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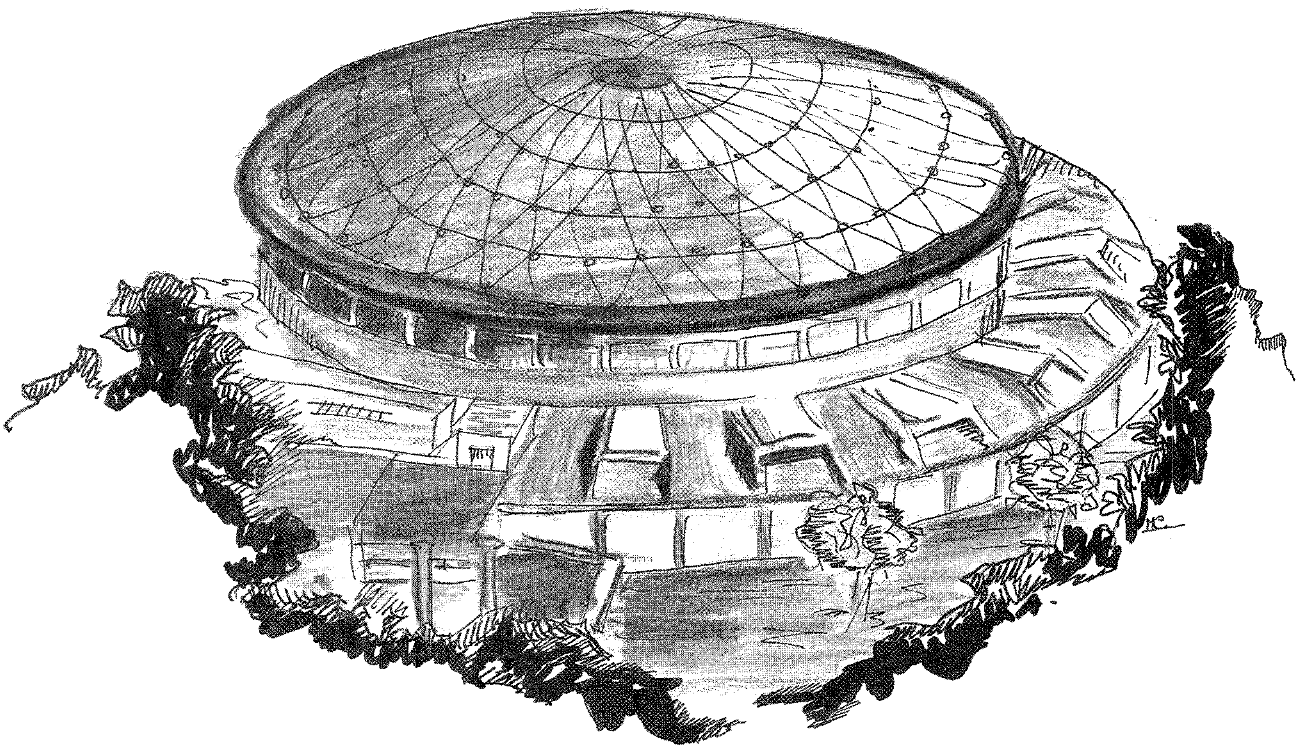

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(1,0) Thirring Models and the Coupling of Spin-0 Fields to the Heterotic String[1]

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Abstract

The (1,0) supersymmetrization of the lefton-righton formulation of massless Thirring models is presented. A general version of the NSR D=4, heterotic string propagating in a background containing the graviton, Kalb-Ramond field, dilaton, gauge vectors *and* massless scalars is given.

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1. Introduction

The question of coupling background spin-zero fields to $D < 10$ heterotic strings (and/or superstrings) has been considered for some time [1]. Only recently [2], within the context of the bosonic string, has progress toward a solution been suggested which:

- allows the consistent introduction of *independent* right and left gauge groups,
- possesses manifest d=2 Lorentz invariance, and
- may be interpreted as the general vertex operator for the emission or absorption of massless spin-zero states in strings.

To accomplish these goals, it has been necessary to construct a new type of two-dimensional, scale-invariant field theory: the lefton-righton Thirring model. The existence of such a model was suggested by a number of well-known facts. The usual Thirring model [3] is very familiar in terms of fermionic fields. Free fermions can be subjected to either abelian [4] or non-abelian [5] bosonization, which at the level of lagrangians leads to abelian [6] or non-abelian [7] chiral boson models. For constant values of the Thirring coupling [8], the Thirring model is equivalent to a set of free fermions. Progress was made towards the construction of purely bosonic Thirring models when it was shown [7] how half the fermions in the model could be bosonized. The complete bosonization, for both the abelian and the non-abelian cases, was then carried out for the non-supersymmetric case [2]. All that remained was the extension to supersymmetric theories, especially to (1,0) theories since our main interest is heterotic strings.

We give the solution in the present work and thus provide further evidence which supports the conjecture that two dimensional lagrangians can be constructed which contain β-function coupling constants (equivalent to vertex operators) for *every* massless mode of string theories. We emphasize that the lefton-righton Thirring model (LRTM) is not just a convenient way to demonstrate explicit spin-0 couplings to the heterotic string. If we demand manifest world-sheet supersymmetry (both classically and quantum mechanically) and complete generality, the LRTM is the only way to couple massless spin-0 background fields. Furthermore, in the case of spinning superstrings, demanding the same conditions imply spin-0 coupling can *only* be written with the use of the LRTM.

2. (1,0) Lefton-Righton Thirring Models: Abelian

From previous studies [7] we recall that the (1,0) supersymmetric lefton L in the presence of a specified background field Γ_{--} is described by

$$S_L^*(L, \Lambda_+{}^{--}, \Gamma_{--}) = i\tfrac{1}{2}\int d^2\sigma d\varsigma^- \Big[(D_+L)(\partial_{--}L + 2\Gamma_{--}) - \Lambda_+{}^{--}(\partial_{--}L + \Gamma_{--})^2\Big] \ . \qquad (2.1)$$

Similarly, the righton R in the presence of a background field Γ_+ is described by

$$\begin{aligned} S_R^*(R, \Lambda_{--}{}^{++}, \Gamma_+) = i\tfrac{1}{2}\int d^2\sigma d\varsigma^- \Big[&(D_+R + 2\Gamma_+)(\partial_{--}R) \\ &- i\Lambda_{--}{}^{++}(D_+R + \Gamma_+)D_+(D_+R + \Gamma_+)\Big] \ . \end{aligned} \qquad (2.2)$$

To find the (1,0) supersymmetric Thirring model, we could use our experience from the bosonic case [2], and start from the following ansatz

$$S_{tot} = S_L^* + S_R^* + i\tfrac{1}{2}\int d^2\sigma d\varsigma^-(-4S)(\mathbf{D}_{--}L)(\mathbf{D}_+R) + \cdots \qquad (2.3)$$

and attempt to derive the rest of the action in the ellipses along with a set of Siegel (chiral boson) symmetry transformations laws by use of a Noether method.

However, there is a simpler way [3] to arrive at the action. Recall that in the bosonic case [2], we introduced the backgrounds fields P_{++} and Q_{--} which entered the theory as would gauge fields coupled to the physical degrees of freedom; specifically P_{++} (Q_{--}) was associated with the background field Γ_{++} (Γ_{--}). To introduce interactions between the left and right sectors, we added to the sum of the two free actions a term $P_{++}Q_{--}$ and then performed functional integrations over P_{++} and Q_{--}. This procedure led directly to the lefton-righton formulation of massless Thirring models. This method can be applied to (1,0) superspace, modulo an important modification which we will soon describe.

Just as in the bosonic case, we start with $\tilde{S}_{tot}$ defined by adding to the sum of the free actions a PQ term (from here on we work in the presence of (1,0) supergravity [9]):

$$\begin{aligned}\tilde{S}_{tot} = \; & S_L^*(L, \Lambda_+{}^{--}, \Gamma_{--} + SQ_{--}) + S_R^*(R, \Lambda_{--}{}^{++}, \Gamma_+ + P_+) \\ & + i\tfrac{1}{2}\int d^2\sigma d\varsigma^- E^{-1}(P_+ Q_{--}) \; .\end{aligned} \tag{2.4}$$

The second order (1,0) supersymmetric action for the lefton-righton Thirring model $S_{LR}(L, R, \Lambda_+{}^{--}, \Lambda_{--}{}^{++})$ is defined by

$$\exp[iS_{LR}(L, R, \Lambda_+{}^{--}, \Lambda_{--}{}^{++})] = \int [dP_+][dQ_{--}] \exp[i\tilde{S}_{tot}] \; . \tag{2.5}$$

However, if one tries to integrate the field P_+ out by the usual completion of the square, one runs immediately into problems, the main but not only one being the appearance of the inverse of $\Lambda_+{}^{--}$. So in order to perform the functional 'integration' over P_+, we notice that in the bosonic case, the Gaussian integration was equivalent to finding a stationary value of the action. This corresponds to finding the solution to the coupled system

$$\frac{\delta \tilde{S}_{tot}(P,Q)}{\delta P} = 0 \quad , \quad \frac{\delta \tilde{S}_{tot}(P,Q)}{\delta Q} = 0 \; . \tag{2.6}$$

If we can solve the second equation in terms of Q_{--}, that is find a solution $P_+ = P_+(Q_{--})$ such that the second equation is satisfied, then we can use this solution in the first equation, and in our case transform this first equation back into a functional integration over Q_{--}, which doesn't present any problem. For our specific problem, we take the variation of the action (2.4) with respect to Q_{--}, and set this variation to zero. It gives

$$P_+(Q_{--}) = 2S^2\Lambda_+{}^{--}Q_{--} - 2S(\nabla_+ L - \Lambda_+{}^{--}(\nabla_{--} + \Gamma_{--})) \; . \tag{2.7}$$

Plugging this value [4] of $P_+(Q_{--})$ into (2.5), we are left with the functional integration over Q_{--} which, being bosonic, is easy to perform.

Integrating Q_{--} out, we obtain:

$$\begin{aligned} S_{LR}(L, R, \Lambda_+{}^{--}, \Lambda_{--}{}^{++}) &= i\tfrac{1}{2}\int d^2\sigma d\varsigma^- E^{-1}\mathcal{L}_- \; , \\ \mathcal{L}_- = \Big\{ & (\mathrm{D}_+ L)\,(\nabla_{--}L + \Gamma_{--}) \;+ (\nabla_+ L)\,\Gamma_{--} \\ & + (\mathrm{D}_{--}R)\,(\nabla_+ R + \Gamma_+) + (\nabla_{--}R)\,\Gamma_+ \\ & - 4\,S\mathrm{D}_{--}R\;\; \mathrm{D}_+ L \;- 4\,i\,\Lambda_{--}{}^{++}\,(S\,\mathrm{D}_+ L)\,\nabla_+(S\,\mathrm{D}_+ L) \\ & - 4S^2\,\frac{\Lambda_+{}^{--}}{1 - 4iS^2\Lambda^2}\,\Sigma^2_{--}\Big\} \; . \end{aligned} \tag{2.8}$$

[3] We wish to thank Q-Han Park for this suggestion.

[4] So the 'integration' $\int [dP_+]$ has now been performed

As for the bosonic counterpart, this action is invariant under two independent Siegel symmetries:

$$\begin{aligned} \delta L &= \xi^{--}\Big[(\nabla_{--}L+\Gamma_{--}) - 2S\frac{\Sigma_{--}}{1-4iS^2\Lambda^2} \\ &\qquad + 8iS\Lambda_{+}{}^{--}\Big(\Lambda_{--}{}^{++}\nabla_{+}V_{--} + \tfrac{1}{2}(\nabla_{+}\Lambda_{--}{}^{++})V_{--}\Big)\Big] \ , \\ \delta\Lambda_{+}{}^{--} &= \nabla_{+}\xi^{--} - \Lambda_{+}{}^{--}\nabla_{--}\xi^{--} + \xi^{--}\nabla_{--}\Lambda_{+}{}^{--} \ , \\ \delta R &= -i(\xi^{++}\nabla_{+}(\nabla_{+}R+\Delta_{+}) + \tfrac{1}{2}\nabla_{+}\xi^{++}(\nabla_{+}R+\Delta_{+})) \ , \\ \delta\Lambda_{--}{}^{++} &= -\nabla_{--}\xi^{++} + \xi^{++}\nabla_{++}\Lambda_{--}{}^{++} - \Lambda_{--}{}^{++}\nabla_{++}\xi^{++} - i\tfrac{1}{2}(\nabla_{+}\xi^{++})\nabla_{+}\Lambda_{--}{}^{++} \ , \end{aligned} \tag{2.9}$$

with notation

$$\begin{aligned} \mathbf{D}_{+}L &= \nabla_{+}L - \Lambda_{+}{}^{--}(\nabla_{--}L+\Gamma_{--}) \ , \\ \mathbf{D}_{--}R &= \nabla_{--}R - i[\Lambda_{--}{}^{++}\nabla_{+}(\nabla_{+}R+\Gamma_{+}) + \tfrac{1}{2}(\nabla_{+}\Lambda_{--}{}^{++})(\nabla_{+}R+\Gamma_{+})] \ , \\ \Sigma_{--} &= \mathbf{D}_{--}R + 2i[\Lambda_{--}{}^{++}\nabla_{+}(S\mathbf{D}_{+}L) + \tfrac{1}{2}(\nabla_{+}\Lambda_{--}{}^{++})S\mathbf{D}_{+}L] \ , \\ V_{--} &= S^2\frac{\Sigma_{--}}{1-4iS^2\Lambda^2} \ , \quad \Delta_{+} = \Gamma_{+} - 2S\mathbf{D}_{+}L - 4S^2\frac{\Lambda_{+}{}^{--}}{1-4iS^2\Lambda^2}\Sigma_{--} \ , \\ \Lambda^2 &= \Lambda_{--}{}^{++}\,\nabla_{+}\Lambda_{+}{}^{--} \ . \end{aligned} \tag{2.10}$$

We see immediately that in comparison with the bosonic case, this action is highly asymmetrical with respect to $+ \leftrightarrow -$. This is due to the unidexterity of (1,0) superspace. A point worthy of notice is the consistent appearance of the $(\Lambda_{--}{}^{++}\nabla_{+} + \frac{1}{2}(\nabla_{+}\Lambda_{--}{}^{++}))$ operator, which corresponds to the operator $\lambda_{--}{}^{++}\partial_{++}$ of the bosonic version of the Thirring model. A way to understand the structure of this operator is to look at superscale invariance. The "minimal" (1,0) supersymmetric transcription of $\partial_{--}C + \lambda_{--}{}^{++}\partial_{++}C$ (C bosonic righton [2]) is clearly $\nabla_{--}R - i\Lambda_{--}{}^{++}\nabla_{+}^2R$, in the sense that it gives the correct bosonic limit. But if one tries to get a (1,0) transcription of $\mathbf{D}_{--}C$ that has a definite superscaling weight, one finds that the minimal transcription just given is not enough. It can easily be calculated that the minimal operator that contains the minimal transcription just given *and* possesses a definite scaling weight is the one we defined as being $\mathbf{D}_{--}R$. The very same reasoning holds for the use of the same structure in other places in the action, as for instance in the definition of Σ_{--}, which has a well-defined scaling weight. Note that the whole action is scale invariant, which is required together with supercoordinate invariance in order to have superconformal invariance.

We should mention that we also derived the action and the transformation laws by actually carrying out the Noether procedure. In fact, this was the way we discovered the action! The differentiation-integration procedure provides a concise alternative derivation of the action. We also note that extension of these results to (1,1) superspace is a simple exercise.

To compare these (1,0) supersymmetric extensions to the previous purely bosonic ones (in other words, to check the correctness of the bosonic limit), we first recall the abelian results [2] with no supersymmetry. We found, in an explicit left-right symmetric notation,

$$\begin{aligned} \mathcal{L}_B = -\tfrac{1}{2}\Big\{ & (\mathbf{D}_{++}B)(D_{--}B) + A_{--}\partial_{++}B \\ & + (\mathbf{D}_{--}C)(D_{--}C) + A_{++}\partial_{--}C \\ & - 4S\,\frac{1}{1-4S^2\lambda^2}\Big[(\mathbf{D}_{++}B)(\mathbf{D}_{--}C)\Big] \\ & + 4S^2\frac{1}{1-4S^2\lambda^2}\Big[\lambda_{--}{}^{++}(\mathbf{D}_{++}B)^2 + \lambda_{++}{}^{--}(\mathbf{D}_{--}C)^2\Big]\Big\} \ , \end{aligned} \tag{2.11}$$

with notations $\lambda^2 \equiv \lambda_{++}{}^{--}\lambda_{--}{}^{++}$, $\mathbf{D}_{++}B \equiv \partial_{++}B+\lambda_{++}{}^{--}D_{--}B$, $\mathbf{D}_{--}C \equiv \partial_{--}C+\lambda_{--}{}^{++}D_{++}C$, $D_\mu\phi = \partial_\mu\phi + A_\mu$. In this form, it is not immediately obvious to see how the (1,0) supersymmetric result can produce the bosonic result above upon projection to components. To prove this, it is necessary to note the identity

$$\frac{1}{1-4S^2\lambda^2} = 1+\frac{4S^2\lambda^2}{1-4S^2\lambda^2} \ . \tag{2.12}$$

Upon substitution into the third and the fourth lines in (2.11), this lagrangian takes the equivalent form

$$\begin{aligned}\mathcal{L}_B = & -\tfrac{1}{2}\Big\{(\mathbf{D}_{++}B)(D_{--}B) \ + \ A_{--}\partial_{++}B+(\mathbf{D}_{--}C)(D_{--}C) \ + \ A_{++}\partial_{--}C \\ & - 4S\mathbf{D}_{++}B\mathbf{D}_{--}C \\ & - 4S^2\lambda_{--}{}^{++}(\mathbf{D}_{++}B)^2 - 4(\frac{S^2}{1-4S^2\lambda^2})\lambda_{++}{}^{--}(\Delta_{--})^2\Big\} \ ,\end{aligned} \tag{2.13}$$

with obvious notations. Now it is clear that the (1,0) supersymmetric extension contains precisely the bosonic theory.

3. (1,0) Lefton-Righton Thirring Models: Non-Abelian

Having seen in the last section how abelian bosonic results generalize to the (1,0) supersymmetric case, we now turn to the non-abelian case. The starting point is again [7], with the notations of [2], adapted to superspace. We restrict the leftons and rightons to be group manifold coordinates; defining the non-abelian lefton by L and the non-abelian righton by R, we may decompose the Lie-algebra valued (1,0) superform components $(\nabla_m L)L^{-1}$ and $(\nabla_m R)R^{-1}$ by $(m = +, --)$:

$$(\nabla_m L)L^{-1} = iL_m^{\hat\alpha}t^{\hat\alpha} \ \ , \ \ (\nabla_m R)R^{-1} = iR_m^{\hat I}t^{\hat I} \ . \tag{3.1}$$

Here $t^{\hat\alpha}$ ($t^{\hat I}$) is the generator of the left-(right-) symmetry group. The two group are, of course, not generally the same. For these generators, we choose

$$\begin{aligned} [t^{\hat\alpha},t^{\hat\beta}] = if^{\hat\alpha\hat\beta\hat\gamma}t^{\hat\gamma} \ \ &, \ \ [t^{\hat I},t^{\hat J}] = if^{\hat I\hat J\hat K}t^{\hat K} \ , \\ tr(t^{\hat\alpha}t^{\hat\beta}) = k_L\delta^{\hat\alpha\hat\beta} \ \ &, \ \ tr(t^{\hat I}t^{\hat J}) = k_R\delta^{\hat I\hat J} \ .\end{aligned} \tag{3.2}$$

We next take the free non-abelian lagrangians of [7] (lefton and righton), and put in arbitrary Siegel-gauge invariant background fields $\Gamma_{--}^{\hat I}$ and $\Gamma_{+}^{\hat\alpha}$. The way to put them in is unique under the requirement that both the lefton and righton actions each possess a Siegel symmetry. One finds, after setting $n_L = -1$ and $n_R = 1$ in [7] and putting the gauge invariant functions in, the lagrangian:

$$\begin{aligned}\mathbf{L}_-^0 = & -k_L(L_{--}^{\hat\alpha} + \Gamma_{--}^{\hat\alpha})(L_+^{\hat\alpha} - \Lambda_+{}^{--}(L_{--}^{\hat\alpha} + \Gamma_{--}^{\hat\alpha})) - k_L L_+^{\hat\alpha}\Gamma_{--}^{\hat\alpha} \\ & -k_R(R_+^{\hat I} + 2\Gamma_+^{\hat I})R_{--}^{\hat I} + ik_R\Lambda_{--}{}^{++}(R_+^{\hat I} + \Gamma_+^{\hat I})\nabla_+(R_+^{\hat I} + \Gamma_+^{\hat I}) \\ & \qquad + \frac{2}{3}ik_R f^{\hat I\hat J\hat K}\Lambda_{--}{}^{++}(R_+^{\hat I} + \Gamma_+^{\hat I})(R_+^{\hat J} + \Gamma_+^{\hat J})(R_+^{\hat K} - \tfrac{1}{2}\Gamma_+^{\hat K}) \\ & -\int_0^1 dy[(\frac{d\tilde L}{dy}\tilde L^{-1})[(\nabla_{--}(\nabla_+\tilde L\tilde L^{-1}) - (\nabla_+(\nabla_{--}\tilde L\tilde L^{-1})]] \\ & +\int_0^1 dy[(\frac{d\tilde R}{dy}\tilde R^{-1})[(\nabla_{--}(\nabla_+\tilde R\tilde R^{-1}) - (\nabla_+(\nabla_{--}\tilde R\tilde R^{-1})]] \ .\end{aligned} \tag{3.3}$$

The final two terms in this action are seen to be the usual extension of the WNZW terms including the addition of the redundant coordinate y required to write such terms in closed form.

To obtain the coupled model, we just proceed as in the abelian case, with complications due to the presence of the terms $f\ (R+\Gamma)(R+\Gamma)(R-\frac{1}{2}\Gamma)$. Using the differentiation-integration method presented for the abelian case, we make the redefinitions below to introduce P_+ and Q_{--},

$$\Gamma^{\hat{I}}_+ \rightarrow \Gamma^{\hat{I}}_+ + P^{\hat{I}}_+ \quad , \quad \Gamma^{\hat{\alpha}}_{--} \rightarrow \Gamma^{\hat{\alpha}}_{--} + S^{\hat{\alpha}\hat{I}} Q^{\hat{I}}_{--} \quad , \tag{3.4}$$

in (3.3), add an interaction term $P^{\hat{I}}_+ Q^{\hat{I}}_{--}$, and 'integrate' first $P^{\hat{I}}_+$ out, and then integrate $Q^{\hat{I}}_{--}$ out. It might seem, a priori, that this gives cubic terms in Q_{--}, but this does not happen since these terms come with a coefficient $(\Lambda_+{}^{--})^3 = 0$. So, integrating fields out as described, we find (with $k^2 \equiv k_L k_R$):

$$\begin{aligned} \mathbf{L}^{NLRTM}_- = \mathbf{L}^0_- \ &-4k^2 S^{\alpha\hat{I}} \mathbf{R}^{\hat{I}}_{--} \mathbf{L}^{\hat{\alpha}}_+ + \ 4k^2 k_R \Lambda_+{}^{--} (M^{-1})^{\hat{I}\hat{K}} S^{\hat{\alpha}\hat{I}} S^{\hat{\alpha}\hat{J}} \Sigma^{\hat{J}}_{--} \Sigma^{\hat{K}}_{--} \\ &+4ik^2 k_L \Lambda_{--}{}^{++} S^{\hat{\alpha}\hat{I}} \mathbf{L}^{\hat{\alpha}}_+ \nabla_+ (S^{\hat{\beta}\hat{I}} \mathbf{L}^{\hat{\beta}}_+) \\ &-4ik^2 k_L \Lambda_{--}{}^{++} f^{\hat{I}\hat{J}\hat{K}} (\Gamma^{\hat{I}}_+ + \tfrac{2}{3} k_L S^{\hat{\alpha}\hat{I}} \mathbf{L}^{\hat{\alpha}}_+) S^{\hat{\beta}\hat{J}} \mathbf{L}^{\hat{\beta}}_+ S^{\hat{\gamma}\hat{K}} \mathbf{L}^{\hat{\gamma}}_+ \ . \end{aligned} \tag{3.5}$$

This lagrangian is invariant (up to surface terms) under two Siegel symmetries

$$\begin{aligned} \alpha^{\hat{\alpha}} = \xi^{--} \Big\{ & L^{\hat{\alpha}}_{--} + \Gamma^{\hat{\alpha}}_{--} + 2k_R (M^{-1})^{\hat{I}\hat{J}} S^{\hat{\alpha}\hat{I}} \Sigma^{\hat{J}}_{--} \\ & - 8ik^2 k_R S^{\hat{\alpha}\hat{I}} \Lambda_+{}^{--} \Big[\Lambda_{--}{}^{++} \nabla_+ V^{\hat{I}}_{--} + \tfrac{1}{2} (\nabla_+ \Lambda_{--}{}^{++}) V^{\hat{I}}_{--} \\ & \qquad - \Lambda_{--}{}^{++} f^{\hat{I}\hat{J}\hat{K}} (\Gamma^{\hat{J}}_+ + 2k_L S^{\hat{\beta}\hat{J}} \mathbf{L}^{\hat{\beta}}_+) V^{\hat{K}}_{--} \Big] \Big\} \ , \end{aligned} \tag{3.6}$$

$$\alpha^{\hat{I}} = -i(\xi^{++}\nabla_+ + \tfrac{1}{2}(\nabla_+ \xi^{++}))(R^{\hat{I}}_+ + V^{\hat{I}}_+) - i\tfrac{1}{2}\xi^{++} f^{\hat{I}\hat{J}\hat{K}} (R^{\hat{J}}_+ R^{\hat{K}}_+ - V^{\hat{J}}_+ V^{\hat{K}}_+) \ ,$$

with $\delta\Lambda_+{}^{--}$ and $\delta\Lambda_{--}{}^{++}$ unchanged from (2.9) and with notations

$$\begin{aligned} \mathbf{L}^{\hat{\alpha}}_+ &= L^{\hat{\alpha}}_+ - \Lambda_+{}^{--} (L^{\hat{\alpha}}_{--} + \Gamma^{\hat{\alpha}}_{--}) \ , \\ \mathbf{R}^{\hat{I}}_{--} &= R^{\hat{I}}_{--} - i[\Lambda_{--}{}^{++} \nabla_+ (R^{\hat{I}}_+ + \Gamma^{\hat{I}}_+) + \tfrac{1}{2} (\nabla_+ \Lambda_{--}{}^{++})(R^{\hat{I}}_+ + \Gamma^{\hat{I}}_+) \\ &\qquad + \tfrac{1}{2} \Lambda_{--}{}^{++} f^{\hat{I}\hat{J}\hat{K}} (R^{\hat{J}}_+ + \Gamma^{\hat{J}}_+)(R^{\hat{K}}_+ - \Gamma^{\hat{K}}_+)] \ , \\ \Sigma^{\hat{I}}_{--} &= \mathbf{R}^{\hat{I}}_{--} - 2ik_L [\Lambda_{--}{}^{++} \nabla_+ (S^{\hat{\beta}\hat{I}} \mathbf{L}^{\hat{\beta}}_+) + \tfrac{1}{2} (\nabla_+ \Lambda_{--}{}^{++}) S^{\hat{\beta}\hat{I}} \mathbf{L}^{\hat{\beta}}_+ \\ &\qquad - \Lambda_{--}{}^{++} f^{\hat{I}\hat{J}\hat{K}} S^{\hat{\beta}\hat{J}} \mathbf{L}^{\hat{\beta}}_+ (\Gamma^{\hat{K}}_+ + k_L S^{\hat{\gamma}\hat{K}} \mathbf{L}^{\hat{\gamma}}_+)] \ , \\ (M)^{\hat{I}\hat{J}} &= \delta^{\hat{I}\hat{J}} - 4ik^2 \Lambda^2 S^{\hat{\alpha}\hat{I}} S^{\hat{\alpha}\hat{J}} \ , \ V^{\hat{I}}_{--} = \ (M^{-1})^{\hat{J}\hat{K}} S^{\hat{\alpha}\hat{I}} S^{\hat{\alpha}\hat{J}} \Sigma^{\hat{K}}_{--} \ , \\ V^{\hat{I}}_+ &= \Gamma^{\hat{I}}_+ + 2k_L S^{\hat{\alpha}\hat{I}} \mathbf{L}^{\hat{\alpha}}_+ - 4k^2 \Lambda_+{}^{--} (M^{-1})^{\hat{J}\hat{K}} S^{\hat{\alpha}\hat{J}} S^{\hat{\alpha}\hat{I}} \Sigma^{\hat{K}}_{--} \ . \end{aligned} \tag{3.7}$$

In (3.6) we introduced the notation $\delta L L^{-1} = \alpha = i\alpha^{\hat{\alpha}} t^{\hat{\alpha}}$ that leads to $\delta L_m{}^{\hat{\alpha}} = \nabla_m \alpha^{\hat{\alpha}} - f^{\hat{\alpha}\hat{\beta}\hat{\gamma}} \alpha^{\hat{\beta}} L_m{}^{\hat{\gamma}}$.

4. General Spin-Zero Couplings in $D = 4$ Heterotic String σ-Models

As for the bosonic string [2], the coupling of background spin-zero fields to compactified heterotic string σ-models follows from a specialization of the non-abelian lefton-righton Thirring model. We may write the the complete action in two pieces $S_{SM} + S_{FB}$. The space-time manifold action [7] is the familiar

$$\begin{aligned} S_{SM} &= \frac{1}{2\pi\alpha'} \int d^2\sigma d\varsigma^- E^{-1} [i\tfrac{1}{2}(\eta_{\underline{ab}} + b_{\underline{ab}}(X)) \Pi_+{}^{\underline{a}} \Pi_{--}{}^{\underline{b}} + \ \alpha' \Phi(X) \Sigma^+ \ ] \ , \\ &\Pi_+{}^{\underline{a}} \equiv (\nabla_+ X^{\underline{m}}) e_{\underline{m}}{}^{\underline{a}}(X) \ , \ \Pi_{--}{}^{\underline{a}} \equiv (\nabla_{--} X^{\underline{m}}) e_{\underline{m}}{}^{\underline{a}}(X) \ , \end{aligned} \tag{4.1}$$

where ∇_A and Σ^+ are the usual (1,0) supergravity derivative and supergravity field strength [9]. The fiber bundle action is obtained from our non-abelian Thirring model subject to certain identifications:

$$\begin{aligned} S_{FB} &= i\tfrac{1}{2}\int d^2\sigma d\varsigma^- E^{-1}\mathbf{L}_-^{(NLRTM)}(\Gamma_{--}{}^{*\hat{\alpha}}, \Gamma_+{}^{*\hat{I}}, S_{\hat{\alpha}\hat{I}}{}^*) \ , \\ \Gamma_+{}^{*\hat{I}} &\equiv \frac{1}{\sqrt{2\pi\alpha'}}(\nabla_+ X^m)A_{\underline{m}}{}^{\hat{I}}(X) \ , \ \Gamma_{--}{}^{*\hat{\alpha}} \equiv \frac{1}{\sqrt{2\pi\alpha'}}(\nabla_{--}X^m)A_{\underline{m}}{}^{\hat{\alpha}}(X) \ , \\ S_{\hat{\alpha}\hat{I}}{}^* &\equiv \Phi_{\hat{\alpha}\hat{I}}(X) \ . \end{aligned} \tag{4.2}$$

Here $A_{\underline{m}}{}^{\hat{\alpha}}(X)$ are the "gravi-photons" of extended supergravity theories, $A_{\underline{m}}{}^{\hat{I}}(X)$ are the "matter gauge fields" and $\Phi_{\hat{\alpha}\hat{I}}(X)$ are the "matter scalar fields." For definiteness, we may pick the $N=4$, $SO(44)$ or $E_8\times E_8\times E_6$ models. We need only set $G_L = U(1)^6$ and $G_R = SO(44)$ or $E_8\times E_8\times E_6$. The $SO(44)$ model was described before [7] in the fermionic formulation with forty-four $\eta_-{}^{\hat{I}}$ superfields. For the choice of $SO(44)$, our present work may be regarded as a superfield non-abelian bosonization in the right-moving non-supersymmetric sector of (1,0) superspace. However, complete generality and manifest realization, for example, of the $E_8\times E_8\times E_6$ symmetry requires use of the LRTM. Although we have concentrated on group manifolds, the extension to coset spaces can be obtained by using precisely the construction given by Park [10]. The presence of $N<4$ space-time supersymmetry in these σ-models seems connected to the <u>*consistent*</u> appearance of coset spaces G/H in the righton sector of the (1,0) supersymmetric action. The reason for this is quite simple. The "off-diagonal" elements of the coset space, as seen via their Thirring couplings, are associated with the appearance of spin-0 bosons that are <u>*not*</u> in the adjoint representation of a group. Spin-1 gauge bosons must be in the adjoint representation. So the consistent presence of a coset space signals, at least, a number of $N=2$ hypermultiplets. For these $N<4$ theories, also the left group G_L may be chosen to be a group other than $U(1)^6$.

5. (1,0) Notons

The quantization of chiral bosons has an extremely confusing history [11] (which extends even to the present[12]). But there appear at least two consistent ways ([13] and [14]) to quantize these theories. The simplest of these, to use in conjunction with path-integral quantization, is the introduction of Hull's no-movers [13]. At least on genus zero surfaces [5], this keeps world-sheet Lorentz invariance manifest. Thus, the consistent cancellation of the "Siegel anomaly" [11] requires the addition of a no-mover (noton) sector. Following Hull, we may introduce $N_{\tilde{\eta}}$ no-movers minus spinors $\tilde{\eta}_-$, $N_{\hat{B}}$ pair superfields $\hat{B}_+^{(s',r')} - \hat{C}^{(s',r')}$ and $N_{\tilde{B}}$ pairs $\tilde{B}_{--}^{(r,s)} - \tilde{C}^{(s,r)}$. For an action we take [6]

$$\begin{aligned} S_N = \ & i\tfrac{1}{2}\int d^2\sigma d\varsigma^- E^{-1}\Big\{ \tilde{\eta}_-(\nabla_+ - \Lambda_+{}^{--}\nabla_{--})\tilde{\eta}_- + \tilde{B}_{--}^{(r,s)}(\nabla_+ - \Lambda_+{}^{--}\nabla_{--})\tilde{C}^{(s,r)} \\ & + \hat{B}_+^{(r',s')}[\nabla_{--} + \Lambda_{--}{}^{++}\nabla_{++} - i\tfrac{1}{2}(\nabla_+\Lambda_{--}{}^{++})\nabla_+]\hat{C}^{(s',r')} \\ & - \tfrac{1}{2}[(s'-r')(\nabla_{++}\Lambda_{--}{}^{++})\hat{B}_+{}^{(r',s')}\hat{C}^{(s',r')} + (s-r)\tilde{B}_{--}^{(r,s)}(\nabla_{--}\Lambda_+{}^{--})\tilde{C}^{(s,r)}]\Big\} \ , \end{aligned} \tag{5.1}$$

[5] Since naive Euclideanization does not preserve the self (or anti-self) duality condition of real chiral bosons, they have not been studied on higher genus surfaces. However, for the tree level string effective action, the genus zero contribution suffices.

[6] These results provide (1,0) extensions of results in Hull's second paper.

which possesses the Siegel symmetries

$$
\begin{aligned}
\delta\tilde{\eta}_- &= \xi^{--}\nabla_{--}\tilde{\eta}_- + \tfrac{1}{2}(\nabla_{--}\xi^{--})\tilde{\eta}_- \ , \\
\delta\hat{B}_+^{(r',s')} &= [\xi^{++}\nabla_{++} - i\tfrac{1}{2}(\nabla_+\xi^{++})\nabla_+]\hat{B}_+^{(r',s')} + \tfrac{1}{2}(s'-r'+1)(\nabla_{++}\xi^{++})\hat{B}_+^{(r',s')} \ , \\
\delta\hat{C}^{(s',r')} &= [\xi^{++}\nabla_{++} - i\tfrac{1}{2}(\nabla_+\xi^{++})\nabla_+]\hat{C}^{(s',r')} - \tfrac{1}{2}(s'-r')(\nabla_{++}\xi^{++})\hat{C}^{(s',r')} \ , \\
\delta\tilde{B}_{--}^{(r,s)} &= \xi^{--}(\nabla_{--}\tilde{B}_{--}^{(r,s)}) - \tfrac{1}{2}(s-r-2)(\nabla_{--}\xi^{--})\tilde{B}_{--}^{(r,s)} \ , \\
\delta\tilde{C}^{(s,r)} &= \xi^{--}(\nabla_{--}\tilde{C}^{(s,r)}) + \tfrac{1}{2}(s-r)(\nabla_{--}\xi^{--})\tilde{C}^{(s,r)} \ ,
\end{aligned}
\tag{5.2}
$$

with $\delta\Lambda_+{}^{--}$ and $\delta\Lambda_{--}{}^{++}$ as usual. The notation $\tilde{C}^{(s,r)}$ implies that $\tilde{C}^{(s,r)}$possesses s upper $(+)$ indices and r upper $(-)$ indices and similarly for the other superfields. By choosing $N_{\tilde{\eta}}$, $N_{\tilde{B}}$, $N_{\hat{B}}$, s, r, s' and r' appropriately, cancellation of the "Siegel anomaly" is achieved.

As noted by Hull, there are many noton systems. Another example, is given by [7]

$$
\frac{1}{1-i\Lambda^2}[\mathbf{D}_{--}W + i\tfrac{1}{2}(\nabla_+\Lambda_{--}{}^{++})\nabla_+W][\mathbf{D}_+W - i\nabla_+(\Lambda_{--}{}^{++}\Lambda_+{}^{--}\nabla_+W)] \ . \tag{5.3}
$$

It is invariant under the two Siegel symmetries

$$
\begin{aligned}
\delta W = \xi^{--}\Big[&\frac{1}{1-i\Lambda^2}(\mathbf{D}_{--}W - i\tfrac{1}{2}\Lambda_+{}^{--}[\Lambda_{--}{}^{++}\nabla_+\nabla_{--}W + \tfrac{1}{2}(\nabla_+\Lambda_{--}{}^{++})\nabla_{--}W]) \\
&- i\tfrac{1}{2}\Lambda_+{}^{--}\left(\Lambda_{--}{}^{++}\nabla_+ + \tfrac{1}{2}(\nabla_+\Lambda_{--}{}^{++})\right)\left(\frac{\tilde{\Sigma}_{--}}{1-i\Lambda^2}\right)\Big] \ , \\
\delta W = &-i\tfrac{1}{2}(\xi^{++}\nabla_+(\tilde{\Delta}_+) + \tfrac{1}{2}(\nabla_+\xi^{++})(\tilde{\Delta}_+)) \ ,
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{D}_+W &= \nabla_+W - \Lambda_+{}^{--}\nabla_{--}W \ , \quad \tilde{\Delta}_+ = \nabla_+W - \frac{\Lambda_+{}^{--}}{1-i\Lambda^2}\mathbf{D}_{--}W \ , \\
\mathbf{D}_{--}W &= \nabla_{--}W - i[\Lambda_{--}{}^{++}\nabla_+\nabla_+W + \tfrac{1}{2}(\nabla_+\Lambda_{--}{}^{++})\nabla_+W] \ , \\
\tilde{\Sigma}_{--} &= \mathbf{D}_{--}W - i[\Lambda_{--}{}^{++}\nabla_+(\mathbf{D}_+W) + \tfrac{1}{2}(\nabla_+\Lambda_{--}{}^{++})\mathbf{D}_+W] \ .
\end{aligned}
\tag{5.4}
$$

The work of Fré et.al.in ref.[12], which purports to show the inconsistency of quantizing chiral bosons using the Hull mechanism, uses only the action in (5.3). As already noted by Hull in the second work of ref.[13], cancellation of the "Siegel anomaly" requires the more general system in (5.1).

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[7] The following results correct those given by Fre et.al. Their previously reported results were never verified at the nonlinear level where we have <u>proven</u> by explicit calculation that they fail!

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