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CONSISTENT AND UNIVERSAL INCLUSION OF THE LORENTZ CHERN-SIMONS FORM IN D=10, N=1 SUPERGRAVITY THEORIES

Consistent and Universal Inclusion
of the Lorentz Chern-Simons Form
in D=10, N=1 Supergravity Theories [1]

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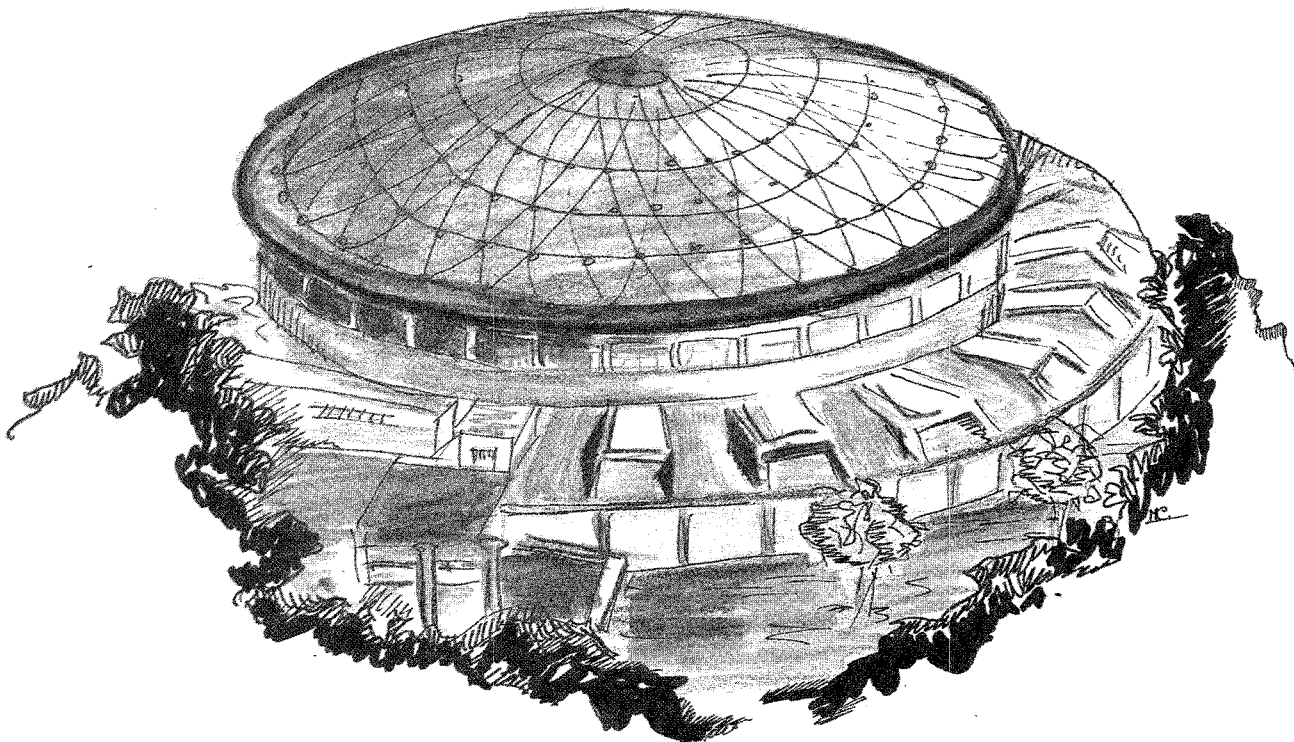
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Abstract

A general method for the manifestly supersymmetric inclusion of the Lorentz Chern-Simons form into all possible consistent formulations of the standard D=10, N=1 supergravity theory is presented. The inequivalence of our results to a construction due to Bonora, Pasti and Tonin and/or Ferrara, Fré and Porrati is discussed.

1. Introduction

A problem that arose in the context of superstring-modified $D = 10, N = 1$ supergravity theories [1] is that of maintaining manifest supersymmetry. The original discussion explicitly breaks spacetime supersymmetry because the modifications and additions suggested to the $D = 10$ supergravity action are not consistent with an on-shell "tensor calculus." To alleviate this problem, two lines of investigation were initiated. One, a component approach [2], was partially successful in extending the Green-Schwarz results. The other [3] utilized superspace and a duality technique and led to complete superspace results. This also served to establish some general principles to use to incorporate string corrections into superspace by use of slope-parameter expansions. This technique [4] was used to describe superstring corrections up to third order in α', the string slope-parameter. To apply directly to the string effective action requires a duality transformation on the dual theory [5]. In subsequent development of the ⌣uperspace method we obtained the instanton number for supersymmetric Yang-Mills theories [6].

After the initial superspace technique was developed, another seemingly similar technique was suggested [7]. It has been claimed that this alternate suggestion offers a consistent description of the the string corrections. However, as we will discuss shortly, this suggestion is unlike any of the conventional Wess-Zumino superspace constructions. This raises questions about its component level consistency.

When we began the superspace-supergravity-superstring program [3] the main reason was to investigate a question about the appearance of the Gauss-Bonnet combination and the Green-Schwarz mechanism. Since then a remarkable development [8] occurred. It has been shown that a covariant quantization of the Green-Schwarz action, when treated as a nonlinear σ-model, leads to superspace torsion and curvature supertensors which must satisfy certain conditions dictated by the conformal invariance of the σ-model. The solutions to these conditions are precisely the superspace torsions and curvatures given as slope-parameter expansions. Thus, the slope-parameter expansion corresponds to the quantum loop expansion of the β-functions in the GS σ-model! This answers a question which was raised in the first work of ref. [4]. There it was noted that some "stringy" principles involving an infinite dimensional group must determine the various terms in the slope-parameter expansion of superspace. It is clear now that the conformal group together with the renormalization group of the GS σ-model are the required "stringy" principles.

The first manifestly consistent Lorentz Chern-Simons form modified theory has only recently been given [9]. To accomplish this, a special formulation (CM) that allowed us to obtain part of the off-shell D=10 supergravity multiplet [10] was used.[4] This formulation contains a symmetry which implies that the spin-connection of the $SO(1,9)$ Lorentz group and the $E_8 \otimes E_8$ gauge-connection may be treated equivalently. This symmetry was known to be possible in lower dimensional theories [12]. However, the superspace approach of ref.[9] demonstrated its presence for the $D = 10$ case. Via the CM system, the consistent inclusion of the Lorentz Chern-Simons form was described to lowest order in the string slope parameter [9]. Using components, Bergshoeff and de Roo extended this [13] beyond linearized order. This latter work (although inspired by the superspace proof) is technically independent of the superspace approach and therefore acts as an independent check of the correctness of the superspace arguments described in reference [9]. In fact, using "gauge completion" [14] one can start from the component level and work backward to superspace.

As noted before [9] the existence of the CM constraints implies that the inclusion of the Lorentz Chern-Simons form must be universally possible in all formulations of $D = 10, N = 1$ supergravity. In the present work we prove this by explicit construction. After the appearance of our previous

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[4] Part of this formulation was suggested by Chau and Milewski [11] who intended to provide a realization of light-like integrability. We proved that their interpretation fails for the coupling of supergravity to matter multiplets and/or string corrections in ref. [10], where we extended the formulation of Chau and Milewski beyond the linearized order and completed their set of constraints.

work [9], a claim [15] was made that it cannot be consistently extended to higher orders. This, of course, also contradicts the component level result [13]. However, this claim is based on the alternate incorporation of string corrections into superspace [7]. We will discuss why this alternate proposal is subject to a fundamental flaw.

2. The Lorentz Chern-Simons Form and Supergravity Bianchi Identities

As noted some time ago [3], the concept of the Lorentz Chern-Simons (denoted by $X^{(LL)}{}_{ABC}$) form has a natural extension to superspace.

$$X^{(LL)}_{\alpha\beta\gamma} = \tfrac{1}{2}\,\omega_{(\alpha}{}^{\underline{ef}}R_{\beta\gamma)\underline{ef}} - \frac{1}{3}\omega_{(\alpha|}{}^{\underline{ef}}\omega_{|\beta|}{}^{\underline{g}}{}_{\underline{e}}\omega_{|\gamma)\underline{gf}} \ ,$$

$$X^{(LL)}_{\alpha\beta\underline{c}} = \omega_{(\alpha}{}^{\underline{ef}}R_{\beta)\underline{cef}} + \omega_{\underline{c}}{}^{\underline{ef}}R_{\alpha\beta\underline{ef}} - \omega_{(\alpha|}{}^{\underline{ef}}\omega_{|\beta)}{}^{\underline{g}}{}_{\underline{e}}\omega_{\underline{cgf}} \ ,$$

$$X^{(LL)}_{\alpha\underline{bc}} = \omega_{\alpha}{}^{\underline{ef}}R_{\underline{bcef}} + \omega_{|\underline{b}|}{}^{\underline{ef}}R_{\underline{c}|\alpha\underline{ef}} - \omega_{|\underline{b}|}{}^{\underline{ef}}\omega_{\underline{c}|}{}^{\underline{g}}{}_{\underline{e}}\omega_{\alpha\underline{gf}} \ ,$$

$$X^{(LL)}_{\underline{abc}} = \tfrac{1}{2}\omega_{|\underline{a}|}{}^{\underline{ef}}R_{\underline{bc}|\underline{ef}} - \frac{1}{3}\omega_{|\underline{a}|}{}^{\underline{ef}}\omega_{|\underline{b}|}{}^{\underline{g}}{}_{\underline{e}}\omega_{|\underline{c}|\underline{gf}} \ . \tag{2.1}$$

By definition, this satisfies the super equations $(dX^{(LL)})_{\underline{ABCD}} = R_{|\underline{AB}|}{}^{\underline{ef}}R_{|\underline{CD})\underline{ef}}$ which imply[5] for the various choice of indices,

$$\tfrac{1}{6}\nabla_{(\alpha}X^{(LL)}{}_{\beta\gamma\delta)} - \tfrac{1}{4}T_{(\alpha\beta|}{}^{\underline{E}}X^{(LL)}{}_{\underline{E}|\gamma\delta)} = \tfrac{1}{4}R_{(\alpha\beta|}{}^{\underline{ef}}R_{|\gamma\delta)\,\underline{ef}} \ ,$$

$$\tfrac{1}{2}\nabla_{(\alpha}X^{(LL)}{}_{\beta\gamma)\underline{d}} - \nabla_{\underline{d}}X^{(LL)}{}_{\alpha\beta\gamma} - \tfrac{1}{2}T_{(\alpha\beta|}{}^{\underline{E}}X^{(LL)}{}_{\underline{E}|\gamma)\underline{d}} + \tfrac{1}{2}T_{\underline{d}(\alpha|}{}^{\underline{E}}X^{(LL)}{}_{\underline{E}|\beta\gamma)}$$

$$= R_{(\alpha\beta|}{}^{\underline{ef}}R_{|\gamma)\underline{d}\,\underline{ef}} \ ,$$

$$\nabla_{(\alpha}X^{(LL)}{}_{\beta)\underline{cd}} + \nabla_{|\underline{c}}X^{(LL)}{}_{\underline{d}]\alpha\beta} - T_{\alpha\beta}{}^{\underline{E}}X^{(LL)}{}_{\underline{Ecd}} - T_{\underline{cd}}{}^{\underline{E}}X^{(LL)}{}_{\underline{E}\alpha\beta}$$

$$- T_{(\alpha|[\underline{c}|}{}^{\underline{E}}X^{(LL)}{}_{\underline{E}|\underline{d}]|\beta)} = 2R_{\alpha\beta}{}^{\underline{ef}}R_{\underline{cd}\,\underline{ef}} + R_{(\alpha|[\underline{c}}{}^{\underline{ef}}R_{\underline{d}]|\beta)\,\underline{ef}} \ , \tag{2.2}$$

$$\nabla_{\alpha}X^{(LL)}{}_{\underline{bcd}} - \tfrac{1}{2}\nabla_{|\underline{b}|}X^{(LL)}{}_{\alpha|\underline{cd}]} - \tfrac{1}{2}T_{\alpha[\underline{b}|}{}^{\underline{E}}X^{(LL)}{}_{\underline{E}|\underline{cd}]} + \tfrac{1}{2}T_{|\underline{bc}|}{}^{\underline{E}}X^{(LL)}{}_{\underline{E}|\underline{d}]\alpha}$$

$$= R_{\alpha[\underline{b}|}{}^{\underline{ef}}R_{|\underline{cd}]\,\underline{ef}} \ ,$$

$$\tfrac{1}{6}\nabla_{[\underline{a}}X^{(LL)}{}_{\underline{bcd}]} - \tfrac{1}{4}T_{[\underline{ab}|}{}^{\underline{E}}X^{(LL)}{}_{\underline{E}|\underline{cd}]} = \tfrac{1}{4}R_{[\underline{ab}|}{}^{\underline{ef}}R_{|\underline{cd}]\,\underline{ef}} \ .$$

In these equations, $R_{\underline{AB}\,\underline{de}}$ denotes components of the superspace curvature tensor. Independent of all constraints, (2.1) defines the superspace Lorentz Chern-Simons form.

To describe an on-shell supergravity theory, constraints consistent with the Bianchi identities on supertensors, $T_{\underline{AB}}{}^{\underline{C}}$, $R_{\underline{AB}\,\underline{cd}}$ and $G_{\underline{ABC}}$ are needed. The forms of the BI's for $T_{\underline{AB}}{}^{\underline{C}}$ and $R_{\underline{AB}\,\underline{cd}}$ are well-known and we will not discuss them further. There is a point of confusion in the literature[6] regarding the Bianchi identities for $G_{\underline{ABC}}$ which merits a comment or two. The Bianchi identities for $G_{\underline{ABC}}$ are always given by

$$\tfrac{1}{6}\nabla_{(\alpha}G_{\beta\gamma\delta)} - \tfrac{1}{4}T_{(\alpha\beta|}{}^{\underline{E}}G_{\underline{E}|\gamma\delta)} = 0 \ ,$$

$$\tfrac{1}{2}\nabla_{(\alpha}G_{\beta\gamma)\underline{d}} - \nabla_{\underline{d}}G_{\alpha\beta\gamma} - \tfrac{1}{2}T_{(\alpha\beta|}{}^{\underline{E}}G_{\underline{E}|\gamma)\underline{d}} + \tfrac{1}{2}T_{\underline{d}(\alpha|}{}^{\underline{E}}G_{\underline{E}|\beta\gamma)} = 0 \ ,$$

$$\nabla_{(\alpha}G_{\beta)\underline{cd}} + \nabla_{[\underline{c}}G_{\underline{d}]\alpha\beta} - T_{\alpha\beta}{}^{\underline{E}}G_{\underline{Ecd}} - T_{\underline{cd}}{}^{\underline{E}}G_{\underline{E}\alpha\beta} - T_{(\alpha|[\underline{c}|}{}^{\underline{E}}G_{\underline{E}|\underline{d}]|\beta)} = 0 \ , \tag{2.3}$$

$$\nabla_{\alpha}G_{\underline{bcd}} - \tfrac{1}{2}\nabla_{[\underline{b}|}G_{\alpha|\underline{cd}]} - \tfrac{1}{2}T_{\alpha[\underline{b}|}{}^{\underline{E}}G_{\underline{E}|\underline{cd}]} + \tfrac{1}{2}T_{[\underline{bc}|}{}^{\underline{E}}G_{\underline{E}|\underline{d}]\alpha} = 0 \ ,$$

$$\tfrac{1}{6}\nabla_{[\underline{a}}G_{\underline{bcd}]} - \tfrac{1}{4}T_{[\underline{ab}|}{}^{\underline{E}}G_{\underline{E}|\underline{cd}]} = 0 \ .$$

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[5] These equations correct some numerical errors which appeared published in ref. [9].

[6] In the presence of matter and/or string corrections, one often sees the vacuous statement that only the BI's for $G + X$ can be used. Apparently the first time this statement appeared was in the superspace construction of the type-II supergravity theory [16].

What changes in the presence of matter superfields and/or superstring corrections are the <u>constraints</u> on G_{ABC}! This is the way all conventional on-shell superspace systems of the Wess-Zumino type work.

Before discussing specifics of our solution there is one important but simple statement to make. As noted before[7], the technical problem with including the Lorentz Chern-Simons form occurs in trying to solve the very first equation in (2.3). The solution to this problem is remarkably simple and resides in the Fierz identity

$$
\begin{aligned}
(\sigma^{[3]\underline{kl}})_{(\alpha\beta|}(\sigma^{[3']}{}_{\underline{kl}})_{|\gamma\delta)}A_{[3]}B_{[3']} = {}& 108\ (\sigma^{\underline{a}})_{(\alpha\beta|}(\sigma^{\underline{b}})_{|\gamma\delta)}A_{\underline{acd}}B_{\underline{b}}{}^{\underline{cd}} \\
& - 6(\sigma^{\underline{a}})_{(\alpha\beta|}(\sigma^{\underline{bcdef}})_{|\gamma\delta)}(A_{\underline{abc}}B_{\underline{def}} + B_{\underline{abc}}A_{\underline{def}})\ ,
\end{aligned}
\tag{2.4}
$$

valid for arbitrary third rank tensors $A_{[3]} \equiv A_{\underline{abc}}$ and $B_{[3']} \equiv B_{\underline{def}}$. This identity permits the Lorentz Chern-Simons form to be included into <u>all</u> possible consistent superspace formulations of $D = 10$, $N = 1$. The reason for this is that it permits us to "factor out" a $(\sigma^{\underline{a}})_{\alpha\beta}$-matrix from $R_{(\alpha\beta|}{}^{\underline{ef}}R_{|\gamma\delta)}{}_{\underline{ef}}$. After this observation, finding the solution to the Bianchi identities in the presence of the Lorentz Chern-Simons form is a just an exercise!

We must choose a set of constraints for our Chern-Simons modified supergravity theory. There are an infinite number available. However, there is a very special set (βFFC [8]) which tremendously simplifies the calculation of the β-function in the Green-Schwarz σ-model calculation. Once the problem of the covariant quantization of the GS action is solved, this set of constraints will be the simplest to test against the result of a two-loop calculation of the β-function. So we will choose the βFFC set. For the sake of simplicity we only work to linear order, where we find[8],

$$
G_{\alpha\beta\gamma} = \beta'X^{(YM)}_{\alpha\beta\gamma} + \gamma'X^{(LL)}_{\alpha\beta\gamma}\ ,\quad F_{\alpha\beta}{}^{\underline{i}} = 0\ ,
$$

$$
G_{\alpha\beta\underline{c}} = i\tfrac{1}{2}(\sigma_{\underline{c}})_{\alpha\beta} + \beta'X^{(YM)}_{\alpha\beta\underline{c}} + \gamma'X^{(LL)}_{\alpha\beta\underline{c}} + i4\gamma'(\sigma^{\underline{b}})_{\alpha\beta}G_{\underline{bde}}G_{\underline{c}}{}^{\underline{de}}\ ,
\tag{2.5a}
$$

$$
G_{\alpha\underline{bc}} = \beta'X^{(YM)}_{\alpha\underline{bc}} + \gamma'X^{(LL)}_{\alpha\underline{bc}} + i2\gamma'\Big[(\sigma_{[\underline{b}|})_{\alpha\beta}T_{\underline{de}}{}^{\beta}G_{|\underline{c}]}{}^{\underline{de}} - 2(\sigma_{\underline{d}})_{\alpha\beta}T_{\underline{e}|\underline{b}}{}^{\beta}G_{\underline{c}]}{}^{\underline{de}} \Big]\ ,
$$

$$
T_{\alpha\beta}{}^{\underline{c}} = i(\sigma^{\underline{c}})_{\alpha\beta}\ ,\quad T_{\alpha\underline{b}}{}^{\underline{c}} = 0\ ,\quad T_{\underline{abc}} = -2L_{\underline{abc}}\ ,\quad F_{\alpha\underline{b}}{}^{\underline{i}} = -i(\sigma_{\underline{b}})_{\alpha\beta}\lambda^{\beta\underline{i}}\ ,
$$

$$
T_{\alpha\beta}{}^{\gamma} = -\big[\ \delta_{(\alpha}{}^{\gamma}\delta_{\beta)}{}^{\delta} + (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}})^{\gamma\delta}\ \big]\chi_{\delta}\ ,\quad T_{\alpha\underline{b}}{}^{\gamma} = \tfrac{1}{48}(\sigma_{\underline{b}}\sigma^{[3]})_{\alpha}{}^{\gamma}A_{[3]}\ ,
$$

$$
R_{\alpha\beta\underline{de}} = -i2(\sigma^{\underline{b}})_{\alpha\beta}\Big[L_{\underline{bde}} - \tfrac{1}{8}A_{\underline{bde}} \Big] + i\tfrac{1}{24}(\sigma^{\underline{abc}}{}_{\underline{de}})_{\alpha\beta}A_{\underline{abc}}\ ,
\tag{2.5b}
$$

$$
R_{\alpha\underline{bde}} = -i(\sigma_{[\underline{d}|})_{\alpha\beta}T_{\underline{b}|\underline{e}]}{}^{\beta} + i(\sigma_{[\underline{b}|})_{\alpha\beta}\Big[\beta'F_{|\underline{de}]}{}^{\underline{i}}\lambda^{\beta\underline{i}} + \gamma'R^{\underline{kl}}{}_{|\underline{de}]}T_{\underline{kl}}{}^{\beta} \Big]\ ,
$$

$$
A_{\underline{abc}} \equiv -i\Big[\beta'(\lambda^{\underline{i}}\sigma_{\underline{abc}}\lambda^{\underline{i}}) + \gamma'(T_{\underline{kl}}\sigma_{\underline{abc}}T^{\underline{kl}}) \Big]\ ,
$$

$$
L_{\underline{abc}} \equiv H_{\underline{abc}} + \gamma'\Big[(R^{\underline{ef}}{}_{[\underline{ab}|} + R_{|\underline{ab}|}{}^{\underline{ef}} - \tfrac{8}{3}G_{\underline{d}}{}^{\underline{e}}{}_{[\underline{a}|}G^{\underline{df}}{}_{|\underline{b}|})G_{|\underline{c}]\underline{ef}} \Big]\ ,
\tag{2.5c}
$$

$$
H_{\underline{abc}} \equiv G_{\underline{abc}} - \beta'X^{(YM)}_{\underline{abc}} - \gamma'X^{(LL)}_{\underline{abc}}\ ,
$$

$$
\nabla_{\alpha}\phi = -\tfrac{1}{2}\chi_{\alpha}\ ,
$$

$$
\nabla_{\alpha}L_{\underline{bcd}} = i\tfrac{1}{4}(\sigma_{[\underline{b}|})_{\alpha\beta}\Big[T_{|\underline{cd}]}{}^{\beta} + 4\beta'\lambda^{\beta\underline{i}}F_{|\underline{cd}|}{}^{\underline{i}} + 4\gamma'T_{\underline{kl}}{}^{\beta}R^{\underline{kl}}{}_{|\underline{cd}]} \Big]\ ,
$$

$$
\nabla_{\alpha}\chi_{\beta} = -i(\sigma^{\underline{c}})_{\alpha\beta}\nabla_{\underline{c}}\Phi + i\tfrac{1}{12}(\sigma^{[3]})_{\alpha\beta}\Big[L_{[3]} - i\tfrac{1}{8}(\chi\sigma_{[3]}\chi) + \tfrac{1}{4}A_{[3]} \Big]\ ,
\tag{2.5d}
$$

$$
\nabla_{\alpha}\lambda^{\beta\underline{i}} = -\tfrac{1}{4}(\sigma^{\underline{ab}})_{\alpha}{}^{\beta}F_{\underline{ab}}{}^{\underline{i}} - \Big[\lambda^{\gamma\underline{i}}\chi_{\gamma}\delta_{\alpha}{}^{\beta} + \lambda^{\beta\underline{i}}\chi_{\alpha} + (\sigma_{\underline{a}})_{\alpha\gamma}(\sigma^{\underline{a}})^{\beta\delta}\lambda^{\gamma\underline{i}}\chi_{\delta} \Big]\ .
$$

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[7] See the first work in [4].

[8] In (2.5b) we are correcting a numerical error published in the first ref. [17].

$$\nabla_\alpha T_{\underline{ab}}{}^\beta = -\tfrac{1}{4}(\sigma^{\underline{cd}})_\alpha{}^\beta R_{\underline{abcd}} - \left[T_{\underline{ab}}{}^\gamma \chi_\gamma \delta_\alpha{}^\beta + T_{\underline{ab}}{}^\beta \chi_\alpha + (\sigma_{\underline{c}})_{\alpha\gamma}(\sigma^{\underline{c}})^{\beta\delta} T_{\underline{ab}}{}^\gamma \chi_\delta \right]$$

$$+ \tfrac{1}{48}\left[2 G_{\underline{abc}}(\sigma^{\underline{c}}\sigma^{\underline{dkl}})_\alpha{}^\beta - (\sigma_{\underline{[a]}}\sigma^{\underline{dkl}})_\alpha{}^\beta \nabla_{\underline{[b]}} \right] A_{\underline{dkl}} \quad ,$$

$$\nabla_\alpha F_{\underline{ab}}{}^f = i(\sigma_{\underline{[a]}})_{\alpha\beta} \nabla_{\underline{[b]}} \lambda^{\beta f} + R_{\alpha\beta \underline{ab}} \lambda^{\beta f} \quad ,$$

$$\nabla_{\underline{a}}\chi_\beta = -i\tfrac{1}{2}(\sigma^{\underline{b}})_{\alpha\beta}\left[T_{\underline{ab}}{}^\alpha + 2\beta' F_{\underline{ab}}{}^f \lambda^{\alpha f} + 2\gamma' R^{\underline{kl}}{}_{\underline{ab}} T_{\underline{kl}}{}^\alpha \right] \quad ,$$

$$(\sigma^{\underline{ab}})_\beta{}^\alpha T_{\underline{ab}}{}^\beta = -i8(\sigma^{\underline{a}})^{\alpha\beta}(\nabla_{\underline{a}}\Phi)\chi_\beta - i\tfrac{2}{3}(\sigma^{\underline{abc}})^{\alpha\beta}\chi_\beta\left[L_{\underline{abc}} + \tfrac{1}{16} A_{\underline{abc}} \right]$$

$$- 3(\sigma^{\underline{ab}})_\beta{}^\alpha \left[\beta' F_{\underline{ab}}{}^f \lambda^{\beta f} + \gamma' R^{\underline{cd}}{}_{\underline{ab}} T_{\underline{cd}}{}^\beta \right] \quad .$$

$$(2.5e)$$

In the limit $\gamma' = 0$, these equations become the matter-coupled version of the βFFC set of ref.[17]. Also of note is that $T_{\underline{ab}}{}^\gamma$ appears in the A-tensor [3,4] in the same way as $\lambda^{\alpha f}$. This implies once again that the gravitino field strength plays the role of the $SO(1,9)$ gaugino $\rho^\gamma{}^{\underline{ab}}$ [9] (here it satisfies $\rho^\gamma{}^{\underline{ab}} \equiv T^{\underline{ab}\,\gamma}$). This is especially clear when we compare the last result in (2.5d) with the first result in (2.5e). The distinguishing feature of the βFFC formulation is that ρ takes such a simple form in comparison to other choices of constraints [9]. This implies that the string effective action takes its simplest form when using the βFFC constraints. We should also emphasize that only (2.5a) and the first two lines of (2.5b) were the assumptions we needed as input. The rest of the above are consequences. Finally, a remarkable result in (2.5c) is the final terms of $L_{\underline{abc}}$. The appearance of the structure $(R^{\underline{ef}}{}_{\underline{[ab]}} + R_{\underline{[ab]}}{}^{\underline{ef}})G_{\underline{[c]ef}}$ is known to be characteristic of the lowest order string corrections in the dual theory [3] and hence is expected to occur. Here it appears as a consequence of the presence of the Lorentz Chern-Simons forms in (2.5a)! In our previous analysis, this structure implied the presence of the Gauss-Bonnet [18] combination in the string effective action.

The results above also allow us to perform at the superspace level the duality transformation to be implemented on our previous discussions of string corrections in superspace [3,4]. The key point to notice is that G appears only in the quantity $L_{\underline{abc}}$ in all torsion and curvature tensors. This is indicative that $L_{\underline{abc}}$ is the auxiliary variable which would appear in a first order formulation of the antisymmetric tensor gauge field. To go to the standard theory, in a second order formulation an algebraic condition must be imposed on $L_{\underline{abc}}$ to eliminate it in favor of $G_{\underline{abc}}$ in all torsion and curvature tensors. This condition is precisely given by the definition of $L_{\underline{abc}}$ in (2.5c). At higher orders, it is likely that this duality relation receives further string corrections. The remaining string correction is then contained in the A-tensor in a form similar to that first suggested by Nishino [19]. Alternately, we can drop the results in (2.5a) along with the definition of $L_{\underline{abc}}$ in (2.5c) and then impose a condition on $L_{\underline{abc}}$ that relates it to the dual of a component level field strength for a six-form gauge field. The superspace version of the corresponding seven-form field strength, $N_{A_1\cdots A_7}$, that is compatible with (2.5) has non-vanishing entries,

$$N_{\alpha\beta[5]} = i\tfrac{1}{2}(\sigma_{[5]})_{\alpha\beta}e^{4\Phi} \quad , \quad N_{\alpha[6]} = -\tfrac{1}{4!}\epsilon_{[6][4']}(\sigma^{[4']})_\alpha{}^\beta e^{4\Phi}\chi_\beta \quad , \tag{2.6}$$

along with $N_{a_1\cdots a_7}$. Finally the relation between $L_{\underline{abc}}$ and $N_{a_1\cdots a_7}$ is then given by

$$L_{\underline{abc}} = -\tfrac{1}{7!}\epsilon_{\underline{abcd_1}\cdots\underline{d_7}}e^{-4\Phi}N^{\underline{d_1}\cdots\underline{d_7}} + i\tfrac{13}{8}(\chi\sigma_{\underline{abc}}\chi) \quad . \tag{2.7}$$

The relation between corrections to the dual theory and the standard theory has now been made explicit. Thus, we find the highly nontrivial result that to the order we are working, the string correction allow the existence of the dual theory. This continues to add to the mystery of why does this duality transformation exist at all [4]?

3. Equations of motion

Having given a complete set of superspace constraints in the last section, we can now follow the usual procedure [20] to derive the equations of motion which are implied by this on-shell superspace formulation. We find for the fermions

$$i(\sigma_{\underline{a}}{}^{bc})_{\alpha\beta}T_{\underline{bc}}{}^{\beta} = -4(\sigma_{\underline{a}}{}^{b})_{\alpha}{}^{\beta}\nabla_{\underline{b}}\chi_{\beta} - i(\sigma^{\underline{bc}}\sigma_{\underline{a}})_{\alpha\beta}\left[\beta'F_{\underline{bc}}{}^{f}\lambda^{\beta f} + \gamma'R^{kl}{}_{\underline{bc}}T_{\underline{kl}}{}^{\beta}\right]$$

$$- 8(\sigma_{\underline{a}}\sigma^{b}\chi)_{\alpha}(\nabla_{\underline{b}}\Phi) - \tfrac{2}{3}(\sigma_{\underline{a}}\sigma^{bcd}\chi)_{\alpha}\left[L_{\underline{bcd}} + \tfrac{1}{16}A_{\underline{bcd}}\right] \ ,$$

$$i(\sigma^{\underline{a}})^{\alpha\beta}\nabla_{\underline{a}}\chi_{\beta} = -i2(\sigma^{b}\chi)^{\alpha}(\nabla_{\underline{b}}\Phi) - i\tfrac{1}{6}(\sigma^{bcd}\chi)^{\alpha}\left[L_{\underline{bcd}} + \tfrac{1}{16}A_{\underline{bcd}}\right]$$

$$+ \tfrac{1}{4}(\sigma^{\underline{ab}})_{\beta}{}^{\alpha}\left[T_{\underline{ab}}{}^{\beta} + \beta'F_{\underline{ab}}{}^{f}\lambda^{\beta f} + \gamma'R^{kl}{}_{\underline{ab}}T_{\underline{kl}}{}^{\beta}\right] \ , \tag{3.1}$$

$$i(\sigma^{\underline{a}})_{\alpha\beta}\nabla_{\underline{a}}\lambda^{\beta f} = i2(\sigma^{\underline{a}}\lambda^{f})_{\alpha}(\nabla_{\underline{a}}\Phi) + i\tfrac{1}{6}(\sigma^{bcd}\lambda^{f})_{\alpha}\left[\ L_{\underline{bcd}} - i\tfrac{1}{8}(\chi\sigma_{\underline{bcd}}\chi) + \tfrac{1}{4}A_{\underline{bcd}}\ \right]$$

$$- \tfrac{1}{2}(\sigma^{\underline{ab}}\chi)_{\alpha}F_{\underline{ab}}{}^{f} \ .$$

These equations are determined by solving the BI's *and* taking linear combinations of the resultants in a way that is consistent the equations arising from an action. It has been misleadingly stated [8,15,22] (see also the second work in [17]) that solving the BI's leads to the equations of motion for the supergravity fields. Technically, this is not correct. On solving the BI's, what one finds are conditions on the fields which are equivalent to linear combinations of the equations of motion that arise from the variation of an action..

To see this explicitly, consider the fourth equation of (2.5e). Multiplying it by a $\sigma^{\underline{a}}$-matrix, we can form the Dirac equation for χ. Now if this was truly the equation of motion, it would correspond to the variation of an action which does not contain any terms of the form $\chi^2\lambda^2$. On the otherhand, the BI's directly give the last result of (3.1). Multiplying this by λ shows that the action must contain terms of the form $\chi^2\lambda^2$. This contradiction resulted from not realizing that the fourth equation of (2.5e) is actually a linear combination of the true gravitino and dilatino field equations. To find the true Dirac equation of motion for χ, we take the 'wrong' Dirac equation for χ and add it to the final equation in (2.5) in such a way that is consistent with an action principle. The equations in (3.1) are the actual equations of motion implied by 'untangling' the conditions found from the BI's. (In a longer future work, we will give a more detailed discussion concerning this point along with a complete description of the theory.)

Some of these results[9] were derived by Raciti, Riva and Zanon [15]. Their analysis was based on the unusual suggestion for solving the G-Bianchi identities by BPT-FFP (Bonora, Pasti and Tonin-Ferrara, Fré and Porrati) [7] to which we now turn.

4. Inequivalence to the BPT-FFP Superspace Construction

A second proposal [7] was made concerning the use of superspace methods to describe anomaly-free $D = 10$, $N = 1$ supergravity by BPT-FFP. While superficially similar to the original proposal, this second suggestion is actually radically different. In fact, it is not a conventional Wess-Zumino superspace type construction. It may even contain a type of inconsistency that makes it impossible to relate the BPT-FFP superspace construction to an explicit component one! These differences can now be clearly demonstrated since the work in the previous sections provides a basis for comparison.

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[9] On comparison with the results given in ref.[15] substantial differences can be seen. The reason for the differences is that we have used the linear combinations required for compatibility with the existence of a component action.

The potential problems with the BPT-FFP construction are caused because it ignores some basic rules inherent in the way Wess-Zumino superspace [20] results are related to component level ones. When solving the superspace Bianchi identities, as implied by the work of Wess and Zumino, the solution should be expressed in terms of quantities which correspond to auxiliary fields, physical non-gauge fields and the supercovariantized field strengths of component gauge fields. It was noted that by regarding the supertensors which correspond to the torsions and curvatures as super-forms, the two-form components with only vector indices correspond to supercovariantized field strengths. The only exception to this rule is the torsion. The reason for this is because already in the case of ordinary nonsupersymmetric gravitational theories, this would-be field strength is an auxiliary field! One way to see that the ordinary torsion corresponds to an auxiliary field is to remember that in a first order formalism, the equation of motion for the spin connection is algebraic. This is comparable to the situation in off-shell supersymmetric theories where the equation of motion for auxiliary fields is also algebraic. [10]

The fact that all superspace field strengths with at least one spinor form-index (and the torsion with all vector indices) can correspond to auxiliary fields means that care must be taken in identifying these with either physical or auxiliary component fields. This identification can be sorted out by first studying the theory without coupling to matter and without modifications from the string. Once the physical fields are determined by this analysis, then the auxiliary fields can be introduced.

A corollary to the basic result of Wess and Zumino is that no constraints (except those already apparent from the component level) can be placed on any supertensor which corresponds to a component supercovariantized field strength. It is the violation of this rule which raises grave doubts about the BPT-FFP construction. The way BPT-FFP violate this occurs in their solution to the G-Bianchi identities. They write a constraint on the vector-vector-vector component of G-field strength! In our notation this takes the form,

$$G_{\underline{abc}} = G^{(0)}_{\underline{abc}} - X_{\underline{abc}} + (d\mathcal{A})_{\underline{abc}} + \mathcal{B}_{\underline{abc}} , \qquad (4.1)$$

where $G^{(0)}_{\underline{abc}}$ is the contribution determined without matter or string corrections, $X_{\underline{abc}}$ is the sum of both Yang-Mills and Lorentz Chern-Simons terms and $\mathcal{A}_{\underline{ab}}$ and $\mathcal{B}_{\underline{abc}}$ are covariant supertensors.

Even if we assume that the Bianchi identities have been correctly solved within the context of the BPT-FFP system, this unconventional constraint most likely implies an inconsistency at the component formulation! The fact that consistent solutions to the superspace Bianchi identities exist that violate the rule concerning component-level supercovariantized field strengths and therefore do not lead to a consistent component level theory was discovered sometime ago [21] in a little-known explicit example. The BPT-FFP construction seems very similar to this example. Since our construction followed the conventional Wess-Zumino approach, we are free of such problems.

The other point which leads to the "nonlinear constraint" reported in ref.[15] is that BPT-FFP [7], as well as its progeny [15,22] confuse the roles of $G_{\underline{abc}}$ with $T_{\underline{abc}}$. Naively, one would say that these are interchangeable, especially since the torsion is an auxiliary field. However, once the string corrections are introduced, there is an important difference between these two supertensors. Since $T_{\underline{abc}}$ is an auxiliary field, it can be written as an explicit power series expansion in γ'. On the other hand, $G_{\underline{abc}}$ cannot be written in terms of such an expansion since it corresponds to a component-level supercovariantized field strength. So in solving the Bianchi identities, in the presence of string corrections, the correct way to proceed is to solve in terms of $G_{\underline{abc}}$! This brings into sharp focus the differences between our solution and that proposed by BPT-FFP[11]. In our solution, we have $G_{\underline{abc}} = G^{(0)}_{\underline{abc}}$ and the proposal by BPT-FFP instead uses (4.1)! Similarly, in

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[10] Another way to see this is to recall that by making an algebraic change of adding a contorsion tensor to the minimal Christoffel connection, one can change the value of the torsion arbitrarily!

[11] It is now also clear why Raciti, Riva and Zanon were not able to find a map in the form of a superconformal Weyl redefinition to relate their "solution" to our previous work [9]. No such map exists!

the BPT-FFP construction T_{abc} is not expanded in terms of γ'. This is precisely opposite to our consistent and conventional WZ-type solution. Within the context of a consistent and conventional solution to the Bianchi identities as carried out in section two, the nonlinear constraint of ref.[15] never appears! Similarly, their purported nonlinear "solution" is meaningless because any attempt to relate their results to a component formulation shows that one must invert equation (17) in the work by Raciti, Riva and Zanon. This can only be done perturbatively.

5. Discussion

There are no obstructions in our approach to proceed to higher orders. We need only expand $G_{\alpha\beta c}$, G_{abc}, L_{abc}, and A_{abc} as power series in β' and γ' and self-consistently solve the superspace Bianchi identities to any desired order. Such an investigation may yield a surprising result. Since the anomaly cancellation mechanism of Green and Schwarz only requires a finite number of terms, it is likely that only a finite number of superspace corrections are required in the presence of the Lorentz Chern-Simons form. However, the reader should be aware that there are additional corrections which are not determined by this process. These additional corrections are; (a.) the "open string corrections" which occur in the Yang-Mills sector [4] and (b.) those common to both type-II and heterotic strings. We already know how to incorporate these latter ones into our slope parameter expansions. This will be the subject of a future presentation. The former corrections are still a puzzle in the superspace-supergravity-superstring framework.

Although our explicit discussion concentrated on the βFFC system, the general features of our arguments can be applied to all superspace formulations of D = 10, N = 1 supergravity. Thus, we have presented the consistent way to introduce the Lorentz Chern-Simons form into all formulations of $D = 10$, $N = 1$ supergravity theories. Our method offers a clear alternative to the BPT-FFP-type construction. For the unconvinced reader, we simply observe that the BPT-FFP construction radically disregards well-established rules concerning the Wess-Zumino type superspace construction of supergravity theories. The component level results of Bergshoeff and de Roo can already be used together with gauge completion to provide an independent check on the superspace consistency of the slope-parameter expansion technique begun in reference [3]. However, the ultimate proof of the correctness of our slope-parameter expansions rests with covariant higher-loop calculations of β-functions for the GS σ-model. We are completely confident that only our superspace results will survive this test. Given the highly unconventional nature of the BPT-FFP construction, all conclusions based on it seem likely doomed not to stand the rigor of detailed further investigation.

6. Acknowledgement

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