



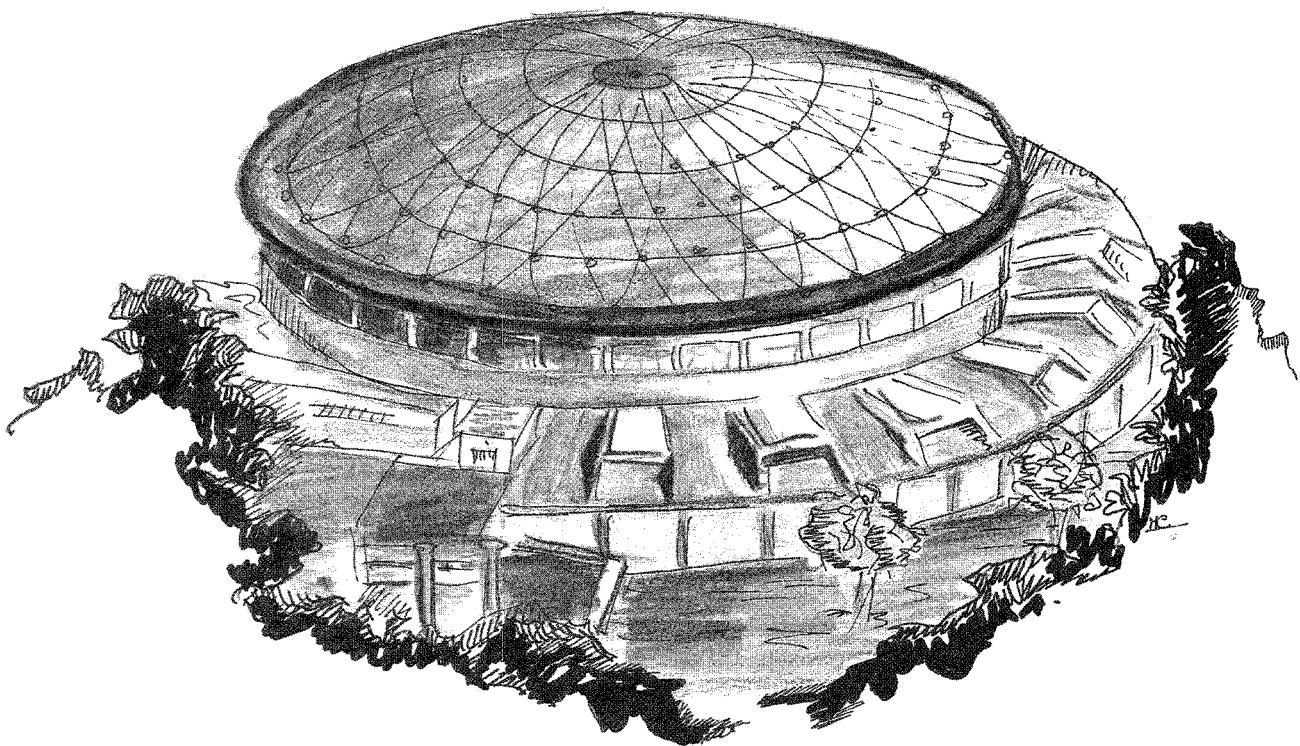
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Bloch Nordsieck Regularization of QCD Transverse Momentum Distributions



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**Bloch Nordsieck Regularization  
of  
QCD Transverse Momentum Distributions**

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**Abstract**

An expression for transverse momentum distributions is presented for Drell-Yan type processes : it is infrared finite, includes soft gluon summation in the leading logarithmic approximation and reproduces the first order perturbative results at large  $p_T$ . In addition, our expression incorporates soft QCD radiative corrections to that part of the cross-section where hard bremsstrahlung dominates. We compute W-transverse momentum distributions and average values for a set of different energies :  $\sqrt{s}=630$  GeV, 1.8 TeV, 6 TeV and 18 TeV.

## 1. Introduction

The question of how final state particles acquire their transverse momentum in hadron-hadron collisions is of great theoretical and phenomenological interest since it can shed light on the dynamics of interaction between hadronic constituents. A typical example is offered by the Drell-Yan process, in which the final state muon pair is seen to acquire a  $Q^2$ -dependent transverse momentum through initial state bremsstrahlung from the colliding quark-antiquark pair<sup>1,2</sup>. This mechanism was shown to hold for Drell-Yan pairs observed at ISR and FNAL<sup>3</sup>. At higher energies, the comparison between theory and experiments for the case of W and Z production is considered to be an important test of QCD at the Cern Collider<sup>4,5</sup>. For transverse momenta of the order of less than 10% of the W-mass, the calculation involves the use of soft gluon summation techniques, while for higher transverse momentum values the usual perturbative expansion is quite adequate. However, since the perturbative calculation diverges at small  $p_T$ , particular care must be taken in joining soft and hard terms, so as to reproduce correctly the perturbative limit, and at the same time avoid double counting in the soft region. We would like to point out that this is a general problem, present also in the case of jet production, where the question of how to include both high and low  $E_T$  jets (the latter are often called, perhaps improperly, mini-jets) has not been completely solved. In this paper we present a formalism for regularizing the transverse momentum divergence which is based on an infrared regulator, rather than the usual delta-function prescription. We believe this method to be numerically simpler than the ones present in the literature and to offer, in addition, a very transparent physical picture which can be applied to other problems such as minijets.

We shall start with the soft QCD bremsstrahlung formula which gives a finite distribution for W-production in the low- $p_T$  region. This formula can be derived using a semi-classical approach based on the Bloch-Nordsieck method for QED, but its validity has also been checked through perturbative calculations.

We then apply the same formula to perform soft radiative corrections to production of a W-boson and one hard jet. The subtractions needed to avoid double counting in the soft region will then show how the soft bremsstrahlung distribution can act as an infrared regulator. We present results at four different energies,  $\sqrt{s} = 630$  GeV, 1.8 TeV, 6 TeV and 18 TeV and show both the normalized transverse momentum distributions as well as the absolute cross-sections.

## 2. Leading log QCD Radiative Corrections to W Production

The 2-vector  $\vec{p}_T$  probability distribution of the W may be written in the parton model

with QCD radiation as

$$\frac{dP}{d\vec{p}_T^W} = \frac{dP}{d\vec{K}_T}$$

where

$$\vec{p}_T^W = - \sum \vec{p}^{gluons} \equiv \vec{K}_T \quad (1)$$

obtains its value from the transverse momentum due to initial state bremsstrahlung. The multiple soft-gluon emission calculation (to all orders in  $\alpha_s$ ) is available in the leading log approximation (LLA)<sup>5,6)</sup>. A calculation which includes hard gluon emission probability is available to order  $\alpha_s$  in ref.(5). A Monte Carlo calculation is available to order  $\alpha_s^2$  in ref.(7).

In the following, we shall approach the problem using a semi-classical 4-dimensional derivation.

For soft massless quanta emitted by a semi-classical source, the 4-dimensional probability distribution is given by <sup>8)</sup>

$$d^4 P(K) = \sum P(\{n_{k'}\}) \delta^4(K - \sum k' n_{k'}) d^4 K \quad (2)$$

The 4-dimensional  $\delta$ -function selects the distributions with the correct energy-momentum loss  $K$ . The above distribution then reduces to

$$d^4 P(K) = \int \frac{d^4 x}{(2\pi)^4} \exp\{iK.x - \int d^3 n(k)[1 - \exp(-ik.x)]\} d^4 K \quad (3)$$

where

$$d^3 n(k) = \frac{d^3 k}{2k_0} |j_\mu(k)|^2 \quad (4)$$

with

$$|j_\mu(k)|^2 = c_{ij} \frac{2\alpha_s}{\pi^2} \frac{1}{k_T^2}$$

and  $c_{ij} = \frac{4}{3}$  or 3 for quark or gluon source respectively.

Thus, the distribution of the vector boson transverse momentum due to soft initial radiation is given by

$$\frac{d^2 P(K)}{d^2 \vec{K}_T} = \int dK_0 \int dK_3 \frac{d^4 P}{d^4 K} \quad (5)$$

Performing the integrals in eq.(3-5), one obtains for the soft radiation

$$\frac{dP^{soft}}{p_T dp_T} = \int_0^\infty b db J_0(p_T b) \exp\{-h(E_W; b)\}, \quad (6)$$

where

$$h(E_W; b) = \frac{8}{3\pi} \int_0^{E_W} \frac{dk_T}{k_T} \alpha_s(k_T^2) \ln \frac{E_W + \sqrt{E_W^2 - k_T^2}}{E_W - \sqrt{E_W^2 - k_T^2}} [1 - J_0(bk_T)] \quad (7)$$

$$\equiv \int_0^{E_W} dk_T (2\pi k_T) f(k_T) [1 - J_0(bk_T)]$$

with

$$f(k_T) = \frac{4\alpha_s(k_T^2)}{3\pi^2} \frac{1}{k_T^2} \ln \frac{E_W + \sqrt{E_W^2 - k_T^2}}{E_W - \sqrt{E_W^2 - k_T^2}} \quad (8)$$

Notice that the soft distribution depends upon the behaviour of  $\alpha_s$ , as  $k_T \rightarrow 0$  as well as from the upper limit  $E_W$ , which is rather arbitrary. For instance, the soft distribution discussed in ref.(6) was obtained with a singular, but integrable  $\alpha_s$  given by

$$\alpha_s(k_T^2) = \frac{12\pi}{25} \frac{p}{\ln[1 + p(k_T^2/\Lambda^2)^p]}$$

with  $p = \frac{5}{6}$  and  $E_W \approx M_W/2^{(6)}$ . This choice of  $\alpha_s$  avoided the introduction of the intrinsic transverse momentum, a parameter needed to justify a non zero  $\langle p_T \rangle$  at very small energies. Different choices for  $\alpha_s$  are given in refs.5 and 9.

The soft contribution to the differential cross-section for W-production can then be written as

$$\frac{d\sigma^{soft}}{d\vec{p}_T dy} = \sigma^0 \int \int F(\vec{Q}_T) \delta(\vec{k}_T) \delta(\vec{p}_T - \vec{k}_T - \vec{Q}_T) d^2\vec{Q}_T d^2\vec{k}_T \quad (9)$$

with

$$\sigma^0 = \frac{d\sigma^W}{dy} = \hat{\sigma}^0 \sum_i [q_i(x_1, m_W^2) \bar{q}_i(x_2, m_W^2) + 1 \leftrightarrow 2] \quad (10)$$

where  $x_1 = \sqrt{\tau}e^y$ ,  $x_2 = \sqrt{\tau}e^{-y}$ ,  $\tau = \frac{m_W^2}{s}$ ,  $\hat{\sigma}^0 = \frac{\pi^2 \alpha}{3 \sin^2 \theta_W} \frac{1}{s}$   
and with

$$F(\vec{Q}_T) \equiv \frac{dP^{soft}}{d^2\vec{Q}_T} \quad (11)$$

In eq.(9) we have folded the soft distribution  $F(\vec{Q}_T)$  with the perturbative contribution, which in this case is just a  $\delta$ -function in transverse momentum.

The above distribution applies to the case when the emitted gluon energy is not larger than 20-30 % of the W-mass. For really hard gluon emission, when  $p_T^W \approx m_W$ , the  $p_T$ -distribution is given by the first and eventually second order perturbative expression. There is however an intermediate range such as

$$\frac{m_W}{4} \leq p_T^W \leq \frac{m_W}{2}$$

where the W-boson acquires its transverse momentum both through soft and hard gluon emission and we shall concentrate in that region in what follows.

### 3. Leading log QCD Radiative Corrections to $W + \text{jet}$ processes

Consider the process drawn in fig.1 to which the subprocess diagrams of fig.2 contribute.

We now apply the formalism of the preceding section to the process of fig.1 and write

$$\begin{aligned} \frac{d\sigma^{W+\text{jet}}}{d^2\vec{p}_T dy} &= \int \frac{dL^{(1)}}{d^2\vec{k}_T dy} F(\vec{Q}_T) d^2\vec{k}_T d^2\vec{Q}_T \delta^2(\vec{p}_T - \vec{k}_T - \vec{Q}_T) \\ &= \int \frac{dL^{(1)}}{d^2\vec{k}_T dy} F(\vec{p}_T - \vec{k}_T) d^2\vec{k}_T \end{aligned} \quad (12)$$

where  $\frac{dL^{(1)}}{d^2\vec{k}_T dy}$  corresponds to the cross-section for  $W + \text{jet}$  emission and  $F(\vec{Q}_T)$  is the probability distribution for soft QCD radiation of total momentum  $\vec{Q}_T$ . In this way the total observed  $W$ - $p_T$  appears as the convolution of two contributions: soft initial state radiation together with hard scattering Compton and  $q\bar{q}$  annihilation in  $Wg$ . However, of the two distributions, while  $F(\vec{p}_T)$  is finite as  $\vec{p}_T \rightarrow 0$ ,  $\frac{dL^{(1)}}{d^2\vec{k}_T dy}$  has to be defined. To do that, we start by considering the differential cross-section for the process shown in fig.1, i.e. order  $\alpha_s$  corrections to  $W$ -production.

This cross-section is given by <sup>10)</sup>

$$\frac{d^2\sigma^{(1)}}{dp_T dy} = \frac{d\sigma^{q\bar{q}}}{dp_T dy} + \frac{d\sigma^{gg}}{dp_T dy} \quad (13)$$

with

$$\begin{aligned} \frac{d\sigma^{q\bar{q}}}{dp_T dy} &= \frac{4\pi\alpha}{9\sin^2\theta_W} \frac{\alpha_s(p_T^2)}{s} \frac{1}{p_T} \int_{x_{min}}^1 dx_1 \\ &\sum_i [q_i(x_1, m_W^2) \bar{q}_i(x_2, m_W^2) + 1 \leftrightarrow 2] \left( \frac{1 + \frac{\tau^2}{x_1^2 x_2^2} - \frac{x_T^2}{2x_1 x_2}}{x_1 - \frac{1}{2}\bar{x}_T e^y} \right) \end{aligned} \quad (14)$$

and

$$\frac{d\sigma^{gg}}{dp_T dy} = \frac{\pi\alpha}{3\sin^2\theta_W} \alpha_s(p_T^2) \frac{p_T}{s^2} \int_{x_{min}}^1 \frac{dx_1}{x_1 - \frac{1}{2}\bar{x}_T e^y} \frac{1}{x_1^2 x_2^2}$$

$$\sum_i \left[ q_i(x_1, m_W^2) G(x_2, m_W^2) \frac{(x_1 x_2 - \tau)^2 + \frac{1}{4}(x_1 x_2 + \tau + V)^2}{x_1 x_2 - \tau + V} + (1 \leftrightarrow 2, V \leftrightarrow -V) \right] \quad (15)$$

and with

$$\tau = \frac{m_W^2}{s} \quad x_T^2 = \frac{4p_T^2}{s} \quad \bar{x}_T^2 = x_T^2 + 4\tau$$

We also have the following definitions :

$$x_2 = \frac{\frac{1}{2}\bar{x}_T e^{-y} x_1 - \tau}{x_1 - \frac{1}{2}\bar{x}_T e^y}, \quad x_{min} = \frac{\frac{1}{2}\bar{x}_T e^y - \tau}{1 - \frac{1}{2}\bar{x}_T e^{-y}}, \quad V = x_1 x_2 + \tau - x_1 \bar{x}_T e^{-y}$$

where  $\sqrt{s}$  is the proton-antiproton center of mass energy,  $G$  and  $q_i$  are the (singular) gluon and quark densities respectively. We shall omit the explicit  $y$ -dependence in the following. All the calculations will be done for  $y=0$ .

In order to get the overall  $W$ -differential cross-section, we have to add the soft contribution of eq.(9) to the hard one of eq.(12) without double counting. Double counting may occur unless the hard contribution in the soft radiation region is subtracted. Let  $q_{max}$  be the maximum value for such soft gluon emission defined as the  $k_T$ -integration limit  $E_W$  in eq.(7). Then one can avoid double counting by writing <sup>11)</sup>

$$\frac{d\sigma}{d^2\vec{p}_T} = \sigma^0 F(\vec{p}_T) + \int_0^{q_{max}} \frac{dL^{(1)}}{d^2\vec{k}_T} [F(\vec{p}_T - \vec{k}_T) - F(\vec{p}_T)] d^2\vec{k}_T + \int_{q_{max}}^{\infty} \frac{dL^{(1)}}{d^2\vec{k}_T} F(\vec{p}_T - \vec{k}_T) d^2\vec{k}_T \quad (16)$$

where, as  $k_T$  becomes soft, the contribution of  $L^{(1)}$  is reduced, and we have an expression for the differential cross-section that joins smoothly the soft and the hard contributions.

This distribution has to be such that the average value of  $p_T^2$  verifies

$$\langle p_T^2 \rangle = \langle p_T^2 \rangle^{1st \ order}$$

and this constraint will define  $\frac{dL^{(1)}}{d^2\vec{k}_T}$

First of all we rewrite eq.(16) as follows

$$\frac{d\sigma}{d^2\vec{p}_T} = (\sigma^0 + \sigma^1) F(\vec{p}_T) + \int_0^{\infty} \frac{dL^{(1)}}{d^2\vec{k}_T} [F(\vec{p}_T - \vec{k}_T) - F(\vec{p}_T)] d^2\vec{k}_T \quad (17)$$

where

$$\sigma^1 \equiv \int_{q_{max}}^{\infty} \frac{dL^{(1)}}{d^2\vec{k}_T} d^2\vec{k}_T \quad (18)$$

and we notice that  $\int d\vec{p}_T \frac{d\sigma}{d\vec{p}_T} = (\sigma^0 + \sigma^1)$  because the soft distribution  $F(\vec{p}_T)$  is normalized to 1.

Then

$$\begin{aligned} \langle p_T^2 \rangle &= \int p_T^2 F(\vec{p}_T) d^2\vec{p}_T + \frac{1}{(\sigma^0 + \sigma^1)} \int p_T^2 \int \frac{dL^{(1)}}{d^2\vec{k}_T} [F(\vec{p}_T - \vec{k}_T) - F(\vec{p}_T)] d^2\vec{k}_T d^2\vec{p}_T \\ &= \int p_T^2 F(\vec{p}_T) d^2\vec{p}_T + \frac{1}{(\sigma^0 + \sigma^1)} \int \frac{dL^{(1)}}{d^2\vec{k}_T} k_T^2 d^2\vec{k}_T \end{aligned}$$

and expanding  $F(\vec{p}_T)$  we have, to first order in  $\alpha_s$

$$\int p_T^2 f(p_T) d^2\vec{p}_T + \frac{1}{(\sigma^0 + \sigma^1)} \int \frac{dL^{(1)}}{d^2\vec{k}_T} k_T^2 d^2\vec{k}_T \equiv \frac{1}{(\sigma^0 + \sigma^1)} \int p_T^2 \frac{d\sigma^1}{d^2\vec{p}_T} d^2\vec{p}_T$$

where  $f(p_T)$  is defined in eq.(8) and  $\frac{d\sigma^1}{d^2 p_T}$  is the first order perturbative cross-section , eq.(13-15). Then,

$$\frac{dL^{(1)}}{d^2 k_T} \equiv \frac{d\sigma^1}{d^2 k_T} - (\sigma^0 + \sigma^1)f(k_T) \quad (19)$$

and from eq.(18)

$$\sigma^1 = \int_{q_{max}}^{\infty} \frac{d\sigma^1}{d^2 k_T} d^2 k_T \quad (20)$$

because  $f(k_T)$  is defined only up to  $k_T=q_{max}$  (the maximum transverse momentum for soft gluon emission).

The full distribution can now be written as

$$\begin{aligned} \frac{d\sigma}{d^2 p_T} &= \left[ \sigma^0 + \int_{q_{max}}^{\infty} \frac{d\sigma^1}{d^2 k_T} d^2 k_T \right] F^{soft}(\vec{p}_T) \\ &+ \int_0^{\infty} \left[ \frac{d\sigma^1}{d^2 k_T} - (\sigma^0 + \sigma^1)f(k_T) \right] \left[ F^{soft}(\vec{p}_T - \vec{k}_T) - F^{soft}(\vec{p}_T) \right] d^2 k_T \end{aligned} \quad (21)$$

where  $q_{max}$  is the only parameter we have to specify.

The maximum transverse momentum for the soft radiation is, for  $y=0$ <sup>3)</sup>,  $q_{max} = \frac{Q(1-z)}{2\sqrt{z}}$  with  $z = \frac{Q^2}{\hat{s}}$ , where  $\hat{s} = x_1 x_2 s$  is the energy squared of the subprocess  $q\bar{q} \rightarrow Wg$  and  $Q^2 = m_W^2$  in the case of W-production. Then the exact expression to calculate would be

$$\begin{aligned} \frac{d\sigma}{d^2 p_T} &= \int dx_1 f(x_1, Q^2) \bar{f}(x_2, Q^2) \left\{ (\hat{\sigma}^0 \delta(x_1 x_2 - \tau) + \hat{\sigma}^1) F^{soft}(\vec{p}_T, q_{max}(x_1, x_2)) + \right. \\ &\left. \int_0^{\infty} d^2 k_T \left[ \frac{d\hat{\sigma}^1}{d^2 k_T} - (\hat{\sigma}^0 \delta(x_1 x_2 - \tau) + \hat{\sigma}^1) f(k_T) \right] \left[ F^{soft}(\vec{p}_T - \vec{k}_T, q_{max}) - F^{soft}(\vec{p}_T, q_{max}) \right] \right\} \end{aligned}$$

$$\text{where } \hat{\sigma}^1 = \int_{q_{max}(x_1, x_2)}^{\infty} \frac{d\hat{\sigma}^1}{d^2 k_T} d^2 k_T$$

However, in order to make the numerical calculations easier, we have calculated the mean value for  $q_{max}$  at different energies in two different ways and computed the distribution as in eq.(21) at each energy for the two different values of  $q_{max}$ . Then we test how sensitive is the result to the value of  $q_{max}$ . For  $y=0$ , the averages are performed as follows:

$$\langle q_{max}^{(a)} \rangle = \frac{\sqrt{s}}{2} \frac{\int_{\sqrt{\tau}}^1 \frac{dx}{x^2} f_i(x) \bar{f}_j(x) (x - \frac{\tau}{x})}{\int_{\sqrt{\tau}}^1 \frac{dx}{x^2} f_i(x) \bar{f}_j(x)} \quad (22)$$

and

$$\langle q_{max}^{(b)} \rangle = \frac{\sqrt{s}}{2} \frac{\int_{\sqrt{\tau}}^1 \frac{dx}{x} f_i(x) \bar{f}_j(x) (x - \frac{\tau}{x})}{\int_{\sqrt{\tau}}^1 \frac{dx}{x} f_i(x) \bar{f}_j(x)} \quad (23)$$

with  $\tau = Q^2/s$  and  $f_i(x)$  is a parton density.

In Table I we show the results for  $\sqrt{s} = .63, 1.8, 6$  and  $18$  Tev. Notice that the value of  $q_{max}$  increases with energy but stabilizes around  $\sqrt{s} = 6$  Tev.

Table I		
$\sqrt{s}$ (Tev)	$q_{max}^{(a)}$ (Gev)	$q_{max}^{(b)}$ (Gev)
.63	18.4	22.6
1.8	23.8	32.6
6	24.8	36.6
18	24.6	36.9

We can now calculate the  $p_T$ -distribution for different  $\sqrt{s}$  values. Using the expression for  $F^{soft}(\vec{p}_T)$  (eq.6,11) we obtain

$$\frac{d\sigma}{dp_T} = (\sigma^0 + \sigma^1) \frac{dP^{soft}}{dp_T} + p_T \int dk_T \left[ \frac{d\sigma^1}{dk_T} - (\sigma^0 + \sigma^1)(2\pi k_T) f(k_T) \right] R(p_T, k_T; q_{max}) \quad (24)$$

where the Bloch Nordsieck infrared regulator has been defined as

$$R(p_T, k_T; q_{max}) = \int bdb J_0(bp_T) e^{-h(b, q_{max})} [J_0(bk_T) - 1]$$

and the integration in eq.(24) extends from zero to all the allowed values.

In the numerical calculation of  $\frac{d\sigma^1}{dk_T}$  we have used  $\alpha_s(k_T^2)$  rather than  $\alpha_s(Q^2)$  as in ref.(5). This has been done so as to be consistent with the argument of  $\alpha_s$  which appears in  $f(k_T)$ . Then for the small  $k_T$  limit we follow ref.(5) and use the expression

$$\alpha_s(k_T^2) = \frac{12\pi}{27} \frac{1}{\ln\left(\frac{k_T^2 + a\Lambda^2}{\Lambda^2}\right)}$$

with  $a=2$ .

In fig. 3 we show the normalized  $p_T$ -distribution for the two values of  $q_{max}$  for the same four values of the total energy as before. We have used the set of parton densities given by Eichten et al. (EHLQ)<sup>12)</sup> with  $\Lambda=0.2$  GeV.

One can see that the two values of  $q_{max}$  give a very similar normalized distribution even at higher energies. This is an indication of how stable our regularization procedure is with regards to the choice of the arbitrary separation between soft and hard regime.

From the above distributions one can numerically calculate the average transverse momentum value acquired by the W-boson at different energies : it is, again, rather independent of the choice of  $q_{max}$ . We show, in Table II, the relative numbers.

Table II

$\sqrt{s}$ (Tev)	$\langle p_T \rangle^{(a)}$ (Gev)	$\langle p_T \rangle^{(b)}$ (Gev)
.63	7.27	7.36
1.8	10.96	11.15
6	14.86	15.22
18	17.8	18.6

While the probability distribution is  $q_{max}$ -independent, this is not true for the differential cross-section, which depends upon the chosen  $q_{max}$  value through the quantity  $\sigma_{tot} = \sigma^0 + \sigma^{(1)}$ . In Table III we show the values of  $\sigma^0$  (eq.10) and  $\sigma^1(q_{max})$  (eq.20) for the four energy values.

Table III

$\sqrt{s}$ (Tev)	$\sigma^0$	$\sigma^1(a)$	$\sigma^1(b)$
.63	1.9 nb	.15 nb	.10 nb
1.8	3.7 nb	.5 nb	.3 nb
6	7.5 nb	2.0 nb	1.1 nb
18	14.6 nb	6.0 nb	3.4 nb

In figs.4 and 5 we plot the differential cross-section from eq.(24) for the two values of  $q_{max}$ . At large  $p_T$  values, all the cross-sections correctly tend to their first order perturbative result. An example of how our regularization procedure works in recovering the first order perturbative result at large  $p_T$  is shown in fig.6, for  $\sqrt{s} = 1.8$  Tev.

To further illustrate our normalization prescription, we plot separately in fig.7, the soft contribution, the hard one conveniently regularized once subtraction is performed, and the total contribution (full line) to the differential cross-section for  $\sqrt{s} = 6$  Tev and  $q_{max} = 36.6$  Gev.

#### ***4. Conclusions***

The formalism which we have presented in this paper regularizes the infrared divergence encountered in QCD for trasverse momentum distributions, through a finite Bloch Nordsieck type function . This regulator is used in the cross section instead of the usual delta-function prescription, to avoid the divergence at small  $p_T$  in the first order expansion. Its physical justification lies in the fact that the first order calculation is meant to reproduce a physical process in which a hard jet is observed : in such a case one must also perform soft radiative corrections and it is through the latter that the Bloch Nordsieck distribution appears. At  $\sqrt{s} = 630 \text{ GeV}$  our method reproduces within a few percent the calculations by Altarelli et al. in the soft region and it can be improved further by adding non leading logarithmic corrections <sup>13)</sup>. At Tevatron energies, our calculation predicts an average transverse momentum for the W-boson lower than that in ref.(5), (see Table II), and not very different from the one found at CERN energies.

#### ***Acknwoledgments***

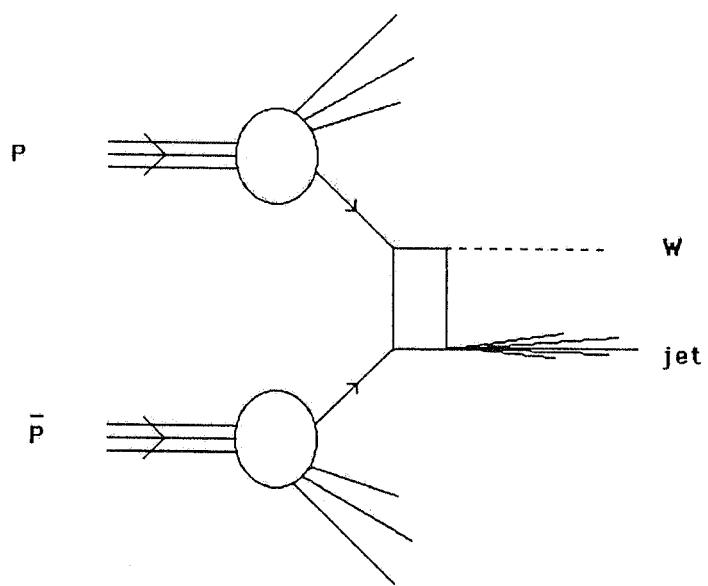
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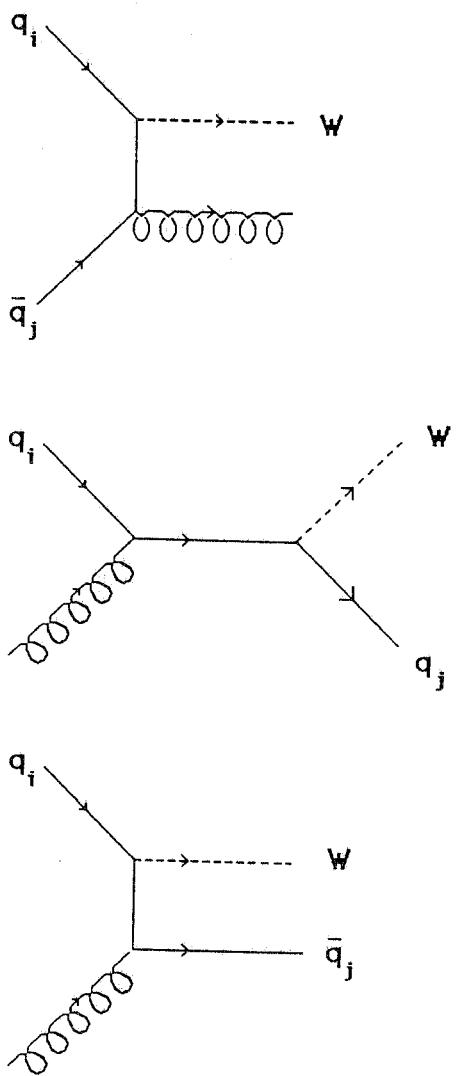
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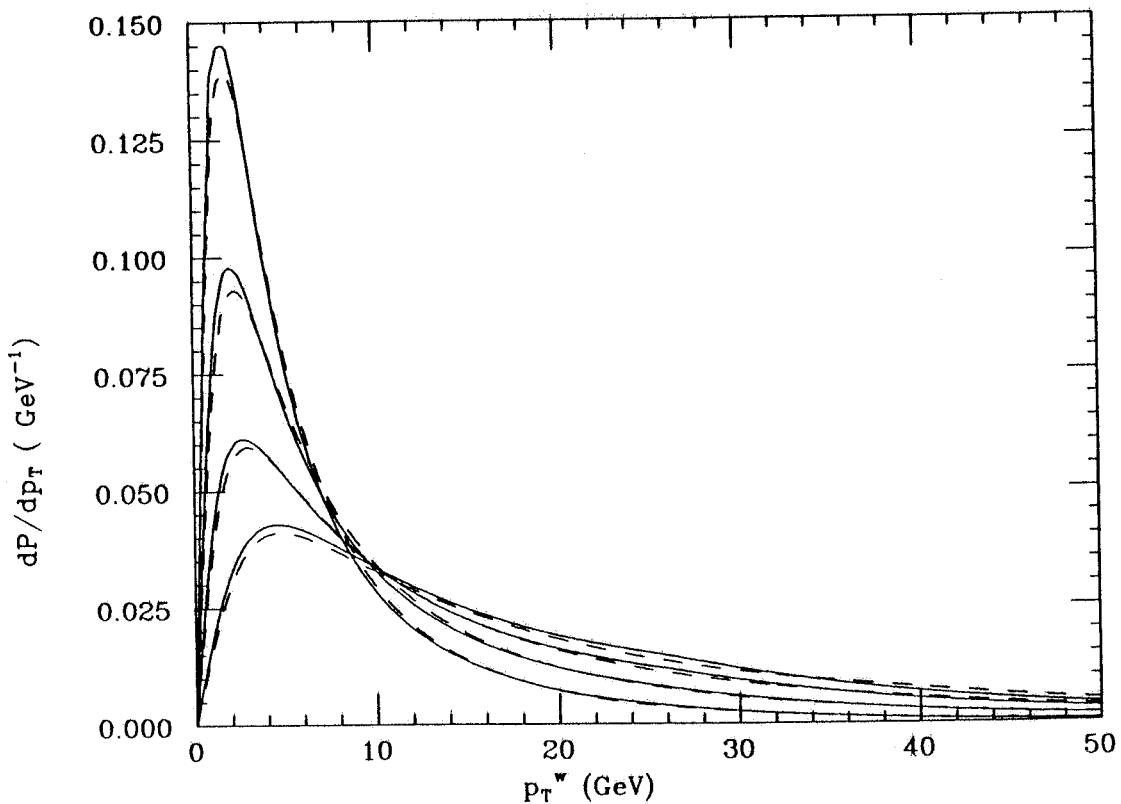
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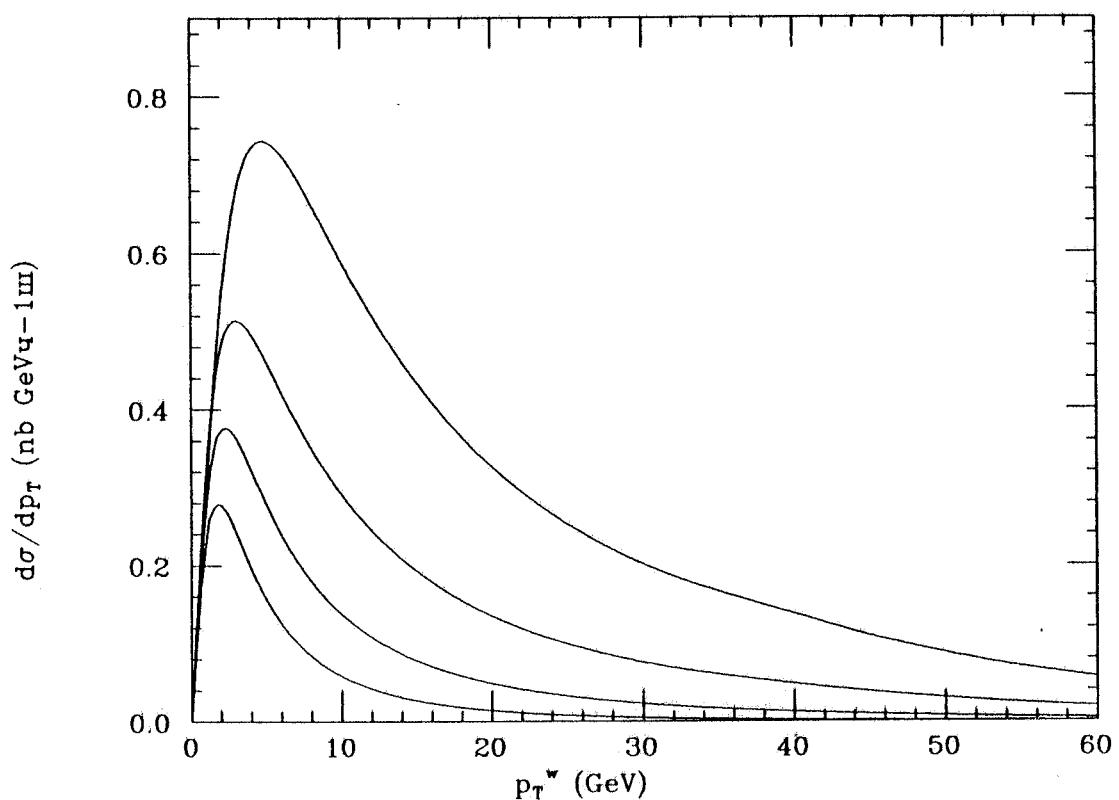
**Fig.1**  $W$ +jet production in  $p\bar{p}$  collisions.



**Fig.2** Feynman diagrams for  $W$ -production through  $q\bar{q}$  annihilation and Compton scattering processes.



**Fig.3** The normalized  $p_T$ -distribution for the two different values of  $q_{max}$  for  $\sqrt{s} = .63, 1.8, 6$  and  $18$  Tev.



**Fig.4** The differential cross-section for the set of higher values of  $q_{max}$ , ( $q_{max}^{(b)}$ ), for the same energy values as in fig.3

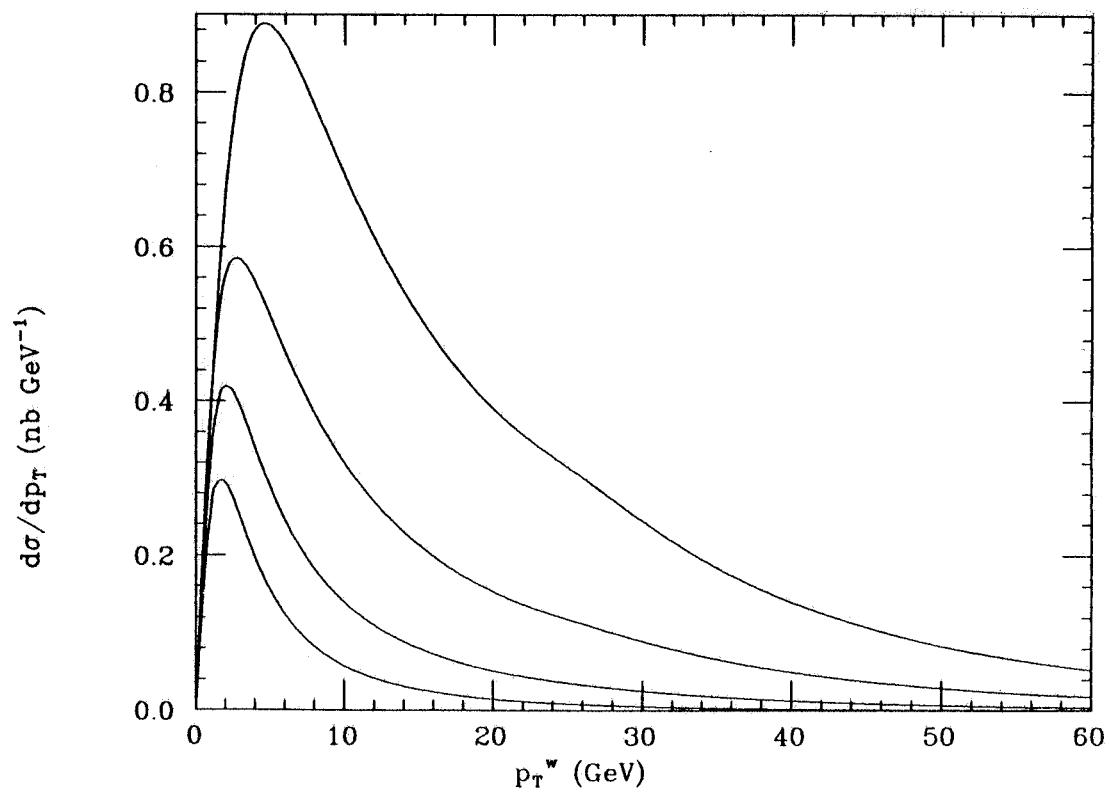


Fig.5 As in fig.4, for the set of lower values of  $q_{max}$ , ( $q_{max}^{(a)}$ ).

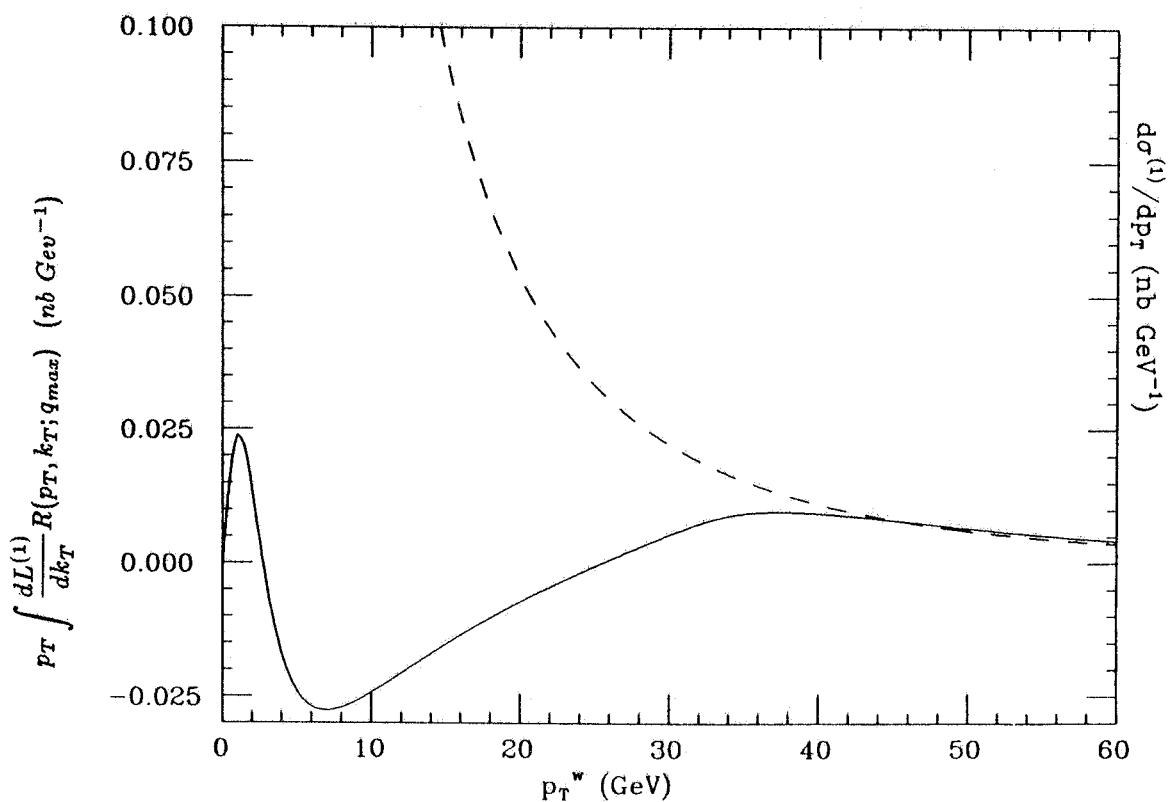
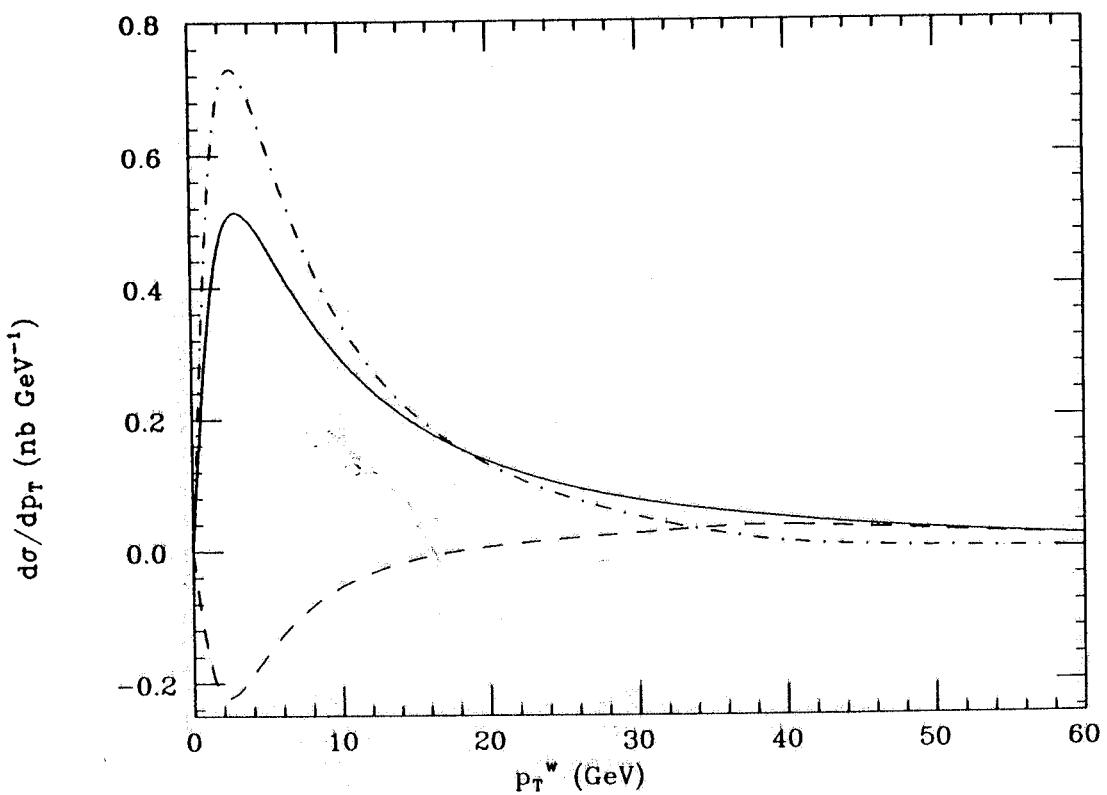


Fig.6 The first order perturbative distribution (dashed line) and the hard part conveniently regularized (full line) for  $\sqrt{s} = 1.8$  Tev and  $q_{max} = 32.6$  Gev.



**Fig.7** The soft contribution (point dashed line), the regularized hard one (dashed line) and the total differential cross-section for  $\sqrt{s} = 6$  Tev and  $q_{max} = 36.6$  Gev.