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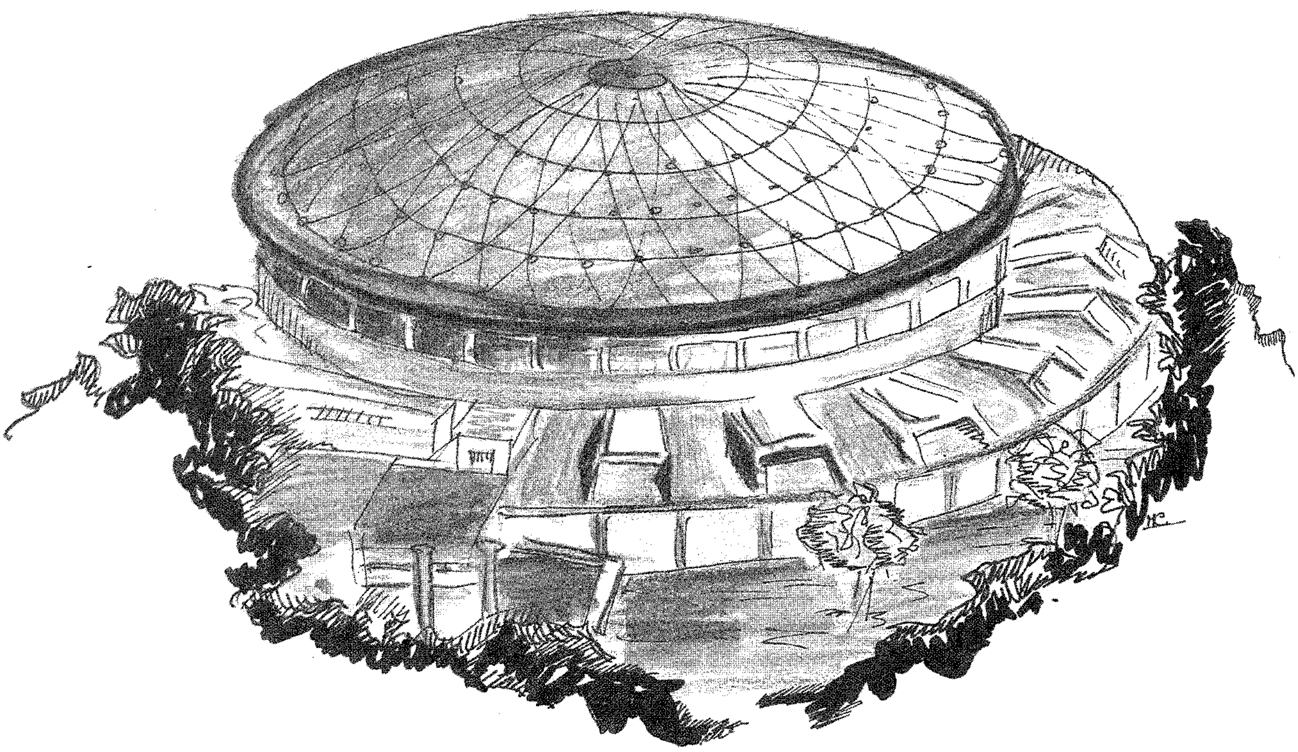

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INTERACTION OF GRAVITY AND MATTER IN THE EXTENDED N=4 SUPERSYMMETRY

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Abstract

We consider a special scheme of reduction from the space M_D of D=10 dimensions of the theory describing N=1 supergravity interacting with nonabelian matter multiplet. As a result the theory is obtained in the Minkowsky space M_4 , which describes interaction of pure N=4 supergravity with only one nonabelian matter N=4 supermultiplet. No additional degrees of freedom are present. The scalar potential, the bosonic part of the lagrangian and supersymmetry transformations are written out explicitly. The general scheme for the construction of the complete lagrangian is given. One of the possible spontaneously broken versions of the theory is also described.

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1. *Introduction.*

In view of the present status of the superstring theory it seems natural to look for another restrictive possibilities in constructing a grand unification theory at scales closed to the Planck mass scale. That which up to now has not yet been studied thoroughly enough is a theory, based on the extended N=4 sypersymmetry (N=4 SUSY). Really only the extended N=4 supergravity (N=4 SUGRA) may be interesting in view of applications to physics, because only the supergravity version of the extended supersymmetry in principle could be broken to the N=1 SUSY in the Minkowsky space M_4 (Only N=1 version of SUSY may be compatible with phenomenology at the scales of order of 1 Tev; cf.ref.[1] for the review).

As a first step in the study of phenomenological applicability of extended SUSY, our problem here is to obtain the interaction of N=4 SUGRA with N=4 SUSY matter multiplet. It is well known, that N=4 SUGRA interacting with matter is to emerge while truncating the N=1 SUGRA from 10 to 4 spacetime dimensions, i.e. from M_{10} to M_4 (see ref.[2]). In such an approach the problem is to separate SUGRA from matter degrees of freedom. (In general six additional N=4 abelian matter multiplets enter in the game in a complicated mixture with gravity multiplet). Here we solve the mixing problem and construct in M_4 the interaction of pure N=4 SUGRA multiplet with only one N=4 SYSY matter multiplet which is in the self-adjoint representation of arbitrary (nonabelian) matter symmetry group G. Our starting point is the N=1 SUGRA interacting with nonabelian N=1 SUSY matter multiplet in M_{10}, described in the ref.[3] . Neglecting the matter degrees of freedom we get the limiting case which is the version of N=4 SUGRA, constructed in [4] . Our method is based on the special compactification scenario from M_{10} to M_4 . Generalizing this scenario in accordance with [5] , it is also possible to obtain in M_4 the localized spontaneously broken version of N=4 SUGRA interacting with one nonabelian matter multiplet. In this case, neglecting matter degrees of freedom, we get the limiting case, which corresponds to the spontaneously broken version of N=4 SUGRA, constructed in [6] .

The problem of constructing of interaction between gravity and nonabelian matter in the framework of N=4 SUSY has been also considered in refs.[7,8] . In the ref.[7] the starting point was the superconformal theory of matter and Weyl supergravity, constructed in M_4 under assumption of the SU(1,1) symmetry of bosonic sector (see [9]). Transition to the case of Poincare gravity includes the imposing of artificial constraints which are solvable only in special cases. In ref.[8] the supersymmetrical generalization of σ-model for scalar fields at the maniold $O(n,6)/(O(n)\otimes O(6))$ was used (here n is the number of matter multiplets) with the gauging of some subgroups of the $O(n,6)$ symmetry group. The beautiful but rather specific methods used in refs.[7,8] allow one to consider as a gauge group G only a few special subgroups of $O(n,6)$ (For n=1 these are $SU(2)*SU(2)$ $O(1,4), O(1,6)$). This is obviously unsatisfactory from the phenomenological viewpoint. Note that the method which is discussed here gives the possibility to consider the case of any gauge group G.

Our paper is based on the ref.[10] , where the case of abelian matter multiplet was considered. The short version of the present paper was published in [11] . We present here explicitly the SYSY algebra of fields and the bosonic part of the lagrangian (the contribution of scalar, vector and tensor fields). The complete lagrangian (including fermionic terms) is rather complicated, but any possible term may be in principle written out imediately using the N=1 SUGRA lagrangian from [3] and the explicit formulae, presented here, which give the relations between fermionic and bosonic fields in M_{10} and M_4.

2. *Supergravity in 10 dimensions.*

Interaction of gravity with one nonabelian matter multiplet in M_{10} was constructed in [3] as a generalization of the result of [12] where the abelian case was considered. The SUGRA multiplet in M_{10} contains the graviton $V_{\hat{M}}{}^{\hat{A}}$ (35 physical components), Majorana-Weyl left-handed gravitino $\Psi_{\hat{M}}$ (56 components), the Majorana-Weil right-handed spinor X (8 components), the antisymmetric tensor gauge field $A_{\hat{M}\hat{N}}$ (28 components) and real scalar field φ (We present in the each case the number of independent components on the mass shell). The matter multiplet in M_{10} contains $A_{\hat{M}}$ ("gluon" field; 8 components) and Λ ("gluino" field, Majorana-Weyl left-handed spinor; 8 components); $A_{\hat{M}}$ and Λ are in the algebra of internal symmetry group G. ($A_{\hat{M}} = (A_{\hat{M}})^a T_a$ and $\Lambda = \Lambda^a T_a$, where T_a are G- group generators in the self-adjoint representation normalized by the condition: $Trace(T_a T_b) = \delta_{ab}$).

The following conventions are used. The signature of the metric is $(+,-,-,\ldots-.)$, the letters $\hat{M},\hat{N},\hat{P},\ldots$ from the central part of the alphabet are world-space indeces, the letters $\hat{A},\hat{B},\hat{C},\ldots$ from the beginning of the alphabet are tangent-space indeces. The flat tangent-space metric tensor is $\eta_{\hat{A}\hat{B}}$. All vector indeces take values 0,1,...,9. As a rule spinor indeces are not written explicitly; $x^{\hat{M}}$ be the coordinates in M_{10} ; indeces in square brackets are antisymmetrized with the unit weight: $x^{[\hat{M}_1}x^{\hat{M}_2}\ldots x^{\hat{M}_n]} = (1/n!)(x^{\hat{M}_1}x^{\hat{M}_2}\ldots x^{\hat{M}_n} \pm$ *permutations*)

The lagrangian of the theory has the form:

$$L = L_g + L_m \tag{2.1}$$

where L_g mainly describes pure gravity, L_m describes matter, interacting with gravity. We shall present L_g and L_m up to the terms of the 4-th order in fermionic fields.

The L_g term is defined by:

$$V^{-1}L_g = -\frac{1}{4k^2}R - \frac{i}{2}\overline{\Psi}_{\hat{M}}\Gamma^{\hat{M}\hat{N}\hat{P}}D_{\hat{N}}(\omega)\Psi_{\hat{P}} + \frac{i}{2}\overline{X}\Gamma^{\hat{M}}D_{\hat{M}}(\omega)X+$$
$$+\frac{9}{32k^2}\varphi^{-2}(\partial_{\hat{M}}\varphi)^2 + \frac{1}{12}\varphi^{-3/2}F_{\hat{M}\hat{N}\hat{P}}F^{\hat{M}\hat{N}\hat{P}} + \frac{3\sqrt{2}}{8}\overline{\Psi}_{\hat{M}}\Gamma^{\hat{N}}\Gamma^{\hat{M}}X\varphi^{-1}\partial_{\hat{N}}\varphi+$$
$$+\frac{ik}{24}\varphi^{-3/4}F_{\hat{P}\hat{Q}\hat{R}}(\overline{\Psi}_{\hat{M}}\Gamma^{\hat{M}\hat{N}\hat{P}\hat{Q}\hat{R}}\Psi_{\hat{N}} - 6\overline{\Psi}^{[\hat{P}}\Gamma^{\hat{Q}}\Psi^{\hat{R}]} + i\sqrt{2}\,\overline{\Psi}_{\hat{M}}\Gamma^{\hat{P}\hat{Q}\hat{R}}\Gamma^{\hat{M}}X) \tag{2.2'}$$

The L_m term is defined by:

$$V^{-1}L_m = -\frac{1}{4}\varphi^{-3/4}F_{\hat{M}\hat{N}}F^{\hat{M}\hat{N}} + \frac{i}{2}\overline{\Lambda}\Gamma^{\hat{N}}D_{\hat{N}}(\omega)\Lambda + \frac{ik}{24}\varphi^{-3/4}\overline{\Lambda}\Gamma^{\hat{M}\hat{N}\hat{P}}\Lambda F_{\hat{M}\hat{N}\hat{P}}+$$

$$+\frac{ik}{2\sqrt{2}}\varphi^{-3/8}\bar{\Lambda}\Gamma^{\hat{M}}\Gamma^{\hat{N}\hat{P}}(\Psi_{\hat{M}}+\frac{i\sqrt{2}}{12}\Gamma_{\hat{M}}X) \tag{2.2''}$$

Here $V = detV_{\hat{M}}{}^{\hat{A}}$ is the determinant of the 10-bein, R is the curvature scalar: $R = R_{\hat{M}\hat{A}}V^{\hat{M}\hat{A}} = R_{\hat{M}\hat{N}\hat{A}\hat{B}}V^{\hat{N}\hat{B}}V^{\hat{M}\hat{A}}$ where $R_{\hat{M}\hat{N}\hat{A}\hat{B}}$ is the curvature tensor

$$R_{\hat{M}\hat{N}\hat{A}\hat{B}} = \partial_{\hat{M}}\omega_{\hat{N}\hat{A}\hat{B}} + \omega_{\hat{M}\hat{A}}{}^{\hat{C}}\omega_{\hat{N}\hat{C}\hat{B}} - (\hat{M}\Leftrightarrow\hat{N}) \tag{2.3}$$

where $\omega_{\hat{N}\hat{A}\hat{B}}$ is the spin connection:

$$\omega_{\hat{N}\hat{A}\hat{B}} = -1/2(\Omega_{\hat{N}\hat{A}\hat{B}} + \Omega_{\hat{B}\hat{N}\hat{A}} - \Omega_{\hat{A}\hat{B}\hat{N}}) \tag{2.4}$$

With our accuracy the "unholonomy coefficients" $\Omega_{\hat{N}\hat{A}\hat{B}}$ are:

$$\Omega_{\hat{M}\hat{N}}{}^{\hat{A}} = -\partial_{\hat{M}}V_{\hat{N}}{}^{\hat{A}} + \partial_{\hat{N}}V_{\hat{M}}{}^{\hat{A}} \tag{2.5}$$

(Actually there are more terms in $\Omega_{\hat{M}\hat{N}\hat{A}}$ which are quadratic in fermion fields. They lead to the terms of four-fermion type in the lagrangian). Covariant derivative are defined by

$$D_{\hat{N}}(\omega) = (\partial_{\hat{N}} + \frac{i}{4}\omega_{\hat{N}\hat{A}\hat{B}}\Gamma^{\hat{A}\hat{B}}) \tag{2.6}$$

The field tensor $F_{\hat{M}\hat{N}\hat{P}}$ in eq.(2.1) is expressed via potentials $A_{\hat{M}\hat{N}}$ but contains also the Chern-Symons-type contribution from gluon matter field:

$$F_{\hat{M}\hat{N}\hat{P}} = 3\partial_{[\hat{M}}A_{\hat{N}\hat{P}]} - 3k\,Trace\,(A_{[\hat{M}}F_{\hat{N}\hat{P}]} - \frac{2ig}{3}A_{[\hat{M}}A_{\hat{N}}A_{\hat{P}]}) \tag{2.7}$$

where $F_{\hat{M}\hat{N}}$ is gluonic field-tensor:

$$F_{\hat{N}\hat{P}} = 2\partial_{[\hat{N}}A_{\hat{P}]} + ig[A_{\hat{N}}, A_{\hat{P}}] \tag{2.8}$$

Here k is the gravitational constant, g is the gauge group constant. The space-time indeces are lowered with help of the metric tensor $G_{\hat{M}\hat{N}} = V_{\hat{M}}{}^{\hat{A}}V_{\hat{N}\hat{A}}$, tangent-space indeces- with help of $\eta_{\hat{A}\hat{B}} = V_{\hat{M}\hat{A}}V^{\hat{M}}{}_{\hat{B}}$, Γ - matrices in (2.2) are: $\Gamma^{\hat{M}} = V^{\hat{M}}{}_{\hat{A}}\Gamma^{\hat{A}}$ and $\Gamma^{\hat{M}\hat{N}...\hat{Q}} = \Gamma^{[\hat{M}}\Gamma^{\hat{N}}...\Gamma^{\hat{Q}]}$, where $\Gamma^{\hat{A}}$ are 32x32 Dirac matrices in M_{10} defined in the appendix.

Our notations differs slightly from that adopted in [3,12]. The corresponding expressions used in [3,12] may be obtained after: 1) changing the signature of the metric, 2) puting $k = \frac{k}{\sqrt{2}}$, 3) changing the definition of $F_{\hat{M}\hat{N}\hat{P}}$: $F_{\hat{M}\hat{N}\hat{P}} \Rightarrow 3F_{\hat{M}\hat{N}\hat{P}}$

We present for further references the infinitisemal symmetry transformations which are possible in the lagrangian (2.1). The general coordinate transformations (GCT) are defined by the following equations:

$$\delta_\xi V_{\hat{M}}{}^{\hat{A}} = \xi^{\hat{N}}\partial_{\hat{N}}V_{\hat{M}}{}^{\hat{A}} + \partial_{\hat{M}}\xi^{\hat{N}}V_{\hat{N}}{}^{\hat{A}}$$

$$\delta_\xi V^{\hat{M}}{}_{\hat{A}} = \xi^{\hat{N}} \partial_{\hat{N}} V^{\hat{M}}{}_{\hat{A}} - \partial_{\hat{N}} \xi^{\hat{M}} V^{\hat{N}}{}_{\hat{A}} \tag{2.9}$$

where $\xi^{\hat{M}}$ are the parameters. The transformations from the gauge group G are:

$$\delta_\Omega A_{\hat{M}} = \partial_{\hat{M}} \Omega + ig[A_{\hat{M}}, \Omega]$$

$$\delta_\Omega \Lambda = ig[\Lambda, \Omega] \tag{2.10}$$

$$\delta_\Omega A_{\hat{M}\hat{N}} = 2k\, Trace(A_{[\hat{M}} \partial_{\hat{N}]} \Omega)$$

where Ω are the parameters. There is also invariance over the abelian gauge transformations U_T which affects only tensor field $A_{\hat{M}\hat{N}}$:

$$\delta_\eta A_{\hat{M}\hat{N}} = 2\partial_{[\hat{M}} \eta_{\hat{N}]} \tag{2.11}$$

where η are the parameters. Finally there is Lorentz-type invariance group $O(1.9)$ with parameters $L_{\hat{A}\hat{B}} = -L_{\hat{B}\hat{A}}$, which affects only tangent- space indeces:

$$\delta_L V_{\hat{M}}{}^{\hat{A}} = L^{\hat{A}\hat{B}} V_{\hat{M}\hat{B}}$$

$$\delta_L X = -\frac{i}{2} L_{\hat{A}\hat{B}} \Sigma^{\hat{A}\hat{B}} X, \; the\ same\ for\ \Psi_{\hat{M}}\ and\ \Lambda. \tag{2.12}$$

Here $\Sigma^{\hat{A}\hat{B}} = \frac{i}{2}\Gamma^{\hat{A}\hat{B}}$ are $O(1.9)$ group generators in spinor representation. We use the Majorana representation of Dirac matrices (see appendix). All matrices $\Gamma^{\hat{A}}$ are imaginary, the Γ_{11}- matrix (the analog of γ_5 in M_4) is real (note, that γ_5 itself is imaginary), the spinor fields $\Psi_{\hat{M}}, X, \Lambda$ are real and restricted by the Weyl condition:

$$\Gamma_{11}\Psi_{\hat{M}} = \Psi_{\hat{M}},\ \Gamma_{11}X = -X,\ \Gamma_{11}\Lambda = \Lambda \tag{2.13}$$

The transformations of N=1 SUSY are parametrized by the Majorana-Weyl real spinor field ε , restricted by the same Weyl condition, as $\Psi_{\hat{M}}$: $\Gamma_{11}\varepsilon = \varepsilon$. They have the form:

$$\delta_\varepsilon A_{\hat{M}} = -\frac{i}{\sqrt{2}} \varphi^{3/8}\, \bar{\varepsilon} \Gamma_{\hat{M}} \Lambda$$

$$\delta_\varepsilon \Lambda = -\frac{1}{2\sqrt{2}} \varphi^{-3/8}\, \Gamma^{\hat{M}\hat{N}} \varepsilon F_{\hat{M}\hat{N}}$$

$$\delta_\varepsilon V_{\hat{M}}{}^{\hat{A}} = -k\, \bar{\varepsilon} \Gamma^{\hat{A}} \Psi_{\hat{M}} + L^{\hat{A}\hat{C}} V_{\hat{M}\hat{C}}$$

$$\delta_\varepsilon \log \varphi = -\frac{2\sqrt{2}k}{3} \bar{\varepsilon} X \tag{2.14}$$

$$\delta_\varepsilon A_{\hat{M}\hat{N}} = \varphi^{3/4}(i\bar{\varepsilon}\Gamma_{[\hat{M}}\Psi_{\hat{N}]} + \frac{1}{2\sqrt{2}}\bar{\varepsilon}\Gamma_{\hat{M}\hat{N}}X) - ik\sqrt{2}\varphi^{3/8} Trace\,\bar{\varepsilon}\Gamma_{[\hat{M}}\Lambda A_{\hat{N}]}$$

$$\delta_\varepsilon X = i\frac{3\sqrt{2}}{8k}\Gamma^{\hat{M}}\varepsilon\partial_{\hat{M}}\log\varphi + \frac{i}{12\sqrt{2}}\varphi^{-3/4}\Gamma^{\hat{P}\hat{Q}\hat{R}}\varepsilon F_{\hat{P}\hat{Q}\hat{R}} - \frac{i}{2}L_{\hat{A}\hat{B}}\Sigma^{\hat{A}\hat{B}}X$$

$$\delta_\varepsilon\Psi_{\hat{M}} = \frac{1}{k}D_{\hat{M}}\varepsilon + \frac{1}{48}\varphi^{-3/4}(\Gamma^{\hat{P}\hat{Q}\hat{R}}{}_{\hat{M}} + 9\delta_{\hat{M}}{}^{[\hat{P}}\Gamma^{\hat{Q}\hat{R}]})\varepsilon F_{\hat{P}\hat{Q}\hat{R}} - \frac{i}{2}L_{\hat{A}\hat{B}}\Sigma^{\hat{A}\hat{B}}\Psi_{\hat{M}}$$

Note, that variations of matter fields have the standard form. The variations of the gravitational multiplet are mainly the same as in the pure supergravity, but with another definition of $F_{\hat{M}\hat{N}\hat{P}}$ and extra term in $\delta_\varepsilon A_{\hat{M}\hat{N}}$. It appears necessary to include into eqs (2.14) the additional local tangent-space Lorentz rotations from O(1.9)-group. The parameters $L_{\hat{A}\hat{B}}$ of these rotations will be established from the requirements for fields in M_4 to have the correct transformation laws. It will appear that $L_{\hat{A}\hat{B}}$ are quadratic in the fermionic fields and thus it will be valid to neglect them in $\delta_\varepsilon\Psi_{\hat{M}}$ and $\delta_\varepsilon X$, $\delta_\varepsilon\Lambda$ but they are essential in the $\delta_\varepsilon V_{\hat{M}}{}^{\hat{A}}$. (We do not consider the cubic fermionic terms in the SUSY transformations according to that we do not consider the 4-order fermionic terms in the lagrangian). The account of higher order terms in fermionic fields leads to a rather cumbersome algebra, though there arise no principal obstacles. On the other hand such terms are inessential when analysing the pattern of symmetry breaking, that is why it seems reasonable to neglect them at the first stage. It is sufficient to have the information, following from the results of refs.[3,12], that they may be included, and SUSY algebra is closed in any order in fermionic fields.

2. *Reduction from M_{10} to M_4. Bosonic Sector*

We admit that $M_{10} = M_4 \otimes Q_6$ i.e. $\hat{M} = (\mu,\ M)$ *and* $\hat{A} = (\alpha,\ A)$. The greek indeces $\alpha, \beta, ..., \mu\ \nu, ...$ take the values 0,1,2,3 and refer to the Minkowsky space M_4, while the latin indices $A, B, ..., M, N, ...$ take the values 4,5,...,9 (or 1,2,...,6- depending on the context) and refer to internal space Q_6. As before the indeces from the beginning of the alphabet refer to the tangent space, the indices from the middle part of the alphabet are world ones. In the following x^μ be the coordinates in M_4, y^M be the coordinates in Q_6.

We assume at the first step that all fields and all parameters of symmetry transformations are independent of coordinates of the internal space Q_6. Exception is possible only for $\xi^{\hat{M}} = (\xi^\mu,\ \xi^M)$. The conditions:

$$\partial_N V_{\hat{M}}{}^{\hat{A}} = \partial_N A_{\hat{M}\hat{N}} = 0 \quad and \quad \partial_N \delta_\xi V_{\hat{M}}{}^{\hat{A}} = \partial_N \delta_\xi A_{\hat{M}\hat{N}} = 0 \tag{3,1}$$

may be satisfied if (cf.[13]):

$$\xi^\mu = \xi^\mu(x)$$

$$\xi^M = a^M{}_N y^N + k\sqrt{2}\omega^N(x) \tag{3.2}$$

where $a^M{}_N$ is the numerical 6x6 matrix from $SL(6, R)$. We shall not find any additional restrictions from this group of transformations, so we shall put $a_M{}^N = 0$ in the following.

Let $\xi^{\hat{M}} = (\xi^\mu(x),\ k\sqrt{2}\omega^M(x)),\quad \Omega = \Omega(x),\quad \eta_{\hat{M}} = (\eta_\mu(x),\ \eta_M(x))$. The $\xi^\mu(x)$ become the parameters of GCT in M_4 (δ_ξ be the corresponding variation), $\omega^M(x)$ become the parameters of the abelian gauge group $U(1)^6$ (δ_ω be the corresponding variation). We shall take $\int d^6y = 1$ and use the same notation for the gravity constant in M_{10} and M_4. This allows one to neglect dimensional factors in the relations between fields in M_{10} and M_4 .

It is possible to impose the condition $V_M{}^\alpha = V_A{}^\mu = 0$, reducing $O(1.6)$ to $O(1.3) \otimes O(6)_T$ (Index T means that group transformations refer to the tangent space of Q_6). From the consistency conditions: $\delta_\varepsilon V_M{}^\alpha = \delta_\varepsilon V_A{}^\mu = 0$ one may fix the components $L_{\alpha B}(-L_{B\alpha})$ of the Lorentz rotation matrix in eq.(2.14) (see below).

We chose the 10-bein in the form:

$$V_{\hat{M}}{}^{\hat{A}} = \begin{pmatrix} V_\mu{}^\alpha(x) & V_\mu{}^A(x) \\ V_M{}^\alpha(x) & V_M{}^A(x) \end{pmatrix} = \begin{pmatrix} (\rho E)^{-1/2} e_\mu{}^\alpha & \sqrt{2}\, k\rho^{1/6} B_\mu{}^N E_N{}^A \\ O & \rho^{1/6} E_M{}^A \end{pmatrix} \tag{3.3}$$

Here $e_\mu{}^\alpha$ is the 4-bein in M_4, $E_M{}^A$ is the 6-bein in Q_6,

$$g_{\mu\nu} = e_\mu{}^\alpha e_{\nu\alpha}, \quad g_{MN} = E_M{}^A E_N{}^B \eta_{AB} \tag{3.4}$$

are metric tensors in M_4 and Q_6; $\eta_{AB} = -\delta_{AB}$. In eq.(3.3): $E = det E_M{}^A$. Note, that:

$$V = det V_{\hat{M}}{}^{\hat{A}} = e(\rho E)^{-1} \tag{3.5}$$

where $e = det e_\mu{}^\alpha$

The scalar factor ρ should be chosen in the form:

$$\rho = \varphi^{-9/4} = E^{-3/4} exp(-\frac{3}{2}kA) \tag{3.6}$$

It is the A-field which will be considered as the scalar field- component of N=4 SUGRA multiplet in M_4. The choice in (3.6) corresponds to the limit $E_M{}^A \Rightarrow \delta_M{}^A$ at $k \Rightarrow 0$, it provides also the correct normalization of the A-field kinetic terms and removes also mixing terms of the type: $\partial_\mu E \partial^\mu A$ (see below).

Let:

$$A_{\hat{M}} = \{A_\mu(x),\ A_M(x)\}, \quad \sqrt{2}\, kA_M \equiv \phi_M$$
$$A_{\hat{M}\hat{N}} = \{A_{\mu\nu}(x),\ A_{\mu N}(x),\ A_{MN}(x)\} \tag{3.7}$$

Now we consider the field $A_{\hat{M}\hat{N}}$ in M_4 and try to minimize the number of its independent components. We impose the condition:

$$A_{MN} = 0 \tag{3.8}$$

The self-consistency requirement $\delta_\varepsilon A_{MN} = 0$ leads to the new condition:

$$\Psi_B = V_B^{\hat{M}} \Psi_{\hat{M}} = \frac{i}{2\sqrt{2}} \Gamma_B X + E^N{}_B Trace(\phi_N \Lambda) \tag{3.9}$$

where ϕ_N is determined in (3.7).

Now we consider the $A_{\mu N}$-components in (3.7). They may be excluded using the condition:

$$\sqrt{2} A_{\mu N} = B_\mu{}^M \eta_{MN} - Trace(a_\mu \phi_N) \tag{3.10}$$

where

$$a_\mu = A_\mu - B_\mu{}^N \phi_N \tag{3.11}$$

is the gluonic field in M_4. Note, that $a_\mu = e_\mu{}^\alpha a_\alpha$ and a_α is defined by the relation $(\rho E)^{1/2} a_\alpha = A_\alpha \equiv V_\alpha{}^{\hat{M}} A_{\hat{M}}$.

The eq.(3.10) is invariant relative to all possible variations: (2.9),(2.10),(2.11) and (2.14). The numerical factor at the $B_\mu{}^M \eta_{MN}$- term in (3.10) is fixed by the supersymmetry if one takes into account (3.8) and (3.9). The variation of the right-hand part of (3.10) under $B_\mu{}^N$-field transformations is compensated by the δ_η-variation of the left-hand side part of (3.10) in accordance with the eq. (2.11). It is possible to check with help of (3.10) that (3.9) is also SUSY invariant condition.

It follows from (3.6),(3.9) and (2.14) that:

$$\delta_\varepsilon g_{MN} = Trace(\phi_M \delta_\varepsilon \phi_N + \phi_N \delta_\varepsilon \phi_M) \tag{3,12}$$

It means that g_{MN} does not contain new independent degrees of freedom. We get:

$$g_{MN} = \eta_{MN} + Trace(\phi_M \phi_N) \tag{3.13}$$

The choice of the integration constant η_{MN} in (3.13) is fixed by the requirement of the $O(6)$ -symmetry, which in this paper we shall try to conserve, and by the normalization of the kinetic terms in the lagrangian.

Consider now the $A_{\mu\nu}$-components of the $A_{\hat{M}\hat{N}}$-field . In the M_4 they describe only one degree of freedom - the pseudoscalar field B(x). The lagrangian for this field is constructed using the duality transformation (see [13], cf. also [10]). We write firstly the tangent-space field- tensor $f_{\alpha\beta\gamma}$ which is invariant relative to GCT and G-group transformations. It is defined by the relation:

$$F_{\alpha\beta\gamma} = (\rho E)^{3/2} f_{\alpha\beta\gamma} \tag{3.14}$$

where $F_{\alpha\beta\gamma} = V_\alpha{}^{\hat{M}} V_\beta{}^{\hat{N}} V_\gamma{}^{\hat{P}} F_{\hat{M}\hat{N}\hat{P}}$. The scaling factor in (3.14) is introduced for convenience. (It comes from the relation: $V_\alpha{}^\mu = (\rho E)^{1/2} e_\alpha{}^\mu$ and insures that the world-space tensor, defined by the equation: $f_{\mu\nu\sigma} = e_\mu{}^\alpha e_\nu{}^\beta e_\sigma{}^\gamma f_{\alpha\beta\gamma}$ will not not contain any scaling factors). We get:

$$f_{\mu\nu\sigma} = 3\,\partial_{[\mu}A'_{\nu\sigma]} - 3k\,B^N_{[\mu}B^M_{\nu\sigma]}\,\eta_{NM} - 3k\,Trace(a_{[\mu}a_{\nu\sigma]} - \frac{2igk}{3}\,a_{[\mu}a_\nu a_{\sigma]}) \qquad (3.15)$$

where:

$$A'_{\mu\nu} = A_{\mu\nu} - 2k\,Trace(\phi_N B^N_{[\mu}a_{\nu]})$$

$$B^N_{\mu\nu} = 2\,\partial_{[\mu}B^N_{\nu]}$$

$$a_{\mu\nu} = 2\partial_{[\mu}a_{\nu]} + ig[a_\mu, a_\nu] \qquad (3.16)$$

and ${B_\mu}^N$ is defined by the eq.(3.3), a_μ by the eq.(3.11).

Starting from the (2.2) one can readily write down all the terms in the M_4 lagrangian, containing the $f_{\mu\nu\sigma}$- field. We shall consider $f_{\mu\nu\sigma}$ as independent variables in the action. It is possible to realize with help of introduction of the constraint, so that the final form of the lagrangian becomes (only $f_{\mu\nu\sigma}$-field contributions are taken into account):

$$L_B = \frac{e}{12}\exp(-4kA)f^2_{\mu\nu\sigma} + e f_{\mu\nu\sigma}X^{\mu\nu\sigma} -$$

$$\frac{1}{6}\,B\,\epsilon^{\mu\nu\rho\sigma}\,\partial_\mu(f_{\nu\rho\sigma} + 3k\,B^N_{[\nu}B^M_{\rho\sigma]}\,\eta_{MN} + 3k\,Trace(a_{[\nu}a_{\rho\sigma]} - \frac{2ig}{3}a_{[\nu}a_\rho a_{\sigma]})) \qquad (3.17)$$

Here $X^{\mu\nu\sigma}$ are fermionic terms. (Note, that the last term in (3.17) is equal to zero due to the eq.(3.15)). Integrating by parts in the action and then solving the algebraic equations of motion for $f_{\mu\nu\sigma}$-field, we obtain:

$$f_{\mu\nu\sigma} = -\exp(4kA)(e\,\epsilon_{\mu\nu\sigma\rho}\partial^\rho B + 6\,X_{\mu\nu\sigma}) \qquad (3.18)$$

Finally, substituting (3.18) to (3.17) and neglecting the 4-order terms in fermions $\sim (X^{\mu\nu\sigma})^2$, we get:

$$L_B = \frac{e}{2}\exp(4kA)(\partial_\mu B)^2 + \frac{ek}{2}B(x)(\eta_{MN}B^M_{\mu\nu}(\tilde{B}^{\mu\nu})^N + a_{\mu\nu}\tilde{a}^{\mu\nu}) + \exp(4kA)\,\partial_\mu B\epsilon^{\mu\nu\rho\sigma}X_{\nu\rho\sigma} \qquad (3.19)$$

where

$$\tilde{B}^N_{\mu\nu} = \frac{e}{2}\,\epsilon_{\mu\nu\rho\sigma}(B^{\rho\sigma})^N$$

$$\tilde{a}_{\mu\nu} = \frac{e}{2}\,\epsilon_{\mu\nu\rho\sigma}a^{\rho\sigma} \qquad (3.20)$$

Here $\epsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric tensor ($\epsilon^{0123} = 1$). Note the relations:

$$g_{\mu\mu'}g_{\nu\nu'}g_{\sigma\sigma'}g_{\rho\rho'}\epsilon^{\mu'\nu'\sigma'\rho'} = e^2\,\epsilon_{\mu\nu\sigma\rho}$$

$$e_\mu{}^\alpha e_\nu{}^\beta e_\sigma{}^\gamma e_\rho{}^\delta \epsilon^{\mu\nu\sigma\rho} = e\epsilon^{\alpha\beta\gamma\delta}$$

where $e^2 = -det\, g_{\mu\nu} = -g$.

We left in M_4 with the $B(x)$-field instead of $A_{\mu\nu}$. Its contribution to the lagrangian is given by the eq.(3.19). Finally we get the following set of independent bosonic fields: the graviton field $e_\mu{}^\alpha$, six abelian gauge fields B_μ^N, the scalar A and the pseudoscalar field B. This is the bosonic part of N=4 SUGRA multiplet. In addition we have six scalar fields ϕ_M (3 scalars and 3 pseudoscalars, as it follows from their interactions with fermions), and gauge gluon field a_μ (a_μ and ϕ_N are in the algebra of G-group). It is the bosonic part of N=4 SUSY matter multiplet.

4. *Bosonic lagrangian in M_4.*

Now it is possible to construct the bosonic lagrangian in M_4, starting from the eq.(2.1). We make some preliminary steps for convenience. For the Einstein term in the M_{10}-lagrangian we use the well-known expression, which is justified up to the total derivative:

$$-\frac{1}{4k^2} V\, R = \frac{V}{16k^2}(\Omega^2_{\hat{A}\hat{B}\hat{C}} - 2\,\Omega^{\hat{A}\hat{B}\hat{C}}\Omega_{\hat{C}\hat{A}\hat{B}} - 4\,(\Omega_{\hat{A}\hat{C}}{}^{\hat{C}})^2) \tag{4.1}$$

where $\Omega_{\hat{A}\hat{B}\hat{C}}$ are defined in (2.5). The nonzero components of this tensor are equal to:

$$\Omega_{\alpha BC} = -\Omega_{B\alpha C} = -e^\mu{}_\alpha (\rho E)^{1/2}(E_B{}^N \partial_\mu E_{NC} + \eta_{BC}\partial_\mu \log \rho^{1/6})$$

$$\Omega_{\alpha\beta C} = -k\sqrt{2}(\rho E)\rho^{1/6} B^N_{\mu\nu} E_{NC} e^\mu{}_\alpha e^\nu{}_\beta$$

$$\Omega_{\alpha\beta\gamma} = (\rho E)^{1/2}(\Omega^{(4)}_{\alpha\beta\gamma} - \eta_{\gamma[\alpha}\partial_{\beta]}\log \rho E) \tag{4.2}$$

Where $\Omega^{(4)}_{\alpha\beta\gamma}$ are the unholonomy coefficients in M_4 , which are defined by the relation, similar to (2.5) (with the substitution $V_{\hat{N}}{}^{\hat{A}} \Rightarrow e_\nu{}^\alpha$). Using (3.5) and (4.1),(4.2) we get the result:

$$-\frac{V}{4k^2} R + \frac{9}{32k^2} V (\partial_{\hat{M}} \log \varphi)^2 =$$

$$-\frac{e}{4k^2} R^{(4)} + \frac{e}{8}\exp(-2kA)(B^N_{\mu\nu})^2 + \frac{e}{2}(\partial_\mu A)^2 - \frac{e}{16k^2}(\partial_\mu g_{MN})(\partial^\mu g^{MN}) \tag{4.3}$$

Here and in the following the contraction of the indices M,N,... (in cases when it is not shown up explicitly) is fulfilled with help of g_{MN}- tensor: $(B^N_{\mu\nu})^2 = B^N_{\mu\nu}(B^{\mu\nu})^M g_{MN}$, etc. As usual: $(\partial_\mu A)^2 = \partial_\mu A \partial^\mu A = \partial_\mu A \partial_\nu A g^{\mu\nu}$, etc; $R^{(4)}$ is the curvature tensor in M_4

To calculate another bosonic terms from (2.1) it is useful to obtain the GCT-covariant tensors in M_4, related to the tangent- space components of the 2-form and 3-form tensor fields $F_{\hat{N}\hat{M}}$ and $F_{\hat{M}\hat{N}\hat{Q}}$ (cf.[13]). Let:

$$F_{\alpha\beta} = (\rho E) e^{\mu}{}_{\alpha} e^{\nu}{}_{\beta} f_{\mu\nu}$$
$$F_{\alpha N} = (\rho E)^{1/2} e^{\mu}{}_{\alpha} f_{\mu N}$$
$$F_{MN} = f_{MN} \tag{4.4}$$

where $F_{\alpha\beta} = V_{\alpha}{}^{\hat{M}} V_{\beta}{}^{\hat{N}} F_{\hat{M}\hat{N}}$ and $F_{\alpha\hat{M}} = V_{\alpha}{}^{\hat{N}} F_{\hat{N}\hat{M}}$. Analogously:

$$F_{\alpha\beta\gamma} = (\rho E)^{3/2} e^{\mu}{}_{\alpha} e^{\nu}{}_{\beta} e^{\sigma}{}_{\gamma} f_{\mu\nu\sigma}$$
$$F_{\alpha\beta N} = (\rho E) e^{\mu}{}_{\alpha} e^{\nu}{}_{\beta} f_{\mu\nu N}$$
$$F_{\alpha MN} = (\rho E)^{1/2} e^{\mu}{}_{\alpha} f_{\mu MN}$$
$$F_{MNQ} = f_{MNQ} \tag{4.5}$$

Using (3.3) we obtain:

$$f_{\mu\nu} = a_{\mu\nu} + B^{N}_{\mu\nu}\phi_N$$
$$k\sqrt{2}\, f_{\mu N} = \partial_{\mu}\phi_N + ig[a_{\mu}, \phi_N] \equiv \nabla_{\mu}(a)\phi_N$$
$$2k^2\, f_{MN} = ig\,[\phi_M, \phi_N] \tag{4.6}$$

The expression for $f_{\mu\nu\sigma}$ has been already presented in (3.15), but

$$f_{\mu\nu N} = \frac{1}{\sqrt{2}}(B^{M}_{\mu\nu} g_{MN} - 2\,Trace(\phi_N f_{\mu\nu}))$$
$$f_{\mu MN} = \frac{1}{2k} Trace(\phi_M \nabla_{\mu}(a)\phi_N - \phi_N \nabla_{\mu}(a)\phi_M)$$
$$f_{MNQ} = -\frac{1}{\sqrt{2}\,k^2} ig Trace(\phi_M[\phi_N, \phi_Q]) = -\frac{\sqrt{2}}{k^2} ig Trace(\phi_{[M}\phi_N\phi_{Q]}) \tag{4.7}$$

Now it is the simple problem to obtain the equations:

$$-\frac{V}{4}\varphi^{-3/4} Trace\, F^2_{\hat{M}\hat{N}} = -\frac{e}{4} Trace(\exp(-2kA) f^2_{\mu\nu} + 2 f^2_{\mu N} + \exp(2kA) f^2_{MN}) \tag{4.8}$$

$$\frac{V}{12}\varphi^{-3/2} F^2_{\hat{M}\hat{N}\hat{Q}} = \frac{e}{12}(\exp(-4kA) f^2_{\mu\nu\sigma} + 3\exp(-2kA) f^2_{\mu\nu N} + 3 f^2_{\mu MN} + \exp(2kA) f^2_{MNQ}) \tag{4.9}$$

(here $f^2_{\mu N} \equiv f_{\mu M} f_{\nu N} g^{\mu\nu} g^{MN}$, etc. (cf. the note after the eq.(4.3)).

Now we are able to calculate all bosonic contributions to the lagrangian in M_4 . The scalar field part is:

$$e^{-1} L_s = e^{-1} L_s^{(g)} + e^{-1} L_s^{(m)} \tag{4.10}$$

where:

$$e^{-1}L_s^{(g)} = \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}\exp(4kA)(\partial_\mu B)^2 \tag{4.11}$$

$$e^{-1}L_s^{(m)} = -\frac{1}{4k^2}Trace\,(\nabla_\mu(a)\phi_M)^2 + \frac{1}{4k^2}(Trace\,\phi_M\nabla_\mu(a)\phi_N)^2 - U_s \tag{4.12}$$

The scalar potential U_s is equal to:,

$$U_s = \frac{g^2}{4k^4}\exp(2kA)(-\frac{1}{4}Trace\,[\phi_M,\phi_N]^2 + \frac{2}{3}(Trace\,\phi_{[M}\phi_N\phi_{Q]})^2) \tag{4.13}$$

To simplify the result we introduce the following matrix notations. We shall consider ϕ_M as a matrix ϕ with matrix elements equal to $\phi_{M\alpha}$, where α is the index of the self-adjoint representation of the internal symmetry group G. Then ϕ^T should have matrix elements $\phi_{\alpha M}$ ($= \phi_{M\alpha}$) The quantity $B^N_{\mu\nu}$ will be considered as the element of column matrix $B_{\mu\nu}$, but $(a_{\mu\nu})_\alpha$ will be the element of row matrix $a_{\mu\nu}$. With help of these notations g_{MN} and $g^{MN} = (g^{-1})_{MN}$ may be represented in the form:

$$g_{MN} \rightarrow -1 + \phi\phi^T \tag{4.14}$$

$$g^{MN} \rightarrow -1 - \phi\frac{1}{1-\phi^T\phi}\phi^T \tag{4.15}$$

Finally we obtain:

$$e^{-1}L_s^{(m)} =$$

$$= \frac{1}{4k^2}Trace(\frac{1}{1-\phi^T\phi}(\nabla_\mu\phi)^T(\nabla^\mu\phi) + \frac{1}{1-\phi^T\phi}\nabla_\mu\phi^T\phi\frac{1}{1-\phi^T\phi}\phi^T\nabla^\mu\phi) - U_s \tag{4.16}$$

New notations make it possible to represent in a rather simple form the contribution of vector particles in the lagrangian:

$$e^{-1}L_v = -\frac{1}{4}\exp(-2kA)(B_{\mu\nu}B^{\mu\nu} + a_{\mu\nu}\frac{1+\phi^T\phi}{1-\phi^T\phi}a^{\mu\nu} +$$

$$+2\,B_{\mu\nu}\phi\frac{1}{1-\phi^T\phi}\phi^TB^{\mu\nu} + 4B_{\mu\nu}\phi\frac{1}{1-\phi^T\phi}a^{\mu\nu})$$

$$+\frac{k}{2}B(a_{\mu\nu}\tilde{a}^{\mu\nu} - B_{\mu\nu}\tilde{B}^{\mu\nu}) \tag{4.17}$$

We present also the another useful representation of $e^{-1}L_v$ in terms of the complex variables:

$$S = \exp(-2kA) - 2ki\,B$$

$$a^+_{\mu\nu} = \frac{1}{2}(a_{\mu\nu} + i\,\tilde{a}_{\mu\nu})$$

$$B^{+}_{\mu\nu} = \frac{1}{2}(B_{\mu\nu} + i\,\tilde{B}_{\mu\nu}) \tag{4.18}$$

We get:

$$e\ ^1L_v = \frac{1}{4}S((a^{+}_{\mu\nu})^2 - (B^{+}_{\mu\nu})^2) - \frac{S+S^{+}}{4}(a^{+}_{\mu\nu} + B^{+}_{\mu\nu}\phi)\frac{1}{1-\phi^T\phi}(a^{+}_{\mu\nu} + \phi^T B^{+}_{\mu\nu}) + \ h.c. =$$

$$-\frac{1}{4}((G^{+}_{B})^{\mu\nu}B^{+}_{\mu\nu} + (G^{+}_{a})^{\mu\nu}a^{+}_{\mu\nu}) + \ h.c. \tag{4.19}$$

where $G^{+}_{B} = -(2/e)\delta L_v/\delta B^{+}$ and $G^{+}_{a} = -(2/e)\delta L_v/\delta a^{+}$. It is the standard type of the lagrangian, discussed in [14], which has large dynamical symmetry at the equations of motion (particulary, it possesses the $SU(1.1)$ symmetry).

5. *Reduction from M_{10} to M_4. Fermionic sector.*

We come to the discussion of the reduction in the fermionic sector of the theory. The Q_6 tangent-space components of $\Psi_{\hat{A}}$ has been already obtained in the eq.(3.9). The M_4 tangent- space components of $\Psi_{\hat{A}}$ may be obtained from the requirement that SUSY transformation of $e_\mu{}^\alpha$ has the standard form:

$$\delta_\varepsilon e_\mu{}^\alpha = -ik\bar{\varepsilon}\gamma^\alpha\psi'_\beta e_\mu{}^\beta \tag{5.1}$$

where ψ'_β is the gravitino field in M_4 (up to some scaling factor; see below). Using (2.14) and the definition of $e_\mu{}^\alpha$ in (3.3) one may get (5.1) if the following conditions are fulfilled:

$$\Psi_\beta = \psi'_\beta + \Gamma_\beta(-\frac{3i}{2\sqrt{2}}X - \frac{1}{2}\Gamma^A\Lambda\, E^M{}_A\phi_M) \tag{5.2}$$

$$L^{\alpha\beta} = ik\bar{\varepsilon}\Gamma^{\alpha\beta}(-\frac{3i}{2\sqrt{2}}X - \frac{1}{2}\Lambda\, E^N{}_A\phi_N) \tag{5.3}$$

$$L^{\alpha B} = ik\,\bar{\varepsilon}\Gamma^\alpha\Psi^B \tag{5.4}$$

(Note, that $L^{\alpha B}$ in the eq.(5.4) also insure the condition $\delta_\varepsilon V_M{}^\alpha = 0$, which was discussed before). The eqs. (5.2),(5.3) define some components of the rotation matrix $L_{\hat{A}\hat{B}}$. But L_{AB}- components has not yet been fixed. The fixation of L_{AB} is possible, when one choses the explicit representation for $E_M{}^A$, which is defined by the eq.(3.13),(3.4) up to the O(6)-rotation over the tangent-space index A. One of the possible forms for $E_M{}^A$ is:

$$E_{MA} = \delta^N_A W_{MN}, \quad E^{MB} = \delta^B_N W^{MN} \tag{5.5}$$

where $W^{MN} = (W^{-1})_{MN}$ and the matrices W and W^{-1} are equal to:

$$W = -1 + \phi\frac{1}{1+\sqrt{H}}\phi^T, \quad W^{-1} = -1 - \phi\frac{1}{\sqrt{H}}\frac{1}{1+\sqrt{H}}\phi^T \tag{5.6}$$

where $H = 1 - \phi^T\phi$. Calculating $\delta_\varepsilon E_M{}^A$ from (5.5) and comparing the result with that, following from (2.14), one may in principle obtain the explicit expression for L_{AB} (For the abelian case the result was given in [10]). But such an expression, as well as the explicit result (5.5) for $E_M{}^A$ will not be needed in the following.

The reduction of the fermionic sector will be completed if one rescales the fermionic fields to obtain the correctly normalised kinetic terms in the M_4 lagrangian. One should simultaneously rescale the SUSY transformation parameter ε to keep the standard form of the $\delta_\varepsilon e_\mu{}^\alpha$ and $\delta_\varepsilon \psi_\mu$ transformations. So we introduce the following fields in M_4 : ψ_α ($\psi_\alpha = \psi_\mu e^\mu{}_\alpha$, where ψ_μ will be the gravitino field in M_4), χ (the neutral spinor field in M_4) and λ (the gluino field). The ψ_μ and χ fields constitute the fermionic part of N=4 SUGRA multiplet in M_4, but λ is the fermionic part of N=4 SUSY matter multiplet. These fields are defined by the equations:

$$\psi'_\alpha = (\rho E)^{1/4}\psi_\alpha$$

$$X = -\frac{1}{2}(\rho E)^{1/4}\chi$$

$$\Lambda = (\rho E)^{1/4}\lambda \tag{5.7}$$

The factor 1/2 in the relation between X and χ insured the correct normalization of kinetic term for χ-field and cancellation of the mixing terms $\sim \partial_\mu \psi_\mu \chi$.

. The supersymmetry parameter ε in M_{10} should be related with the analogous parameter ϵ in M_4 by the equation:

$$\varepsilon = (\rho E)^{-1/4}\epsilon \tag{5.8}$$

We add few words about the interpretation of these fields (cf.[2], see also [10] for the details). Any spinor ψ_α, χ, λ, or ϵ should be considered in M_4 as real Majorana spinor, but each of their Dirac components contains in addition the 4-fold internal index. This follows from the fact of reduction of tangent-space group $O(1.9)$ to $O(1.3) \otimes O(6)$. The explicit formulae may be obtained with help of the following parametrization of Γ-matrices in M_{10}

$$\Gamma^\alpha = \gamma^\alpha \otimes I_8, \quad \alpha = 0,1,2,3$$

$$\Gamma^A = \gamma_5 \otimes T^A, \quad A = 1,2,...,6 \tag{5.9}$$

where I_8 is the unit 8x8 matrix, T^A are the 8x8 Dirac matrices in Q_6 : $\{T_A, T_B\} = 2\eta_{AB}$, and γ_α, γ_5 are Dirac matrices in M_4 .

We use the following parametrisation of T^A: $T^A = (T^a, T^{a+3})$, $a = 1,2,3$, and:

$$T^a = \begin{pmatrix} 0 & \alpha^a \\ \alpha^a & 0 \end{pmatrix}, \quad T^{a+3} = \begin{pmatrix} \beta^a & 0 \\ 0 & -\beta^a \end{pmatrix} \tag{5.10}$$

where α^a and β^a are real antisymmetric 4x4 matrices (see the appendix) with the algebra of SU(2). Then

$$\Gamma_{11} = \Gamma^0\Gamma^1 ... \Gamma^9 = \gamma_5 \times \begin{pmatrix} 0 & iI_4 \\ -iI_4 & 0 \end{pmatrix} \tag{5.11}$$

where I_4 is the unit 4x4 matrix.

Let us now consider the χ- field as an example. Writing out in M_{10} the spinorial (primed) index $\hat{A}' = 1, 2, ..., 32$ explicitly, we get in accordance with (5.9):

$$\chi \equiv \chi^{\hat{A}'} = \chi^{(\alpha', A')} \tag{5.12'}$$

$$\chi^{A'} = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi^{j'} \\ -i\gamma_5 \chi^{j'} \end{pmatrix} \tag{5.12''}$$

where α' is the 4-fold Dirac index, $A' = 1, 2, ..., 8$ and $j' = 1, 2, 3, 4$. (The Dirac index α' is not written explicitly in (5.12'')). The *eq.*(5.12'') follows with the help of (5.11) from the Weyl condition: $\Gamma_{11}\chi = -\chi$. The same representation exist for the ψ_μ ,λ and ϵ. (The only difference for them is in the positive sign before γ_5 in the equation, analogous to (5.12'') due to another sign in the Weyl condition).

Note that $O(6)$ generators are $\Sigma^{AB} = (i/2)T^{[A}T^{B]}$. The subset $(\Sigma^{ab}, \Sigma^{a+3,b+3})$ consists of the generators of $O(4) \sim SU(2) \otimes SU(2)$- subgroup. They are diagonalized in the representation (5.10):

$$X^s = \frac{1}{2}\epsilon^{sjk}\Sigma^{jk} = -\frac{i}{2} \begin{pmatrix} \alpha^s & 0 \\ 0 & \alpha^s \end{pmatrix}$$

$$Y^s = \frac{1}{2}\epsilon^{sjk}\Sigma^{3+j,3+k} = -\frac{i}{2} \begin{pmatrix} \beta^s & 0 \\ 0 & \beta^s \end{pmatrix} \tag{5.13}$$

So $\chi^{j'}$ and $\psi_\mu^{j'}$, $\lambda^{j'}$, $\epsilon^{j'}$ are transformed as a spinorial representation of $O(4)$ algebra.

We shall use below only the $O(6)$- notations, which correspond to the decomposition (5.9) and (5.12') (analogously for ψ_μ and λ ,ϵ), and shall not use the explicit form (5.12'') which corresponds to $O(4)$-notations.

6. *Supersymmetry transformations*

Now we are in position to obtain the fermionic terms in the lagrangian in M_4 . This may be achieved directly by substitution of (3.9), (5.2), (5.7) in the M_{10} lagrangian in (2.1). But final result is rather cumbersame, so we restrict ourselves here only to SUSY transformations in M_4 . They are:

$$\delta_\epsilon A = \bar{\epsilon}[\frac{1}{\sqrt{2}}\chi + \frac{i}{2}Trace\,(\hat{\phi}\lambda)]$$

$$\delta_\epsilon B = \frac{1}{\sqrt{2}} \exp(-2kA)\bar{\epsilon}\gamma_5[i\,\chi - Trace\,(\hat{\phi}\lambda)]$$

$$\delta_\epsilon e_\mu{}^\alpha = -ik\,\bar{\epsilon}\gamma_\alpha\psi_\mu$$

$$\delta_\epsilon B^N_\mu = -\frac{i}{\sqrt{2}}\exp(kA)E^N{}_B\bar{\epsilon}[\Gamma^B\psi_\mu - \frac{i}{\sqrt{2}}\gamma_\mu\Gamma^B\chi + \gamma_\mu Trace\,((\phi^B + \frac{1}{2}\Gamma^B\hat{\phi})\lambda)]$$

$$\delta_\epsilon\phi_M = -ikE_M{}^B\bar{\epsilon}\Gamma_B\lambda$$

$$\delta_\epsilon a_\mu = \frac{i}{\sqrt{2}}\exp(kA)\bar{\epsilon}[-\gamma_\mu\lambda + \hat{\phi}\psi_\mu - \frac{i}{\sqrt{2}}\gamma_\mu\hat{\phi}\chi + \phi_B\gamma_\mu Trace((\phi^B + \frac{1}{2}\Gamma^B\hat{\phi})\lambda)]$$

$$\delta_\epsilon\psi_\mu = \frac{1}{k}D'_\mu\epsilon - \frac{i}{2}\exp(2kA)\gamma_5\epsilon\,\partial_\mu B + \frac{1}{4}\Gamma^{AB}\epsilon\, f_{\mu MN}E^M{}_A E^N{}_B -$$
$$-\frac{1}{4\sqrt{2}}\exp(-kA)\gamma^{\nu\sigma}\gamma_\mu\Gamma^B\epsilon\; E^M{}_B(g_{MN}B^N_{\nu\sigma} - Trace\, f_{\nu\sigma}\phi_M) +$$
$$+\frac{1}{24}\exp(kA)\gamma_\mu\Gamma^{ABC}\epsilon\, E^M{}_A E^N{}_B E^P{}_C f_{MNP}$$

$$\delta_\epsilon\chi = -\frac{i}{\sqrt{2}}\gamma^\mu\epsilon\,\partial_\mu A + \frac{1}{\sqrt{2}}\exp(2kA)\gamma^\mu\gamma_5\epsilon\,\partial_\mu B - i(2\sqrt{2}k)^{-1}\gamma^\mu\epsilon\,\partial_\mu\log\, E -$$
$$-i(6\sqrt{2})^{-1}[\exp(kA)E^M{}_A E^N{}_B E^P{}_C f_{MNP}\Gamma^{ABC} + 3E^N{}_B E^P{}_C f_{\mu NP}\gamma^\mu\Gamma^{BC} +$$
$$+3\exp(-kA)E^P{}_C f_{\mu\nu P}\gamma^{\mu\nu}\Gamma^C]\epsilon$$

$$\delta_\epsilon\lambda = -\frac{1}{2\sqrt{2}}[\exp(kA)E^M{}_A E^N{}_B f_{MN}\Gamma^{AB} +$$
$$+2E^N{}_B f_{\mu N}\gamma^\mu\Gamma^B + \exp(-kA)f_{\mu\nu}\gamma^{\mu\nu}]\epsilon \tag{6.1}$$

Here we introduce the following notations:

$$\phi_B \equiv E^M{}_B\phi_M, \quad \hat{\phi} \equiv \phi_B\Gamma^B$$
$$D'_\mu\epsilon = D_\mu\epsilon + \frac{1}{4}E^N{}_A\partial_\mu E_{NB}\Gamma^{AB}\epsilon \tag{6.2}$$

ϕ_M is defined in (3.7), $f_{\ldots}$-tensors are defined in (4.6),(4.7).

The SUSY algebra is closed (with GCT- transformations) up to: 1) equations of motion for fermionic fields, 2) gauge transformations over the field a_μ (the gauge group G), 3) gauge transformations over the field B^N_μ (the abelian gauge group $U(1)^6$), 4) Lorentz rotations from the $O(1.3)$. Namely:

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = i\delta_\xi + \delta_\Omega + \delta_\omega + \delta_L \tag{6.3}$$

The parametrs are:

$$\xi^\mu = \xi^\mu_{21}, \quad \Omega = -\xi^\mu_{21} a_\mu - \frac{i}{\sqrt{2}k} \exp(kA) E^N{}_C \phi_N \xi^C_{21}$$

$$\omega^N = -i\xi^\mu_{21} B^N_\mu + \frac{i}{\sqrt{2}k} \exp(kA) E^N{}_C \xi^C_{21}$$

$$L^{\alpha\beta} = -\frac{ik}{2\sqrt{2}} \exp(-kA) E^M{}_B \bar{\epsilon}_2 (\gamma^{\alpha\beta\nu\sigma} + 2\eta^{\nu[\alpha}\eta^{\beta]\sigma}) \Gamma^B \epsilon_1 \times$$

$$\times (g_{MN} B^N_{\nu\sigma} - Trace\, \phi_M f_{\nu\sigma}) \tag{6.4}$$

where $\xi^\mu_{21} = \bar{\epsilon}_2 \gamma^\mu \epsilon_1, \quad \xi^B_{21} = \bar{\epsilon}_2 \Gamma^B \epsilon_1$.

Note that $\delta_\epsilon B$-transformation can not be found directly from (2.14). We get our result for $\delta_\epsilon B$ in (6.1) considering the restrictions, which follow from (6.2).

7. *Spontaneously broken version of $N = 4$ supersymmetry.*

It is possible in our approach to obtain the spontaneously broken version of the theory, which will be the generalization of the result of [6] due to the inclusion of matter degrees of freedom. The result may be obtained by imploying the specific compactification procedure from M_{10} to M_4 , which was discussed in [5]. (But with another interpretation of the 3-form tensor field f_{MNP})

One should make the following substitutions in the preceeding formulae:

$$E_M{}^A \Rightarrow E_N{}^A U^N{}_M(y)$$

$$B^M_\mu \Rightarrow (U^{-1}(y))^M{}_N B^N_\mu \tag{7.1}$$

i.e. the tensor fields (in the indices $M, N, ..$) are y-dependent, but everywhere (with the exeption of A_{MN}- field, see below) this dependence appears only through the D-functions of some symmetry group S, acting on the Q_6 world- space indeces. The part of the y-dependence immediately disappears due to the contraction over the indices, but terms with derivatives require a separate consideration. Because only 2-form structures $\sim (\partial_{\hat{M}} X_{\hat{N}} - \partial_{\hat{N}} X_{\hat{M}})$ enter in the lagrangian, the y-dependence appears only in the form:

$$\frac{1}{\sqrt{2}k} C^N{}_{PQ} = (U^{-1})^M{}_P (U^{-1})^L{}_Q (\partial_L U^N{}_M - \partial_M U^N{}_L) \tag{7.2}$$

Imposing the condition:

$$[L_M, L_N] = C^Q{}_{MN} L_Q \tag{7.3}$$

where $L_Q = \sqrt{2}k (U^{-1})^N{}_Q \partial_N$, one may prove that $C^Q{}_{MN}$ are y-independent and should be considered as the structural constants of some subgroup S' of the S -group.

Particulary, considering $S' = O(4)$ one may get:

$$U^M{}_N(y) = \begin{pmatrix} U^m{}_n & 0 \\ 0 & U^{\hat{m}}{}_{\hat{n}} \end{pmatrix} \tag{7.4}$$

and:

$$C^Q{}_{MN} = \frac{1}{\sqrt{2}}(\lambda_v \, \epsilon^q{}_{mn}, \; \lambda_a \, \epsilon^{\dot{q}}{}_{\dot{m}\dot{n}}) \tag{7.5}$$

where $m, n, q = 1,2,3$; $\dot{m}, \dot{n}, \dot{q} = 4,5,6$; λ_v *and* λ_a are constants and ϵ^{mnq} is totally antisymmetric tensor ($\epsilon^{123} = \epsilon^{456} = 1$).

We consider the case, where the Killing tensor: $-C^Q{}_{MP}C^P{}_{NQ}$ is equal to $\eta_{MN} = -\delta_{MN}$, and $C_{MNP} = \eta_{PL}C^L{}_{MN}$ is completely antisymmetric.

Now some of the $f_{...}$- tensors, defined in the eqs.(4.4),(4.5) will be proportional to $U^M{}_N$- factors. So it is convenient to introduce new quantities:

$$f'_{\mu N} = \overline{U}^R{}_N f_{\mu R}, \quad f'_{MN} = \overline{U}^Q{}_M \overline{U}^R{}_N f_{QR} \tag{7.6}$$

And:

$$f'_{\mu\nu P} = \overline{U}^S{}_P f_{\mu\nu S}$$

$$f'_{\mu NP} = \overline{U}^R{}_N \overline{U}^S{}_N f_{\mu RS}$$

$$f'_{MNP} = \overline{U}^Q{}_M \overline{U}^R{}_N \overline{U}^S{}_P f_{QRS} \tag{7.7}$$

where $f...$ are defined as before in eqs (4.5),(4.4) (but with the substitution (7.1) for A_M, $A_{\mu M}$ A_{MN}- fields).

Now the condition $A_{MN} = 0$ will not be self-consistent. But we do not need new degrees of freedom, related with A_{MN} , because we do not want new N=4 SUSY multiplets in M_4. The only possibility is to consider A_{MN} depending only on y. Then the contribution from A_{MN} to the physical quantity f'_{MNP} should be constant. If we try to keep the largest possible symmetry group, then the only possibility is:

$$\overline{U}^Q{}_M \overline{U}^R{}_N \overline{U}^S{}_P \, 3\partial_{[Q}\tilde{A}_{RS]} = \frac{a}{2k^2} C_{MNP} \tag{7.8}$$

where $\tilde{A}_{RS} = U^M{}_R U^N{}_S \, A_{MN}$, and a is some constant. Then calculating $f'_{...}$- tensors in (7.6) we get:

$$f'_{\mu N} = \frac{1}{\sqrt{2}k} \nabla(a,B)_\mu \phi_N$$

$$f'_{MN} = \frac{1}{2k^2}(-\phi_Q C^Q{}_{MN} + ig\,[\phi_M, \phi_N]) \tag{7.9}$$

and:

$$f'_{\mu\nu P} = \frac{1}{\sqrt{2}}(g_{PM}B^M_{\mu\nu} - 2\phi_P f_{\mu\nu})$$

$$f'_{\mu NP} = \frac{1}{2k}(\phi_N \nabla_\mu(a,B)\phi_P - \phi_P \nabla_\mu(a,b)\phi_N) +$$

$$+ \frac{1}{\sqrt{2}k}[C^Q{}_{NP}(A_{\mu Q} + \frac{1}{\sqrt{2}} Trace(a_\mu \phi_Q) - a\eta_{MQ}B^M_\mu)]$$

$$f'_{MNP} = \frac{a}{2k^2}C_{MNP} + \frac{1}{2\sqrt{2}k^2}Trace(3\phi_{[M}C^Q{}_{NP]}\phi_Q - 4ig\,\phi_{[M}\phi_N\phi_{P]}) \tag{7.10}$$

Here:

$$\nabla_\mu(a,B)\phi_N = \partial_\mu\phi_N + ig\,[a_\mu,\phi_N] + \phi_Q C^Q{}_{MN}B^M_\mu \tag{7.11}$$

$$B^M_{\mu\nu} = \partial_\mu B^M_\nu - \partial_\nu B^M_\mu - C^M{}_{PQ}B^P_\mu B^Q_\nu \tag{7.12}$$

The expressions for $f_{\mu\nu\sigma}$ and $f_{\mu\nu}$ defined as in eqs.(4.4),(4.5) will be the same as before (cf.(3.15),(4.6)) but with the replacement of $B^N_{\mu\nu} = 2\partial_{[\mu}B^N_{\nu]}$ to the Yang-Mills field-strength (7.12).

Gauge invariance of $f'_{\mu NP}$ leads to the additional constraint:

$$A_{\mu M} + \frac{1}{\sqrt{2}}Trace(a_\mu\phi_M) = a\eta_{MN}B^N_\mu \tag{7.13}$$

Comparing (7.13) with (3.10) we may expect, that

$$a = \frac{1}{\sqrt{2}} \tag{7.14}$$

This expectation may be confirmed by the direct application of SUSY transformation to the eq.(7.13). In this way we obtain that really eqs. (3.9), (3.10) are justified and so is the eq.(7.14).

We find that the difference from the previous case consistts in the appearance of structures, covariant over the Yang-Mills symmetry group transformations over the indeces $M, N, ..$ (instead of abelian ones), and in the appearance of some additional terms with C_{MNP}- tensor. Particulary the formulae (4.8), (4.9) will take place, as before, but with the substitution of $f'_{...}$-tensors instead of corresponding $f_{...}$. The eq.(4.3) is also justified but with the nonabelian field-strength (7.12) and covariant derivatives $\nabla_\mu(B)$ instead of usual ones:

$$\nabla_\mu(B)E_N{}^A = \partial_\mu E_N{}^A + E_Q{}^A C^Q{}_{MN}B^M_\mu$$
$$\nabla_\mu(B)E^N{}_A = \partial_\mu E^N{}_A - C^N{}_{PQ}B^P_\mu E^Q{}_A \tag{7.15}$$

The same changements should be done in the eqs.(4.2), but in addition new nonzero components of $\Omega_{\hat{A}\hat{B}\hat{C}}$ will appear:

$$\Omega_{ABC} = \frac{1}{\sqrt{2}k}\rho^{-1/6}C^Q{}_{MN}E^M{}_A E^N{}_B E_{QC} \tag{7.16}$$

Now it is possible to derive the bosonic part of the lagrangian in M_4 . It has the same form as in the eqs.(4.10) , (4.17) but with the substitution of nonabelian field - tensor $B^N_{\mu\nu}$ instead of abelian one and covariant derivatives of the type of (7.15) over the indeces $M, N, ...$. In addition we get instead of (4.13) another form of the scalar potential:

$$U_s = \frac{1}{4k^4}\exp(2kA)(-\frac{1}{8}C^L{}_{MN}(C^{L'}{}_{M'N'}g^{MM'}g^{NN'}g_{LL'} + 2C^N{}_{LM'}g^{MM'})+$$

$$+\frac{1}{4}Trace(-\phi_L C^L{}_{MN} + ig[\phi_M, \phi_N])^2-$$

$$-\frac{1}{24}(C_{MNP} + Trace(3\phi_{[M}C^L{}_{NP]}\phi_L - 4ig\,\phi_{[M}\phi_N\phi_{P]}))^2) \qquad (7.17)$$

Here $\phi^2_M = g^{MN}\phi_M\phi_N$ as before, $(C^L{}_{MN})^2 = C^L{}_{MN}C^{L'}{}_{M'N'}g_{LL'}g^{MM'}g^{NN'}$, *etc.*

The SUSY transformations and SUSY algebra for the considered case was presented in [11] The term $\sim \gamma_\mu\epsilon$ appears in the transformation $\delta_\epsilon\psi_\mu$ which indicates the spontaneously supersymmetry breaking. All 4 supersymmetries are broken.

Note in the conclusion, that the only nonabelian symmetry group S' which may be considered should be $O(4)$ or some subgroups of $O(4)$. It is due to the fact, that only six vector fields B^N_μ are in our disposition.

The detailed investigation of the potentials (4.13) and (7.17) is a separate problem which has not yet been solved. But some facts may be established easilly. There is no mass term for the ϕ_M-field (It is obvious in the case (4.13), but it is the result of cancellations of contributions from different terms in (7.17)). The potential (4.13) has a flat valleys in the space of ϕ_M -fields in the directions, where $[\phi_M, \phi_N] = 0$. There is negative cosmological term in (7.17) . It is equal to:

$$-(8k^4)^{-1}\exp(2kA)(\lambda_v^2 + \lambda_a^2), \qquad (7.18)$$

It follows if we use (7.5) and the simplest anzatz: $\phi_M = \Phi T_M$, where T_M are generators of some $O(4)$-subgroup of internal symmetry group G.

8. *Conclusion*

We have considered the reduction scenario from ten dimensional space M_{10} to the Minkowsky space M_4 of the theory described in [3], that makes it possible to obtain in M_4 the "minimal" version of the theory, describing N=4 supergravity and N=4 nonabelian matter multiplet (without any additional abelian matter multiplets). The internal-space metric tensor g_{MN} is expressed in such a case in terms of scalar fields ϕ_M of nonabelian N=4 matter multiplet. The A_{MN} and $A_{\mu N}$ components of the 2-form field $A_{\hat{N}\hat{M}}$ do not enter in the game due to conditions (3.8) and (3.10) (or (7.8) and (7.13),(7.14) in the spontaneously broken version). The scalar potential may be written explicitly in such a "minimal" sceme. That provides the possibility to investigate the inherent (spontaneous) breaking of the N=4 supersymmetry in Minkowsky space or to consider some mechanisms of the soft explicit breaking.

All that corresponds to a rather restrictive but nevertheless fenomenological level from the point of view of string theory but it might be much easier to attack, than the string theory itself.

Appendix

1) The 32x32 Dirac matrices in M_{10} satisfy the conditions:

$$\{\Gamma_{\hat{A}},\Gamma_{\hat{B}}\} = 2\eta_{\hat{A}\hat{B}}, \quad \Gamma^0\Gamma^{\hat{A}}\Gamma^0 = (\Gamma^{\hat{A}})^+, \quad C\Gamma^{\hat{A}}C^{-1} = -(\Gamma^{\hat{A}})^T \qquad (A1)$$

where C is the charge conjugation matrix. We use the Majorana representation, where $C = \Gamma^0$ and all matrices $\Gamma^{\hat{A}}$ are pure imaginary. The matrix Γ_{D+1} in D-dimensional space which is the anolog of γ_5 in M_4 is chosen to satisfy the condition $\Gamma^2_{D+1} = 1$, thus:

$$\Gamma_{D+1} = (-1)^{\frac{D-2}{4}}\Gamma^0\Gamma^1...\Gamma^{D-1} \qquad (A2)$$

Thus, Γ_{11} is pure real symmetrical matrix. (As a contrast to γ_5 which is pure imaginary and antisymmetrical matrix).

The Majorana spinors Ψ are defined by the condition:

$$\overline{\Psi} = \tilde{\Psi} \qquad (A3)$$

where $\overline{\Psi} = \Psi^+\Gamma^0$, $\tilde{\Psi} = \Psi C$. In the Majorana representation they are real.

Following the ref. [12] we also widely use the matrices of the form

$$\Gamma^{A_1A_2...A_n} = \Gamma^{[A_1}\Gamma^{A_2}...\Gamma^{A_n]} \qquad (A4)$$

For all different indeces: $\Gamma^{A_1...A_n} = \Gamma^{A_1}...\Gamma^{A_n}$

2) The real antysymmetric matrices α^j, *and* β^j in eq. (5.10) are defined by the relations:

$$\{\alpha^j,\alpha^k\} = \{\beta^j,\beta^k\} = 2\eta^{jk}, \quad [\alpha^j,\beta^k] = 0 \qquad (A5)$$

$$[\alpha^j,\alpha^k] = 2\epsilon^{jks}\alpha_s$$
$$[\beta^j,\beta^k] = 2\epsilon^{jks}\beta_s, \quad j,k,s, = 1,2,3. \qquad (A6)$$

Here $\epsilon^{123} = 1$, $\alpha_s = \alpha^k\eta_{ks} = -\alpha^s$, *etc.*

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