



Laboratori Nazionali di Frascati

LNF-89/069(PT)
9 Ottobre 1989

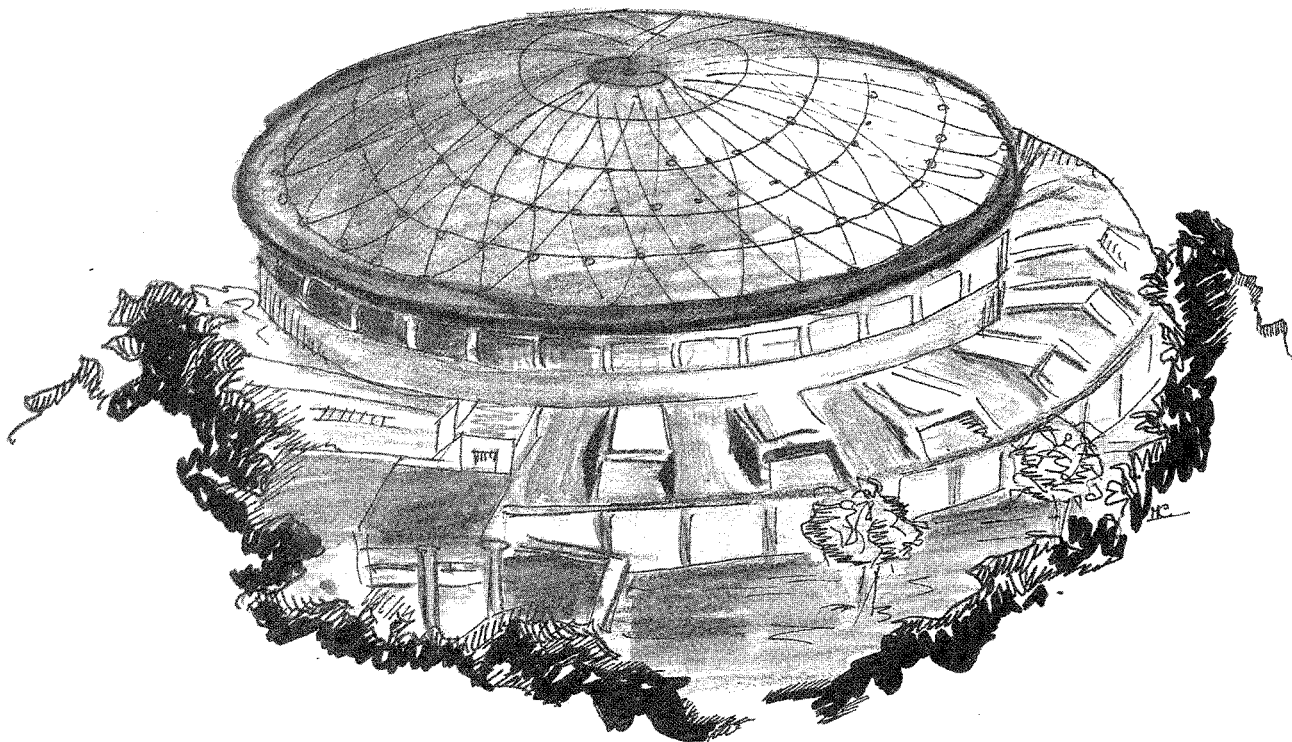
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COHERENT STATES AND STRUCTURE FUNCTIONS IN QED

Invited talk presented at the Workshops
"QED Structure Functions" - Ann Arbor (May 22-25, 1989)

and

"Radiative Corrections: Results and Perspectives" Brighton (July 10-13, 1989)



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P.O. Box, 13 - 00044 Frascati (Italy)

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COHERENT STATES AND STRUCTURE FUNCTIONS IN QED

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ABSTRACT

The methods of coherent states and of structure functions in QED are considered in detail for a precise evaluation of the radiative effects at LEP/SLC. They are explicitly shown to give identical results for the exponentiated infrared factors and the $O(\alpha)$ terms corresponding to the initial and final state radiation. Furthermore interference effects from initial and final state radiation are introduced within the formalism of structure functions in a way which is suitable for Monte Carlo applications. The final formulae improve to $O(0.1\%)$ the evaluation of the e.m. effects.

The important role played by QED radiative corrections at LEP/SLC energies for precision tests of the standard model is well known^[1,2]. The understanding and the detailed description of the multiphoton effects closely follow the pioneering work of Touschek and collaborators^[3-5] at Frascati, more than twenty years ago, when a similar problem of precision had to be faced in testing Quantum Electrodynamics at "large" transverse momenta, i.e. at $Q^2 \sim 0$ (1 GeV^2).

The strategy was very clear: (i) to sum to all orders the large double and single soft and collinear logarithms of perturbation theory; (ii) to calculate exactly to one loop all remaining terms of $O(\alpha)$. The discovery of the J/ψ and other narrow resonances introduced further complications into the problem, due to the very nature of the resonant process and the subtle interference effects between initial and final leptonic states. Those were successfully described^[6] along the same lines, in particular by exploiting the technique of the coherent states, introduced earlier^[4,7] with the aim of having a realistic QED S-matrix.

The study^[8] of genuine weak radiative effects at LEP/SLC energies clearly indicated the necessity of controlling e.m. corrections to a high degree of accuracy. Then the earlier analytical results were generalized to the case of Z_0 production to exact one-loop accuracy^[9,10] and to all orders^[9] in the leading logarithmic approximation, reaching a level of $O(1\%)$ precision. More recently initial state radiation effects have been evaluated^[11] to two loops, pushing the theoretical accuracy - for the line shape measurements - to $O(0.1\%)$.

To this aim the method of the structure functions^[12], extended also to final states^[13] in the reaction $e^+e^- \rightarrow \mu^+\mu^-$, has been shown to be quite powerful, suggesting a systematic approach to other processes as, for example, Bhabha scattering. However, only numerical solutions have been obtained so far in the case of a resonant cross section, leaving unclear the connection to the previous analytical approach of the problem. Furthermore the full extension of this technique to leptonic final states clearly demands an appropriate treatment of initial - and final - state interference effects which, on the other hand, are described to all orders in the coherent state formalism.

In this talk I will address the above questions, discussing analytical solutions of the structure functions method, including initial-final state interference effects, which are explicitly introduced in the formalism. More in detail I will explicitly show that the method of structure functions coincides with the coherent state approach with an accuracy of $O(1\%)$, giving explicitly in addition the $O(\alpha^2)$ corrections needed to improve further the theoretical precision.

The basic formula which describes the reaction $e^+e^- \rightarrow \mu^+\mu^-$ in the approach of structure functions without interference effects is the following^[13]

$$d\sigma(s) = \int_0^\epsilon dx d\sigma_0((1-x)s) H_e(x, s) F_\mu(\epsilon - x, (1-x)s), \quad (1)$$

where $\epsilon \equiv \Delta E/E$ is the energy resolution, and the initial and final state radiation kernels are given in terms of the electron and muon structure functions as

$$H_e(x, s) = \int_{1-x}^1 \frac{dz}{z} D_e(z, s) D_e\left(\frac{1-x}{z}, s\right), \quad (2)$$

$$F_\mu(x, s) = \int_0^x dy H_\mu(y, (1-y)s), \quad (3)$$

with $H_\mu(x, s)$ defined as in eq. (2). Eq. (1) describes the factorizable corrections only, corresponding to real and virtual photon emission from the initial and final legs, with no relative interference. By taking into account the effect of the soft radiation to all orders and of the hard one up to $O(\alpha^2)$ one obtains^[13]

$$H_e(x, s) = \Delta_e(s) \beta_e x^{\beta_e-1} - \frac{1}{2} \beta_e (2-x) + \frac{1}{8} \beta_e^2 \left\{ (2-x) [3 \ln(1-x) - 4 \ln x] - 4 \frac{\ln(1-x)}{x} + x - 6 \right\}, \quad (4)$$

with $\beta_e = \frac{2\alpha}{\pi} (L_e - 1)$, $L_e = \ln\left(\frac{s}{m_e^2}\right)$ and

$$\begin{aligned} \Delta_e(s) &= 1 + \frac{\alpha}{\pi} \left[\frac{3}{2} L_e + 2(\zeta(2) - 1) \right] + \left(\frac{\alpha}{\pi} \right)^2 \left\{ \left[\frac{9}{8} - 2\zeta(2) \right] L_e^2 \right. \\ &+ \left. \left[3\zeta(3) + \frac{11}{2} \zeta(2) - \frac{45}{16} \right] L_e + \left[-\frac{6}{5} \zeta^2(2) - \frac{9}{2} \zeta(3) - 6\zeta(2) \ln 2 + \frac{3}{8} \zeta(2) + \frac{57}{12} \right] \right\} \quad (5) \\ &\equiv 1 + \frac{\alpha}{\pi} \Delta_e^{(1)} + \left(\frac{\alpha}{\pi} \right)^2 \Delta_e^{(2)} \end{aligned}$$

By insertion of (4) in eqs. (2) and (1) one easily obtains

$$\sigma(s) = \int_0^\varepsilon dx \sigma_0(s(1-x)) \{ \Delta_e(s) \Delta_\mu(s) \beta_e x^{\beta_e-1} (\varepsilon-x)^{\beta_\mu} + R(x, \dots) \} \quad (6)$$

where the first term in the r.h.s. of eq. (6) is proportional to the leading soft contribution, while $R(x, \dots)$ give further correction terms of order (β^2) and β_e in the final cross section. We will assume the fractional energy resolution $\varepsilon \equiv \Delta E/E$ of order $10^{-1} - 10^{-2}$.

By splitting the Born cross section $\sigma_0(s)$ as $\sigma_0 = \sigma_0^{\text{QED}} + \sigma_0^{\text{INT}} + \sigma_0^{\text{RES}}$, with

$$\begin{aligned} \sigma_0^{\text{QED}}(s) &= A \frac{1}{s} \\ \sigma_0^{\text{INT}}(s) &= B \operatorname{Re} \left\{ \frac{1}{s - M^2 + i\Gamma M} \right\} \\ \sigma_0^{\text{RES}}(s) &= C \frac{s}{(s - M^2)^2 + \Gamma^2 M^2} \end{aligned} \quad (7)$$

the corresponding radiatively corrected cross sections are found^[14] to be

$$\sigma^{\text{QED}}(s) = \sigma_0^{\text{QED}}(s) \Delta_e(s) \Delta_\mu(s) \varepsilon^{\beta_e + \beta_\mu} + \dots \quad (8)$$

$$\sigma^{\text{INT}}(s) = \sigma_0^{\text{INT}}(s) \Delta_e(s) \Delta_\mu(s) \varepsilon^{\beta_\mu} \frac{\Gamma(1+\beta_e) \Gamma(1+\beta_\mu)}{\Gamma(1+\beta_e+\beta_\mu)} \frac{1}{\cos \delta_R} \cdot \text{Re} \left\{ e^{i\delta_R} \left[\frac{\varepsilon}{1 + \left(\frac{\varepsilon s}{M\Gamma}\right) \sin \delta_R e^{i\delta_R}} \right]^{\beta_e} \right\} + \quad (9)$$

$$\sigma^{\text{RES}}(s) = \sigma_0^{\text{RES}}(s) \Delta_e(s) \Delta_\mu(s) \varepsilon^{\beta_\mu} \frac{\Gamma(1+\beta_e) \Gamma(1+\beta_\mu)}{\Gamma(1+\beta_e+\beta_\mu)} \cdot \left| \frac{\varepsilon}{1 + \left(\frac{\varepsilon s}{M\Gamma}\right) \sin \delta_R e^{i\delta_R}} \right| (\cos \beta_e \phi - \cot \delta_R \sin \beta_e \phi) + \dots \quad (10)$$

where $(M_R^2 - s)^{-1} \equiv \frac{\sin \delta_R e^{i\delta_R}}{M\Gamma}$, $\tan \delta_R = \frac{M\Gamma}{(M^2 - s)}$, $\phi = \arctan \left[\frac{\varepsilon s + M^2 - s}{M\Gamma} \right] - \arctan \left[\frac{M^2 - s}{M\Gamma} \right]$

and the dots indicate next to leading terms corresponding to $R(x, \dots)$ in eq. (6).

Comparing with the analogous expressions obtained in the framework of coherent states^[2,9], one finds that exponentiated infrared and the $O(\alpha)$ factors coincide with those in eqs. (8-10). On the other hand extra terms are contained in eqs. (8-10) of $O(\beta^2)$ - in particular the factor $-\beta_e^2 (\phi^2/2)$ in eq. (13) arising from the expansion of $\cos \phi \beta_e$ - and $O(\varepsilon\beta)$, not written explicitly^[15].

Concerning the remaining terms, including initial-final states interference, box diagrams, etc. they can also be included to $O(\alpha)$ in the approach of structure functions. However a general treatment of interference effects, to all orders, can be obtained as follows. For pure QED processes the simple rescaling^[16] $s \rightarrow s(t/u)$, where s, t and u are the Mandelstam variables, gives the usual result

$$d\sigma^{\text{QED}} \approx d\sigma_0^{\text{QED}} e^{\beta_e + \beta_\mu + 2\beta_{\text{int}}}, \quad (11)$$

where $\beta_{\text{int}} = (4\alpha/\pi) \ln tg \theta/2$. However this simple rule does not work for a resonant process. Then, more generally, a full account of all radiative effects can be simply obtained by replacing the Born cross section $d\sigma_0 [(1-x)s]$ in eq. (1) by $d\sigma_0 [(1-x)s] \cdot K(x)$, where the K -factor includes all not-factorizable corrections, which can be determined to all orders for the soft contribution and to $O(\alpha)$ for the nonleading terms.

Indeed one obtains^[17] the soft contributions, corresponding to initial-final state interference, the following expressions for the $K^{(i)}(x)$ factors

$$\begin{aligned}
K^{\text{QED}}(x) &= \frac{(\beta_e + \beta_{\text{int}})}{\beta_e} [x(\epsilon - x)]^{\beta_{\text{int}}} \\
K^{\text{INT}}(x) &= \frac{(\beta_e + \beta_{\text{int}})}{\beta_e} \left[\frac{sx}{s(1-x) - M_R^2} (\epsilon - x) \right]^{\beta_{\text{int}}}
\end{aligned} \tag{12}$$

$$K^{\text{RES}}(x) = \frac{(\beta_e + 2\beta_{\text{int}})}{\beta_e} \left| \frac{sx}{s(1-x) - M_R^2} \right|^{2\beta_{\text{int}}} .$$

The above result is based on the observation^[6] that in the pure QED process the virtual matrix element M_V^{QED} scales as $\{\lambda^2/s\}^{(\beta_e + \beta_\mu + 2\beta_{\text{int}})/4}$, while for a resonant process one has $M_V^{\text{RES}} \sim \{\lambda^2/s\}^{(\beta_e + \beta_\mu)/4} \cdot \{\lambda^2/(s - M_R^2)\}^{\beta_{\text{int}}/2}$, where λ is the minimum energy cutoff. Eqs. (12) allow us to obtain the complete analytical solution within the method of the structure functions, generalizing the results of ref. [14].

Then, after the substitution $d\sigma_0^{(i)}[(1-x)s] \rightarrow d\sigma_0^{(i)}[(1-x)s] \cdot K^{(i)}(x)$ in eq. (1), using eq. (12), and following ref. [14], one then finds^[17] for the leading terms - corresponding to the resummation of the soft contributions:

$$\begin{aligned}
d\sigma^{\text{QED}}(s) &= d\sigma_0^{\text{QED}}(s) \left\{ \Delta_e(s) \Delta_\mu(s) \epsilon^{\bar{\beta}_e + \bar{\beta}_\mu} \right. \\
&\cdot \left. \frac{\Gamma(1 + \bar{\beta}_e) \Gamma(1 + \bar{\beta}_\mu)}{\Gamma(1 + \bar{\beta}_e + \bar{\beta}_\mu)} {}_2F_1(1, \bar{\beta}_e; 1 + \bar{\beta}_e + \bar{\beta}_\mu; \epsilon) + \dots \right\},
\end{aligned} \tag{13}$$

$$\begin{aligned}
d\sigma^{\text{INT}}(s) &= d\sigma_0^{\text{INT}}(s) \Delta_e(s) \Delta_\mu(s) \epsilon^{\bar{\beta}_\mu} \frac{\Gamma(1 + \bar{\beta}_e) \Gamma(1 + \bar{\beta}_\mu)}{\Gamma(1 + \bar{\beta}_e + \bar{\beta}_\mu)} \frac{1}{\cos \delta_R} \cdot \\
&\cdot \text{Re} \left\{ e^{i\delta_R} \left[\frac{\epsilon}{1 + \left(\frac{\epsilon s}{M\Gamma}\right) \sin \delta_R e^{i\delta_R}} \right]^{\beta_e} \left[\frac{\epsilon}{\epsilon + \left(\frac{M\Gamma}{s}\right) \frac{e^{-i\delta_R}}{\sin \delta_R}} \right]^{\beta_{\text{int}}} \right\} + \dots,
\end{aligned} \tag{14}$$

$$\begin{aligned}
d\sigma^{\text{RES}}(s) &= d\sigma_0^{\text{RES}}(s) \Delta_e(s) \Delta_\mu(s) \varepsilon^{\beta_\mu} \frac{\Gamma(1+\bar{\beta}_e) \Gamma(1+\beta_\mu)}{\Gamma(1+\bar{\beta}_e+\beta_\mu)} \left| \frac{\varepsilon}{\varepsilon + \left(\frac{M\Gamma}{s}\right) \frac{e^{-i\delta_R}}{\sin \delta_R}} \right|^{2\beta_{\text{int}}} \\
&\left| \frac{\varepsilon}{1 + \left(\frac{\varepsilon s}{M\Gamma}\right) \sin \delta_R e^{i\delta_R}} \right|^{\beta_e} (\cos \beta_e \phi - \cot \delta_R \sin \beta_e \phi) + \dots,
\end{aligned} \tag{15}$$

where $\bar{\beta}_{e,\mu} = \beta_{e,\mu} + \beta_{\text{int}}$, $\bar{\beta}_e = \beta_e + 2\beta_{\text{int}}$.

A few comments are in order here. First of all the main β_{int} -dependence in eqs. (13-15) appears through exponentiated factors, which coincide with those found in refs. [6, 9], using the method of coherent states. Of course they also reproduce the exact one-loop calculations^[9, 10]. Furthermore the β_{int} -dependence drops out completely in the limiting case of narrow resonance production ($\Gamma \ll \Delta\omega$), as for example the J/ψ .

Physically this can be understood through the observation that the initial and final state can no longer interfere since the long time delay ($\tau \sim 1/\Gamma$) due to the resonance formation and decay.

Grouping together all non-infrared factors coming from $\Delta_e(s)$, $\Delta_\mu(s)$ and the Γ -functions in eqs. (13-15), as well as those coming from the non-soft terms of the electron and muon radiators, one can then write, as in ref. [14],

$$\frac{d\sigma}{d\Omega} = \sum_i^{\text{QED, INT, RES}} \frac{d\sigma_0^{(i)}}{d\Omega} \left\{ C_{\text{infra}}^{(i)} (1 + \bar{C}_F^{(i)}) + C_F^{(i)} \right\} \tag{16}$$

where the infrared factors $C_{\text{infra}}^{(i)}$ are simply obtained from eqs. (8-10), and

$$\begin{aligned}
\bar{C}_F^{(\text{QED})} &= \left(\frac{\alpha}{\pi}\right) \left[\Delta_e^{(1)} + \Delta_\mu^{(1)} \right] - \beta_\mu \varepsilon \\
&+ \left(\frac{\alpha}{\pi}\right)^2 \left[\Delta_e^{(2)} + \Delta_\mu^{(2)} + \Delta_e^{(1)} \Delta_\mu^{(1)} \right] - \frac{\pi^2}{6} \bar{\beta}_e \bar{\beta}_\mu - \frac{1}{4} \beta_e \bar{\beta}_e \varepsilon^{1-\bar{\beta}_e}
\end{aligned} \tag{17}$$

$$\begin{aligned} \bar{C}_F^{(INT)} &= \left(\frac{\alpha}{\pi}\right) \left[\Delta_e^{(1)} + \Delta_\mu^{(1)} \right] - \beta_\mu \varepsilon + \left(\frac{\alpha}{\pi}\right)^2 \left[\Delta_e^{(2)} + \Delta_\mu^{(2)} + \Delta_e^{(1)} \Delta_\mu^{(1)} \right] - \frac{\pi^2}{6} \bar{\beta}_e \bar{\beta}_\mu \\ &- \bar{\beta}_e \frac{\cos \phi (\beta_e + 1) + \tan \delta_R \sin \phi (\beta_e + 1)}{\cos \phi \beta_e + \tan \delta_R \sin \phi \beta_e} \left| \frac{\varepsilon}{1 + \frac{\varepsilon s}{M_R^2 - s}} \right| \end{aligned} \quad (18)$$

$$- \frac{1}{4} \beta_e \bar{\beta}_e \frac{\cos \phi + \tan \delta_R \sin \phi}{\cos \phi \beta_e + \tan \delta_R \sin \phi \beta_e} \left| \frac{\varepsilon}{1 + \frac{\varepsilon s}{M_R^2 - s}} \right|^{1 - \beta_e}$$

$$\begin{aligned} \bar{C}_F^{(RES)} &= \left(\frac{\alpha}{\pi}\right) \left[\Delta_e^{(1)} + \Delta_\mu^{(1)} \right] - \beta_\mu \varepsilon + \left(\frac{\alpha}{\pi}\right)^2 \left[\Delta_e^{(2)} + \Delta_\mu^{(2)} + \Delta_e^{(1)} \Delta_\mu^{(1)} \right] - \frac{\pi^2}{6} \bar{\beta}_e \bar{\beta}_\mu \\ &- 2 \bar{\beta}_e \frac{\cos \phi (\beta_e + 1) - \cot \delta_R \sin \phi (\beta_e + 1)}{\cos \phi \beta_e - \cot \delta_R \sin \phi \beta_e} \left| \frac{\varepsilon}{1 + \frac{\varepsilon s}{M_R^2 - s}} \right| \end{aligned} \quad (19)$$

$$- \frac{1}{4} \beta_e \bar{\beta}_e \frac{\cos \phi - \cot \delta_R \sin \phi}{\cos \phi \beta_e - \cot \delta_R \sin \phi \beta_e} \left| \frac{\varepsilon}{1 + \frac{\varepsilon s}{M_R^2 - s}} \right|^{1 - \beta_e}$$

Finally the factors $C_F^{(i)}$ contain other $O(\alpha)$ finite terms, coming from bremsstrahlung and box diagrams, odd in the exchange $\theta \leftrightarrow \pi - \theta$, etc., and can be obtained from refs. [2,14].

The above equations represent our final result, which describes the radiative correction factors to an accuracy better than (1%). The effect of the new terms of $O(\beta^2, \varepsilon\beta)$ in eqs. (18) is shown^[14] in Figs. (1, 2), where we plot the ratios

$$R^{(i)} = \frac{d\sigma^{(i)} \left[1 + O(\alpha) + O(\beta\varepsilon) + O(\alpha^2) \right]}{d\sigma^{(i)} \left[1 + O(\alpha) \right]}$$

for $i = \text{QED, INT, RES}$, for $\varepsilon = 0.01$ and $\varepsilon = 0.05$.

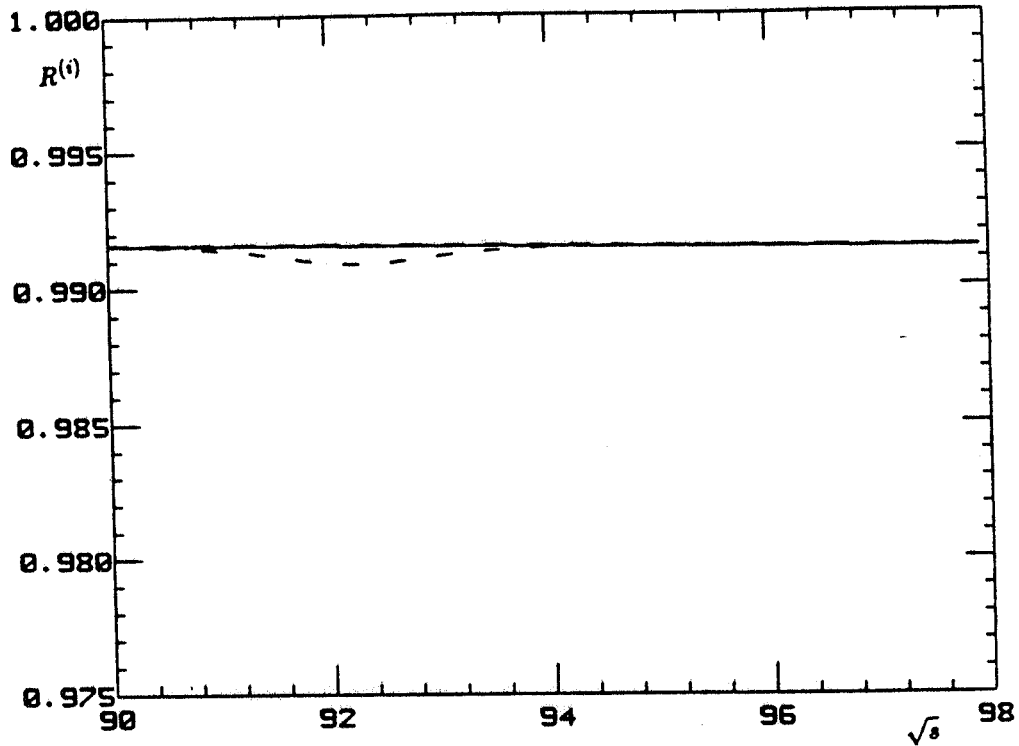


Fig. 1 Ratio $R^{(i)} = \frac{d\sigma^{(i)}[1+o(\alpha)+o(\beta\epsilon)+o(\alpha^2)]}{d\sigma^{(i)}[1+o(\alpha)]}$ for $e^+e^- \rightarrow \mu^+\mu^-$, with $i=QED$ (solid line), INT (dashed line), RES (dotted-dashed line) for $\epsilon = 0.01$. The values of M and Γ are taken to be 92 Gev and 2.6 Gev respectively.

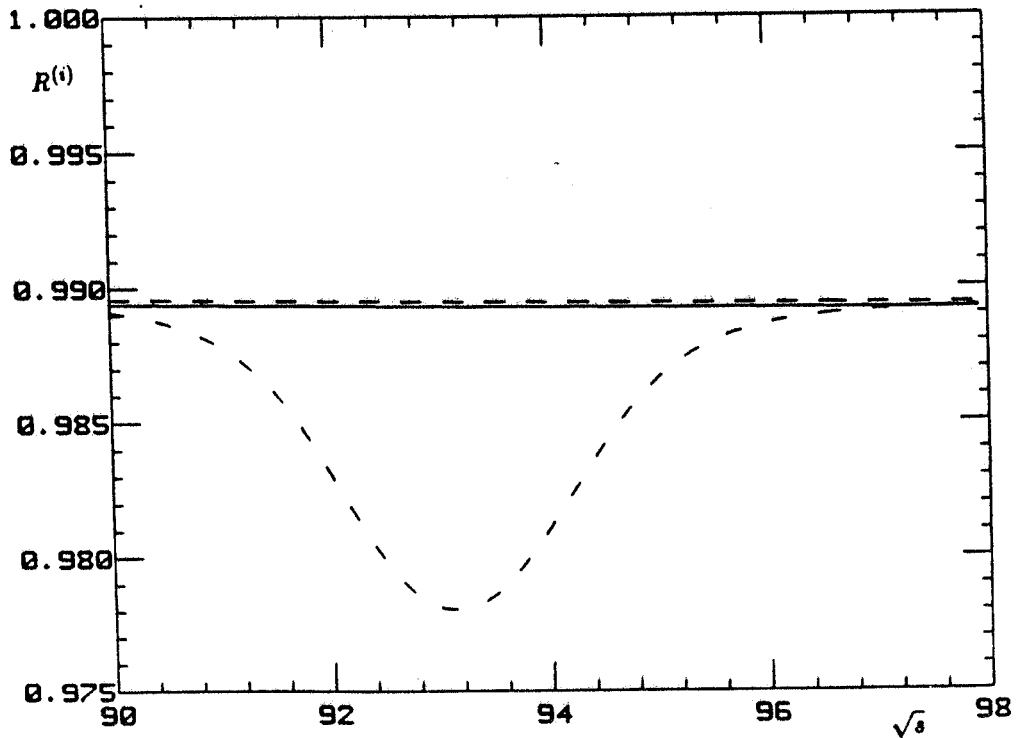


Fig. 2 Same as in fig. (1) for $\epsilon = 0.05$.

Notice that the factors $C_{\text{infra}}^{(i)}$ do not appear in the ratios $R^{(i)}$. We have taken the scattering angle $\theta = \pi/2$ in the factors $C_F^{(i)}$ to automatically cancel the box diagram and other non factorizable contributions.

As is clear from Figs. (1,2) the QED and INT corrections are practically constant in the resonant region to a value of about -0.01, while the RES correction is modulated essentially by the term $1 - (\beta_e^2 \phi^2/2)$, with an extra factor of $O(0.01 - 0.02)$.

The extension of our results to the case of the Z line shape is straightforward. It simply corresponds to take the limit $\beta_\mu \beta_{\text{int}} = 0$, $\Delta_\mu = 1$ and $\varepsilon = 1 - (4\mu^2/s)$ in eq. (15). Then one simply obtains for $s \approx M_R^2$

$$\sigma(s) \approx \sigma_0(s) \left| \frac{M_R^2 - s}{M_R^2} \right|^{\beta_e} \frac{\sin(1 - \beta_e) \delta_R}{\sin \delta_R} \frac{\pi \beta_e}{\sin \pi \beta_e} \Delta_e(s) \quad (23)$$

which agrees with refs [6, 9, 18] up to constant factors of $O(\beta^2)$.

To conclude, we have explicitly shown that the approach of the coherent states and that of the structure functions offer two complementary methods in QED to achieve the theoretical accuracy required for precision measurements at LEP/SLC energies. They give identical results for the exponentiated and finite $O(\alpha)$ factors relative to the initial and final states radiation. Moreover we have also shown how to include the interference effects coming from initial and final state radiation within the approach of structure functions in QED. The additional $O(\alpha^2)$ corrections improve the theoretical accuracy to 0 (0.1%). The overall picture provides simple analytical formulae which can be easily extended to e^+e^- reactions other than $e^+e^- \rightarrow \mu^+\mu^-$, and in addition the method is suitable for MonteCarlo applications.

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