



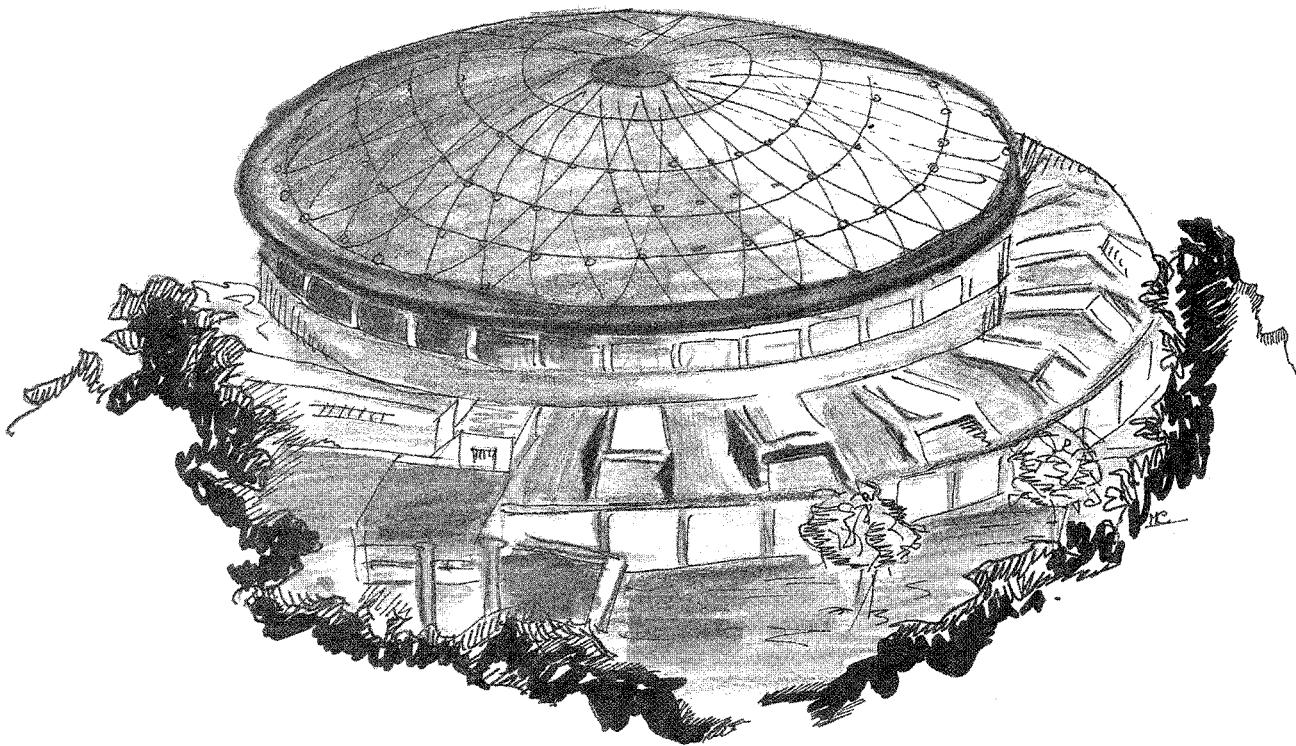
# **Laboratori Nazionali di Frascati**

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## Bremsstrahlung and the Underlying Event Structure

Invited talk at the Inter. Conf. on  
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## Bremsstrahlung and the Underlying Event Structure

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In this talk we address the problem of the structure of the underlying event and its dependence upon a specific trigger. Generally speaking, events selected without any special trigger, like the minimum bias events, need not have the same characteristics as those selected by the jet trigger or through the W and Z-boson selection criteria. We use perturbative QCD in order to unfold part of the structure. In particular we concentrate on the contribution to the underlying event structure which comes from initial state bremsstrahlung. In the present paper we discuss the relationship between transverse momentum and transverse energy for W and Z-boson production both in the low as well as in the high transverse momentum sector. In order to join smoothly these two regions we develop a formalism based on a Bloch Nordsieck regulator in the infrared sensitive region.

### 1. The Role of QCD in Hadronic Interactions

So far experimental<sup>[1]</sup> and theoretical<sup>[2,3]</sup> studies of the mini- (low- $E_T$ ) jet production cross section have been crucial in narrowing the gap between hard phenomena, where perturbative QCD can be safely applied, and the soft region where we expect fully non-perturbative mechanisms to be dominant. A basic point in narrowing this gap was to show that the mini-jet cross-section can very well be accounted for by perturbative QCD with reasonable values of the parameters. Indeed, a jet yield as large as the one observed by UA1, can be obtained through a first order QCD calculation, by assuming that the two partons scatter with a center of mass energy  $\sqrt{\hat{s}} \approx 2 \div 3 \text{ GeV}$ , which corresponds to a strong coupling constant  $\alpha_s \approx 0.3$ , a value still quite safe from the point of view of perturbative QCD. Of course,  $\alpha_s$  being smaller than one is not the only guiding line in approaching the low- $p_T$  region. In general we should distinguish between the following three regions :

(1)  $\alpha_s(\frac{\hat{s}}{\Lambda^2})$  small and

$\ln(\frac{s}{\hat{s}})$  small :

this is the region characterizing the production of high- $p_T$  jets which appear as the final state product of hard parton-parton scattering processes.

(2)  $\alpha_s(\frac{\hat{s}}{\Lambda^2})$  small and

$\ln(\frac{s}{\hat{s}})$  large :

since  $\alpha_s$  is still small, this region corresponds to perturbative parton-parton scattering. However we are approaching the large logarithms regions and multiple parton-parton scattering as well as multisoft gluon emission from initial state bremsstrahlung may become important. Summation techniques<sup>[4]</sup> can be used to describe global QCD energy emission.

(3)  $\alpha_s(\frac{\hat{s}}{\Lambda^2})$  large : fully non perturbative regime applies.

The study of the mini-jet cross section and the W and Z-transverse momentum distribution belong to the second region, minimum bias events to the third. Comparative studies of minimum bias and hard events can be useful in any attempt to understand the third region. In fact, the UA1<sup>[1]</sup> collaboration had analyzed - in a comparative fashion - the characteristics of minimum bias and hard scattering events. Some general characteristics of hard scattering versus minimum bias can be summarized as follows :

- (i) The average charged particle multiplicity is larger in hard events ( $E_T^{jet} \geq 5 \text{ GeV}$ ) than in minimum bias events
- (ii) The KNO particle distribution is narrower
- (iii) The single particle  $p_T$  distribution is flatter.

Many models have been constructed to explain some or all of these features : these include a statistical multi-source analysis by Meng Ta-Chung<sup>[5]</sup> and his collaborators, geometrical description within the multibranch approach by Hwa<sup>[6]</sup>, multiparton scattering in Montecarlo approaches etc. For instance, in the multisource statistical approach, the

narrowing of the KNO distribution is attributed to the method of event selection. In geometrical models, it is argued that impact parameter distributions carry information on average multiplicity, KNO distributions, underlying event shapes , etc. In bremsstrahlung type models<sup>[7]</sup>, type and energy of initial state partons are responsible for these quantities. For instance, in this model, the ratio of multiplicities in the minimum bias (mb) versus minijets (mj) is mainly determined through the  $q\bar{q}$  and gg coupling to gluons:

$$\frac{\langle n \rangle_{mb}}{\langle n \rangle_{mj}} \approx \frac{\langle n \rangle_{\text{only quarks}}}{\langle n \rangle_{\text{gluons}}} \approx \frac{\frac{4}{3}\alpha_s}{3\alpha_s} \approx \frac{1}{2} \quad (1.1)$$

a value close to the one observed experimentally.

Other models, like Pythia, combine a geometrical approach, through a specific input for the spatial distribution of quark and gluons inside the proton, with a QCD viewpoint through the introduction of multiparton scattering formalism.

To understand further and distinguish between different models, we suggest to select a hard scattering event with no final state interaction as is the case for W and Z-production and study that part of the event which can be calculated strictly from QCD, like the multigluon emission region. In minimum bias events, some of the quantities of interest are the average charged particle multiplicity,  $\langle p_T \rangle$  per charged particle,  $\langle \Sigma p_T^{ch} \rangle$ , KNO distribution both for the particle multiplicity as well as for the scalar  $E_T$ . These same quantities can be studied in the underlying event for the W and Z cases. In particular, one can study them as a function of the Intermediate Boson transverse momentum. It is to be expected that a high- $p_T$  W may be accompanied by an underlying event which looks quite different from minimum bias, whereas a low- $p_T$  boson event may not be so different. The main difference between a high  $p_T$  and a low  $p_T$  W lies in the colour structure of the event : the latter may very well have the same structure as minimum bias, since a low- $p_T$  W is produced with many soft gluons accompanying it while a high  $p_T$  W will be accompanied by a hard gluon, which can therefore be counted (as a jet) and carries an "observable" colour. By that we mean that the rest of the event, all the produced particles minus the W-boson and the hard gluon, now are not colourless anymore. The event is constructed through two quarks which have disappeared in a singlet state, one hard jet and soft particles which as a whole are not in a colourless state. On the contrary a low- $p_T$  W is accompanied by many soft gluons, which by definition are in a state over which there has been a colour averaging process. The underlying event , in this case, therefore may be similar to minimum bias in that it has the same colour structure. It would be interesting to compare W-events at high and low- $p_T$  values with mini-jet events or even high  $E_T$  events, where the colour structure has changed.

In what follows we shall set up a formalism needed to compare transverse energy distributions and average values for the W and the minimum bias case .

## 2. Bremsstrahlung Distributions in W - production

We shall start by making a separation between scalar  $E_T$  produced from initial state bremsstrahlung by the two quarks participating in the hard scattering process which leads to W-production and that produced by the other partons into which the original protons have broken up. We shall indicate quantities related to the breaking up of the proton as pertaining to the Hadronic Underlying Event (HUE) structure. The  $E_T$  and  $E_T/p_T$  distributions can be written as

$$\frac{dP}{dE_T} \approx \int \Pi_{HUE}(E_T - E'_T) \frac{dP_{brems}}{dE'_T} dE'_T \quad (2.1)$$

$$\frac{d^2P}{dE_T dp_T^W} \approx \int \Pi_{HUE}(E_T - E'_T) \frac{d^2P_{brems}}{dE'_T dp_T^W} dE'_T \quad (2.2)$$

so that

$$\langle E_T(p_T^W) \rangle = \langle E_T(p_T^W) \rangle_{HUE} + \langle E_T(p_T^W) \rangle_{brems} \quad (2.3)$$

In this talk we shall concentrate only on the bremsstrahlung distribution, for which no assumptions need to be made concerning the Underlying event. Thus we shall drop the subscript *brems* from all the distributions which follow. The average scalar  $E_T$  as calculated from this formalism will then be directly compared with experimental data, modulo assuming a suitable model for fragmentation into charged particles. We shall return to this point later, and concentrate for the time being in obtaining the W-transverse energy and momentum distributions from QCD.

To calculate  $\langle E_T \rangle_{brems}$  as a function of  $p_T^W$ , one needs the theoretical distribution in the two variables,  $E_T$  and  $p_T^W$ .

According to QCD, collinear partons, emitted from the hadrons, acquire their transverse momentum through gluon bremsstrahlung, prior to their annihilation into a given final state. These bremsstrahlung gluons will then hadronize and produce hadronic transverse energy. As is well known, however, the first order perturbative QCD calculation diverges logarithmically as  $p_T \rightarrow 0$ . This divergence is a fictitious one, since it arises from assuming that one can count individual zero momentum gluons. The realistic physical picture is rather that of an imbalance in energy and momentum due to the emission of an indistinguishable number of soft gluons. This distribution is finite as  $p_T \rightarrow 0$ , as shown by many authors. In general, one can write for the 4-dimensional distribution

$$d^4P(K) = \sum_k P(n_k) \delta^4(K - \sum_k n_k) = \int \frac{d^4x}{(2\pi)^4} \exp[iKx - \int d^3n(k)(1 - e^{-ikx})] \quad (2.4)$$

from which one can then obtain the transverse distribution, integrating over the energy and longitudinal momentum variables. One obtains [4,8,9,10,11]

$$\frac{dP}{dp_T^W} = p_T^W \int bdb J_0(p_T^W b) \exp[- \int \frac{d^3\vec{k}}{2k} |j_\mu(k)|^2 (1 - J_0(k_T b))] \quad (2.5)$$

The transverse energy distribution has been written as<sup>[12,13]</sup>

$$\frac{dP}{dE_T^W} = \int \frac{dt}{2\pi} e^{iE_T^W t} \exp[- \int \frac{d^3 \vec{k}}{2k} |j_\mu(k)|^2 (1 - e^{-ik_T t})] \quad (2.6)$$

while the double distribution in  $E_T$  and  $p_T$  can be written as<sup>[15]</sup>

$$\frac{dP}{dE_T^W dp_T^W} = p_T^W \int \frac{dt}{2\pi} e^{iE_T^W t} \int bdb J_0(p_T^W b) \exp[- \int \frac{d^3 \vec{k}}{2k} |j_\mu(k)|^2 (1 - J_0(k_T b) e^{-ik_T t})] \quad (2.7)$$

For arbitrary values of transvere momentum, one needs a distribution which is valid both in the soft as well as in the perturbative region, i.e. one which sums soft gluons to all orders in  $\alpha_s \ln(\frac{s}{\delta})$  and which reproduces the perturbative calculation at large  $p_T$ -values. Notice that in the perturbative region and to first order in  $\alpha_s$ ,

$$\frac{d\sigma^{\text{pert}}}{dE_T} = \frac{d\sigma^{(1)}}{dp_T} \quad (2.8)$$

and therefore

$$\frac{d\sigma^{\text{pert}}}{d^2 \vec{p}_T dE_T} = \frac{d\sigma^{(1)}}{d^2 \vec{p}_T} \delta(E_T - p_T) \quad (2.9)$$

One can write in general

$$\frac{d^3 \sigma}{d^2 \vec{p}_T dE_T} \approx \frac{d^3 \sigma^{\text{soft}}}{d^2 \vec{p}_T dE_T} + \frac{d^2 \sigma^{\text{hard}}}{d^2 \vec{p}_T} \quad (2.10)$$

where the first term at the right hand side describes the contribution of many soft gluons for which  $E_T \geq |\vec{p}_T|$ , while the second term represents the perturbative contribution. The problem then becomes that of avoiding double counting in the soft region. In the following we shall first present a formalism which deals with the transverse momentum distribution alone and in which the question of double counting is described in detail. Subsequently we shall extend the formalism to the  $E_T$ -distribution and finally to the double distribution.

### 3. Bloch – Nordsieck Regularization Method for $p_T$ – distributions

For the transverse momentum distribution of QCD radiation accompanying a hard scattering process like W-production, we propose the following expression<sup>[14]</sup> :

$$\frac{d\sigma}{d^2 \vec{p}_T} = \sigma^0 F(\vec{p}_T) + \int_0^{q_{\max}} \frac{dL^{(1)}}{d^2 \vec{k}_T} [F(\vec{p}_T - \vec{k}_T) - F(\vec{p}_T)] d^2 \vec{k}_T + \int_{q_{\max}}^\infty \frac{dL^{(1)}}{d^2 \vec{k}_T} F(\vec{p}_T - \vec{k}_T) d^2 \vec{k}_T \quad (3.1)$$

where

$$F(\vec{p}_T) = \frac{d^2 P_{soft}}{d^2 \vec{p}_T} \quad (3.2)$$

is the soft transverse momentum distribution discussed in the preceding session and ,upon integration over the azimuthal angle, is given by

$$\frac{dP_{soft}}{p_T dp_T} = \int_0^\infty bdb J_0(p_T b) \exp\{-h(q_{max}; b)\} \quad (3.3)$$

In eq.(3.3),

$$h(q_{max}; b) = \frac{8}{3\pi} \int_0^{q_{max}} \frac{dk_T}{k_T} \alpha_s(k_T^2) \ln \frac{q_{max} + \sqrt{q_{max}^2 - k_T^2}}{q_{max} - \sqrt{q_{max}^2 - k_T^2}} [1 - J_0(bk_T)] \quad (3.4)$$

$$\equiv \int_0^{q_{max}} dk_T (2\pi k_T) f(k_T) [1 - J_0(bk_T)] \quad (3.5)$$

with

$$f(k_T) = \frac{4\alpha_s(k_T^2)}{3\pi^2} \frac{1}{k_T^2} \ln \frac{q_{max} + \sqrt{q_{max}^2 - k_T^2}}{q_{max} - \sqrt{q_{max}^2 - k_T^2}} \quad (3.6)$$

Notice that the soft distribution depends upon the behaviour of  $\alpha_s$  as  $k_T \rightarrow 0$  as well as from the upper limit  $q_{max}$  which is arbitrary. As for the other quantities defined in eq.(3.1),  $\sigma^0$  is the lowest order W-production cross-section, while  $\frac{dL^{(1)}}{d^2 k_T}$  contains the perturbative contribution . Then, the third term in eq.(3.1) represents the folding of a perturbative contribution with a soft multigluon term. It corresponds physically to a situation in which the W-boson is produced in association with a jet and its transverse momentum is acquired both through initial state radiation as well as through the balancing hard jet. In order to avoid double counting in the soft region, we have subtracted, in the integrand, the soft contribution in the interval  $0 \div q_{max}$ . This subtraction appears in the second term at the right hand side of eq.(3.1). Thus ,as  $k_T$  becomes soft, the contribution of  $L^{(1)}$  is reduced and we have an expression for the differential cross-section that joins smoothly the soft and the hard contributions. It is important to notice that this procedure not only avoids double counting in the soft region, but *at the same time* regularizes the perturbative contribution. This method substitutes therefore the usual  $\delta$ -function subtraction procedure in QCD with a finite Bloch-Nordsieck regularization method.

So far,we have not defined the perturbative function  $\frac{dL^{(1)}}{d^2 k_T}$ . To do so, we first notice that

$$F(\vec{p}_T) \xrightarrow{\alpha_s \rightarrow 0} f(p_T)$$

and then rewrite eq.(3.1) as

$$\frac{d\sigma}{d^2 \vec{p}_T} = (\sigma^0 + \sigma^1) F(\vec{p}_T) + \int_0^\infty \frac{dL^{(1)}}{d^2 k_T} [F(\vec{p}_T - \vec{k}_T) - F(\vec{p}_T)] d^2 k_T \quad (3.7)$$

where

$$\sigma^1 \equiv \int_{q_{max}}^{\infty} \frac{dL^{(1)}}{d^2 k_T} d^2 k_T \quad (3.8)$$

and we notice that  $\int d\vec{p}_T \frac{d\sigma}{d\vec{p}_T} = (\sigma^0 + \sigma^1)$  because the soft distribution  $F(\vec{p}_T)$  is normalized to 1. With these definitions one can see that if

$$\frac{dL^{(1)}}{d^2 k_T} \equiv \frac{d\sigma^1}{d^2 k_T} - (\sigma^0 + \sigma^1) f(k_T) \theta(q_{max} - k_T) \quad (3.9)$$

then

$$\langle P_T^2 \rangle = \langle p_T^2 \rangle_{soft} + \langle p_T^2 \rangle_{hard} = \langle p_T^2 \rangle_{1st \ order} \quad (3.10)$$

The  $\theta$  function in eq.(3.9) appears because  $f(k_T)$  is defined only in the soft region. One can also rewrite the entire distribution as

$$\begin{aligned} \frac{d\sigma}{dp_T} &= (\sigma^0 + \sigma^1) \frac{dP_{soft}}{dp_T} \\ &+ p_T \int dk_T \left[ \frac{d\sigma^1}{dk_T} - (\sigma^0 + \sigma^1)(2\pi k_T) f(k_T) \theta(q_{max} - k_T) \right] R(p_T, k_T; q_{max}) \end{aligned} \quad (3.11)$$

where the Bloch Nordsieck infrared regulator has been defined as

$$R(p_T, k_T; q_{max}) = \int bdb J_0(bp_T) e^{-h(b, q_{max})} [J_0(bk_T) - 1] \quad (3.12),$$

and the integration in eq.(3.11) extends from zero to all the allowed values.

In fig.1 we show the normalized  $p_T$ -distribution for various energy values of the proton-antiproton system.

#### 4. Bloch – Nordsieck Regularization of Transverse Energy – Momentum Distributions

The formalism of the previous session extends easily to the transverse energy distribution. We start by defining, as in section 2, the soft  $E_T$ - distribution

$$H(E_T; q_{max}) = \frac{dP_{soft}}{dE_T} \quad (4.1)$$

with its limit

$$H(E_T; q_{max}) \xrightarrow{\alpha \rightarrow 0} (2\pi E_T) f(E_T) \quad (4.2)$$

as well as the first order perturbative contribution

$$\frac{d\sigma^{(1)}}{dE_T} \equiv \frac{d\sigma^{(1)}}{dp_T} \quad (4.3)$$

. Next, the infrared regulator is defined as

$$R_E(E_T, E'_T; q_{max}) = H(E_T - E'_T; q_{max}) - H(E_T; q_{max}) \quad (4.4)$$

where the subscript  $E$  refers to the energy variable and the total cross-section

$$\sigma_W = \sigma^0 + \sigma^1 = \sigma^0 + \int_{q_{max}}^{\infty} \frac{d\sigma^{(1)}}{dE_T} dE_T \quad (4.5)$$

The following step consists in constructing the hard part of the function to be folded with the Bloch-Nordsieck regulator, i.e.

$$\frac{dL^{(1)}}{dE_T} = \frac{d\sigma^{(1)}}{dE_T} - (\sigma^0 + \sigma^1)(2\pi E_T) f(E_T) \theta(q_{max} - E_T) \quad (4.6)$$

The differential cross-section will then be written as

$$\frac{d\sigma}{dE_T} = \sigma_W H(E_T; q_{max}) + \int_0^{\infty} \frac{dL^{(1)}}{dE'_T} R_E(E_T, E'_T; q_{max}) dE'_T \quad (4.7)$$

The same procedure allows us to obtain the double distribution. Since this is less known than either the momentum or the energy distribution, we shall precede the presentation of the full cross section, by discussing in more detail the soft double distribution<sup>[15]</sup>.

The double distribution in the soft region can be obtained by writing an expression analogous to that of eq.(2.4) with a  $\delta$  function which imposes the following constraint on the emitted radiation :

$$\Sigma \vec{k}_T = \vec{p}_T \quad \Sigma k_T = E_T \quad (4.8)$$

We can then readily convert the  $E_T$  and  $\vec{p}_T$  conserving  $\delta$ -functions,  $\delta(E_T - \sum n_k k) \delta^{(2)}(\vec{p}_T - \sum n_k \vec{k}_T)$ , to obtain the following double distribution from the soft radiation

$$\frac{dP^{soft}}{d^2 \vec{p}_T dE_T} = \int \frac{dt}{2\pi} \int \frac{d^2 \vec{b}}{(2\pi)^2} \exp\{iE_T t - i\vec{p}_T \cdot \vec{b} - h(q_{max}; t; \vec{b})\}, \quad (4.9)$$

$$h(q_{max}; t, \vec{b}) = \frac{4}{3\pi^2} \int_0^{q_{max}} \frac{d^2 \vec{k}}{k^2} \alpha_s(k^2) \{1 - \exp(-ikt + i\vec{k} \cdot \vec{b})\} \ln \frac{q_{max} + \sqrt{q_{max}^2 - k^2}}{q_{max} - \sqrt{q_{max}^2 - k^2}} \quad (4.10)$$

This is an interesting expression with only the upper limit  $q_{max}$  as an unknown parameter. We use  $q_{max} \approx M_W/4$  as in ref.(10). In fig.2, we show the soft distribution in  $p_T^W$  for some

characteristic values of  $E_T = 5, 7.5, 10, 15, 20$  Gev. Since the hard contribution is not yet included, these results are reliable only for reasonably small values of  $E_T$ . Note the rather rapid variation in the shape of the  $p_T$ -distribution as  $E_T$  is varied. It would be interesting to check if such a behavior is experimentally visible. As discussed in the later section, a complete check with experimental data can be effected only after the underlying event (UE) has also been folded in.

Another quantity which can be calculated easily via eq.(4.9-4.10) is  $\langle E_T^W \rangle_{soft}$  vs.  $p_T$ . This is shown in fig.3 for  $\Lambda = 0.1$  and  $0.2$  Gev.

Once the expression for the soft distribution is given, we can proceed to write the full (soft plus hard) expression for the double distribution and then to calculate the average  $E_T$  as a function of  $p_T^W$ . We define

$$G_{soft}(E_T, \vec{p}_T; q_{max}) = \frac{d^3 P}{dE_T d^2 \vec{p}_T} \quad (4.11)$$

and the first order perturbative cross-section

$$\frac{d^3 \sigma^{(1)}}{dE_T d^2 \vec{p}_T} = \frac{d^2 \sigma^{(1)}}{d^2 \vec{p}_T} \delta(E_T - p_T) \quad (4.12)$$

The infrared regulator will now be given by

$$R_{E,p}(E_T, E'_T, \vec{p}_T, \vec{k}_T; q_{max}) = G_{soft}(E_T - E'_T, \vec{p}_T - \vec{k}_T; q_{max}) - G_{soft}(E_T, \vec{p}_T; q_{max}) \quad (4.13)$$

The hard contribution is

$$\frac{dL^{(1)}}{dE_T d^2 \vec{p}_T} = \frac{dL^{(1)}}{d^2 \vec{p}_T} \delta(E_T - p_T) \equiv \left( \frac{d^2 \sigma^{(1)}}{d^2 \vec{p}_T} - \sigma_W \lim_{\alpha_s \rightarrow 0} F_{soft} \right) \delta(E_T - p_T) \quad (4.14)$$

and the full distribution is given by

$$\begin{aligned} \frac{d^3 \sigma}{dE_T d^2 \vec{p}_T} &= \sigma_W G_{soft}(E_T, \vec{p}_T; q_{max}) + \\ &+ \int \frac{dL^{(1)}}{dE'_T d^2 \vec{k}_T} R_{E,p}(E_T, E'_T, \vec{p}_T, \vec{k}_T; q_{max}) dE'_T d^2 \vec{k}_T \end{aligned} \quad (4.15)$$

Integrating eq.(4.15) we obtain the average energy for a given  $p_T$  value. The result cannot be immediately compared with data as the measured quantity of interest is the  $\Sigma p_T$  of all charged particles as mentioned earlier. Using isotopic spin considerations, one could say that

$$E_T(\text{into charged pions}) \approx \frac{2}{3} E_T(\text{from gluons}) \quad (4.16a)$$

whereas

$$E_T(\text{into kaons}) \approx \frac{1}{2} E_T(\text{from gluons}) \quad (4.16b)$$

In fig.4 we have indicated these two possibilities by drawing two different curves. We expect the experimental data to lie in between these two, if the entire contribution were to come from bremsstrahlung alone. Notice that at high  $p_t$ -values, phase space considerations suggest the  $\frac{k}{\pi}$  ratio to be higher than at low  $p_T$ -values.

### 5. Conclusions

In the above, we have presented a formalism to deal with infrared sensitive quantities like transverse energy and momentum distributions, using an infrared regulator which substitutes the usual  $\delta$  function subtraction method with a finite distribution which, in the  $\alpha_s$  going to zero limit, is in itself a  $\delta$  function. We have then applied our method to calculate the average transverse energy emitted through initial state bremsstrahlung when a W-boson of given transverse momentum  $p_T^W$  is produced. This quantity can then be compared with experimental data to extract further information on the particle structure which pertains solely to the breaking up of proton, after the two colliding partons have been emitted.

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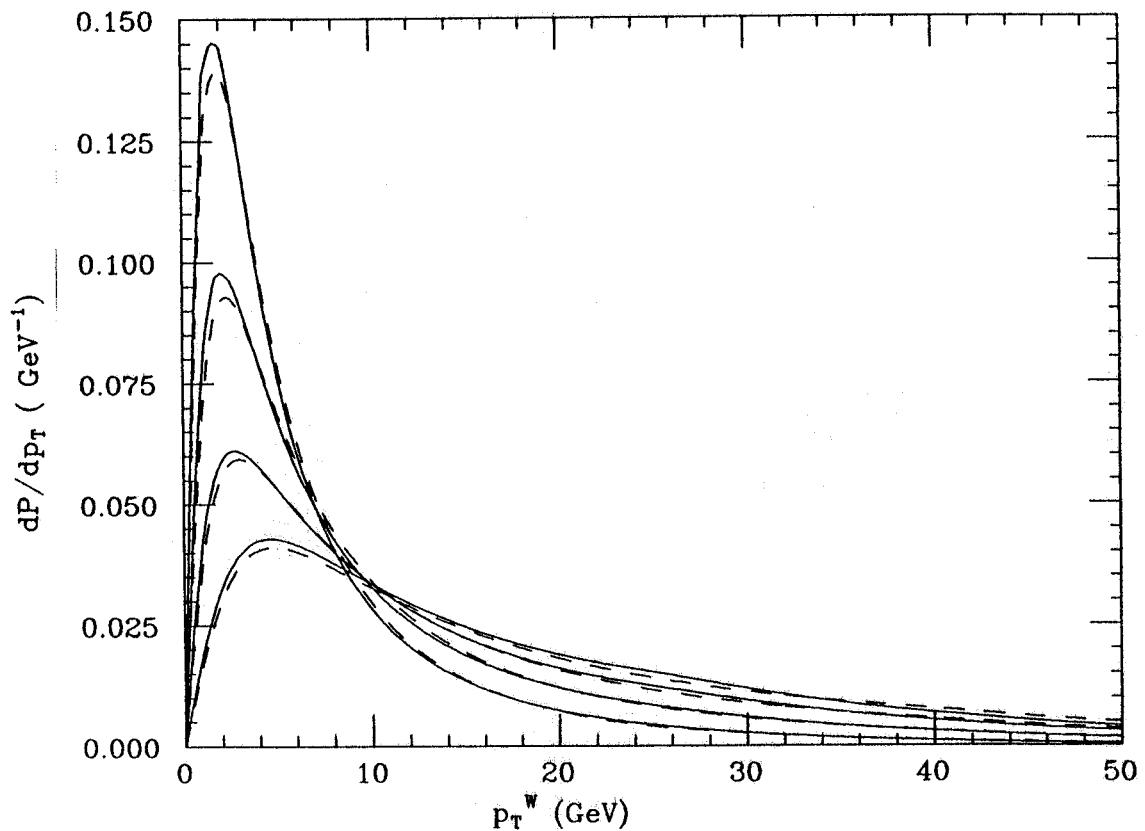
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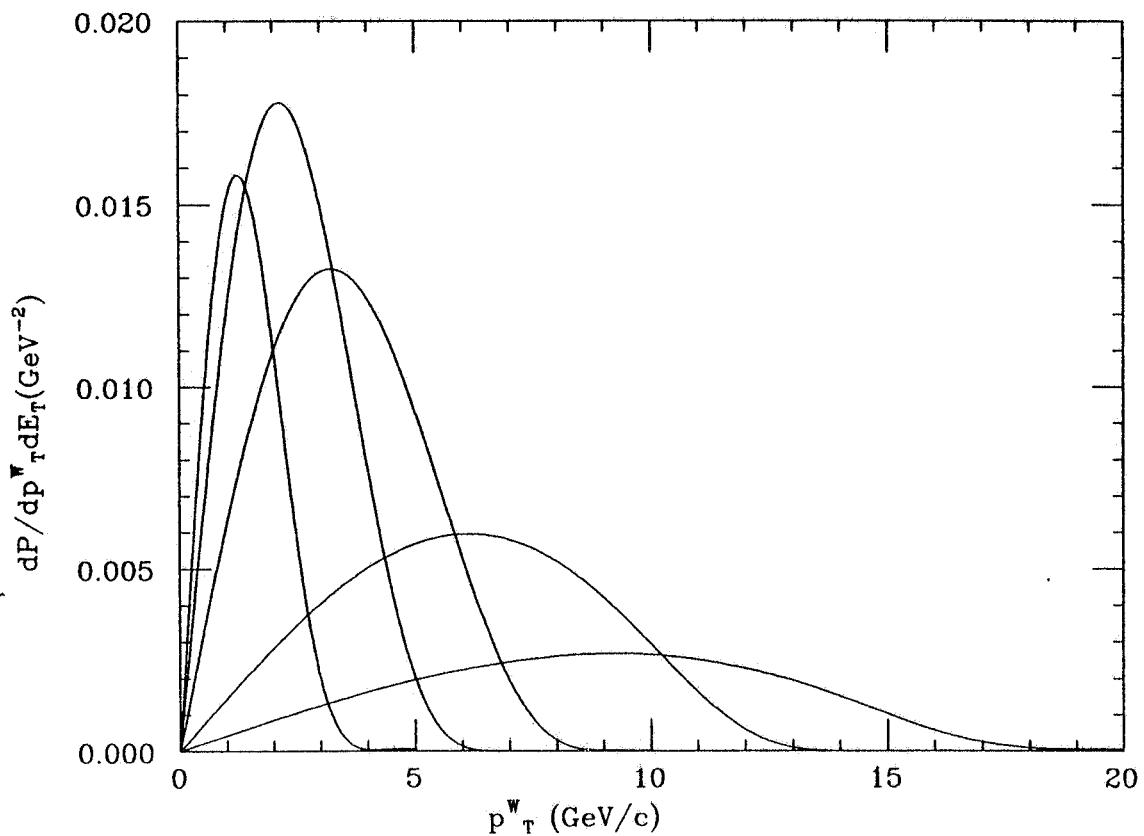
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**Fig.1** The normalized  $p_T$ -distribution for two different values of  $q_{max}$  for  $\sqrt{s} = .63, 1.8, 6$  and 18 Tev.  $q_{max} = 18$  and 24 GeV respectively.



**Fig.2** The double distribution for soft radiation is shown as a function of  $p_T^W$  for different  $E_T$  values  $E_T = 5, 7.5, 10$  and 20 Gev.

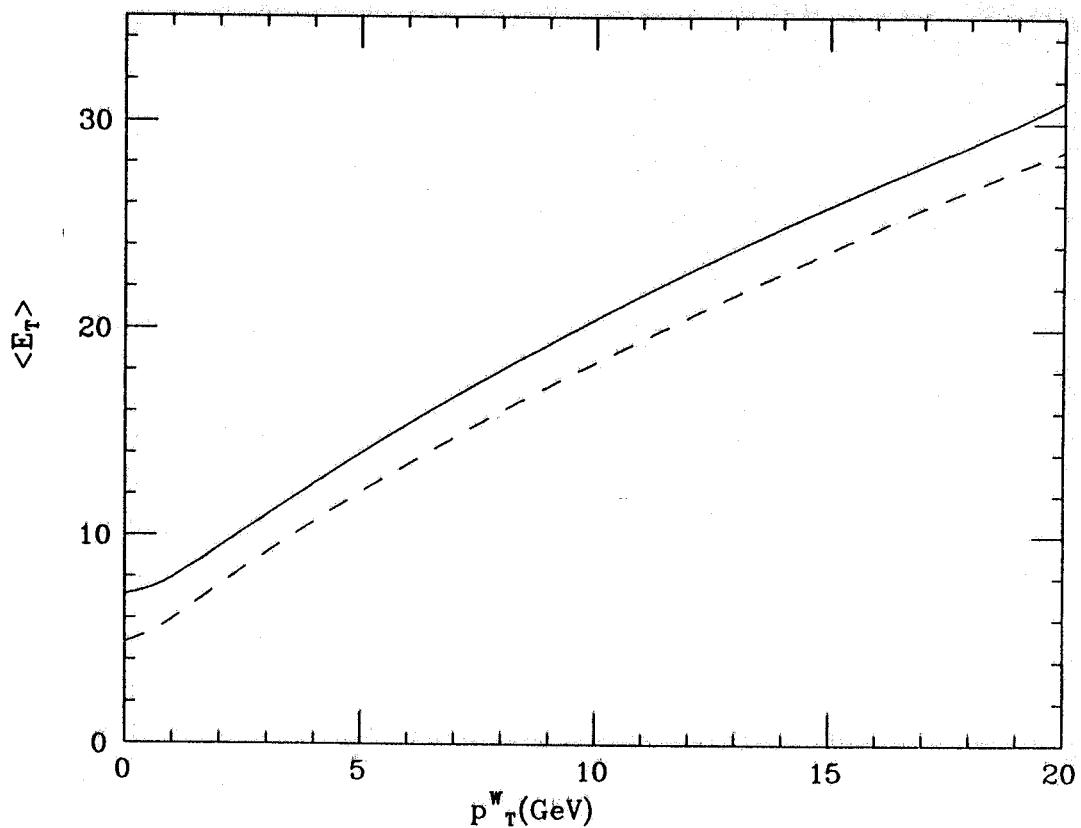


Fig.3  $\langle E_T^W \rangle_{soft}$  vs.  $p_T$  for  $\Lambda = 0.1$  (dashed line) and  $\Lambda = 0.2$  GeV (full line).

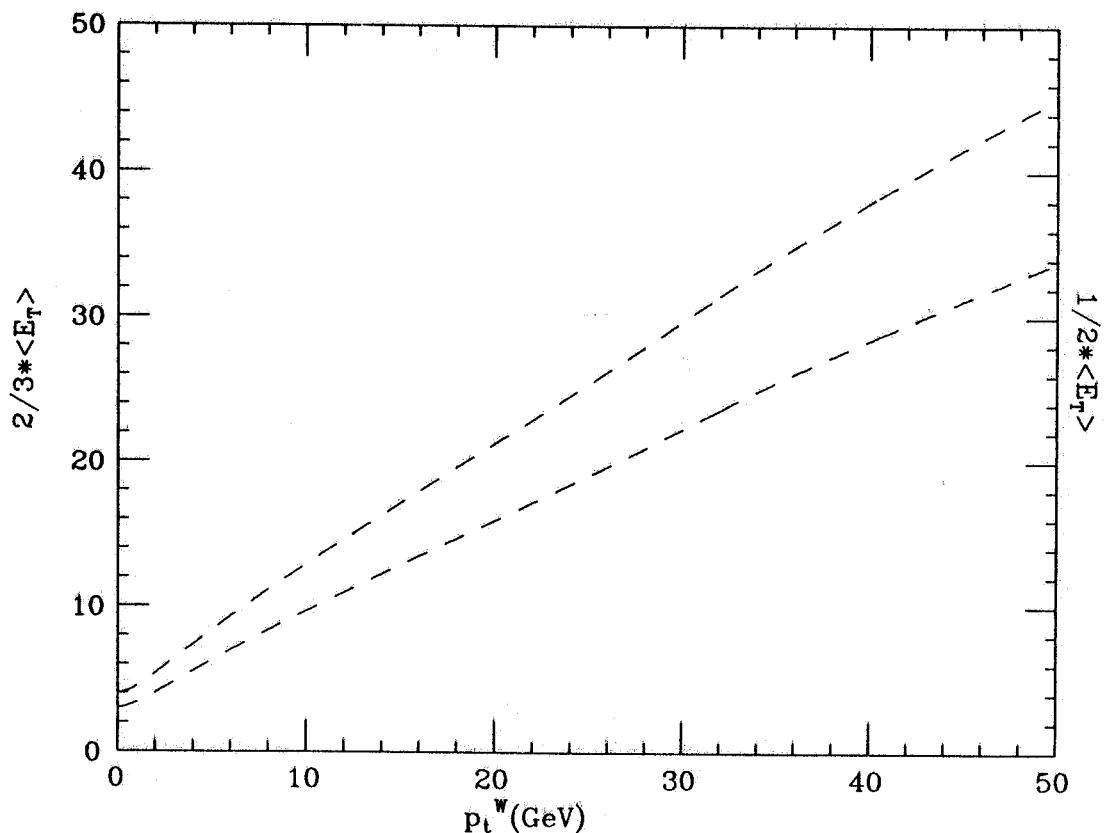


Fig.4 Average transverse energy emitted through (hard and soft) initial state bremsstrahlung as a function of W-boson transverse momentum  $p_T^W$ , produced at a proton-antiproton collider  $\sqrt{s} = 1.8$  TeV,  $q_{max} = 32.6$  GeV. Two dashed curves corresponds to transverse energy carried entirely by charged pions (upper curve) or kaons (lower curve).