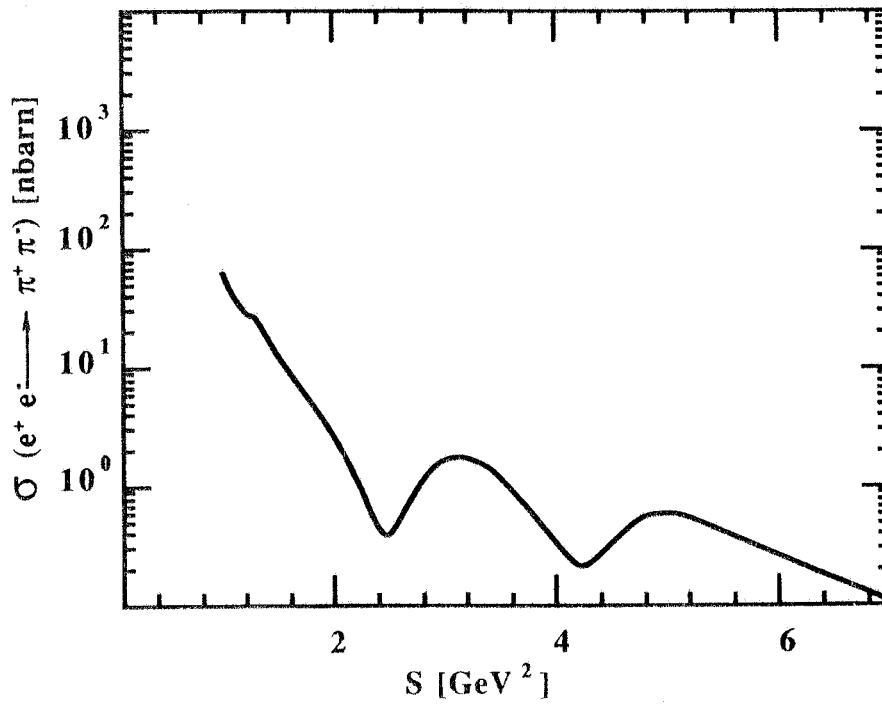FIG. 2 - Same as Fig. 1 but with the contributions of four vector mesons, i.e. ρ(770), ρ'(1450), ρ"(1700) and .ρ'"(2150).

FIG. 3 -The VMD prediction for the total cross sections (a) $\sigma(e^+e^- \to \pi^+\pi^-)$, (b) $\sigma(e^+e^- \to K^+K)$ with four contributing isovector mesons.

REFERENCE

(1) R. Baldini-Ferroli; Proceedings of the Nucleon Structure Workshop: FENICE Experiment and Investigation of the Neutron Form Factor; Frascati, 1988

(2) PS 170 Collaboration, Proc. of the Nucleon Structure Workshop: FENICE Experiment and Investigation of the Neutron Form Factor, Frascati, 1988 (in press).

(3) R.J. Abrams, R.L. Cool, G. Giacomelli, T.F. Kycia, B.A. Leontic, K.K. Li and D.N. Michael; Phys. Rev. Letters 18, 1209 (1967).

(4) S.Dubnicka, Nuovo Cimento A100, 1 (1988).

(5) A.B. Clegg and A. Donnachie; Manchester Preprint M/C - TH 89/27 (Manchester, 1989). See also: A. Donnachie and H. Mirzaie, Z. Phys. C33, 407 (1987); A. Donnachie and A.B. Clegg, Z. Phys. C34, 257 (1987).

(6) D.Bollini et al., Lett. Nuovo Cimento 14, 418 (1975).

(7) Baltrusaitis et al., Phys. Rev. D32, 566 (1985).

(8) S. Dubnicka and E. Etim; Proc. of the Nucleon Structure Workshop: FENICE Experiment and Investigation of the Neutron Form Factor, Frascati, 1988. See Frascati Preprint: LNF-89/013(PT).

(9) F.James, M.Roos, MINUIT; Cern Library D506.
F.James, M.Roos, Comp. Phys. Comm. 10, 343 (1975).

(10) S.J. Brodsky and G.R. Farrar; Phys. Rev. Lett. 31, 1153 (1973), G.R. Farrar and Jackson; Phys. Rev. Lett. 43, 24B (1979). G.P. Lepage and S.J. Brodsky, Phys. Lett. 87B, 359 (1979); Phys. Rev. D22, 2157 (1980) and D24, 2848 (1980).
V.A. Matreev, R.M. Murodyaw and A.N. Tavkhelidze; Lettere Nuovo Cimento 7, 719 (1973).

(11) C. Voci and P. Sartori; Proc. of the Nucleon Structure Workshop: FENICE Experiment and Investigation of the Neutron Form Factor, Frascati, 1988.