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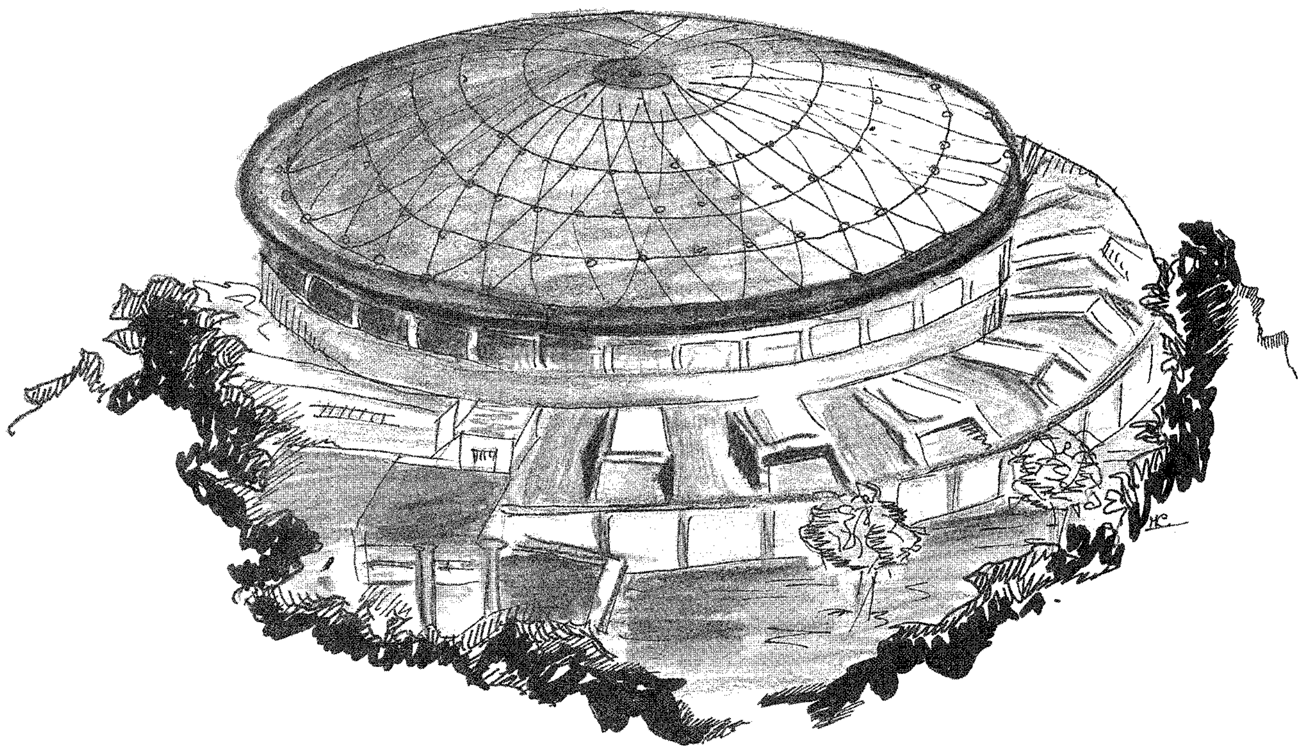

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IMPROVED SUPERGEOMETRIES FOR TYPE-II GREEN-SCHWARZ NON-LINEAR σ-MODEL[1]

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Abstract

New supergeometries which are tailored to simplify calculations of the one-loop quantum corrections of Green-Schwarz nonlinear σ-models are described.

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1. Introduction

The Green-Schwarz actions [1] for type-II superstrings have been known for some time. Similarly the superspace geometries for the $D = 10$, $N = 2a$ [2] and $N = 2b$ [3] supergravity theories have been found. The Green-Schwarz action for the $D = 10, N = 1$ heterotic string has been quantized as a nonlinear σ-model using methods suggested by Kallosh et.al. [5]. However, the type-II theories have not been investigated in this regard. As shown by Grisaru et.al. [4], the calculation of the one-loop quantum corrections to these theories may be simplified tremendously if a special class of constraints are chosen for the background $D = 10$ supergeometries. In this brief note, we will construct the corresponding geometries for the type-II theories.

2. Improved Type-II A Supergeometry

We are considering a superspace as a supermanifold with ten even coordinates $x^{\underline{a}}, \underline{a} = 1, ..., 10$ and a pair of Majorana-Weyl spinors with 16 coordinates each, which are Grassmann variables and have opposite chirality. We denote them by θ^{α} and $\theta^{\dot\alpha}$. Together, they can be put in a compact form: $Z^{\underline{M}} = (x^{\underline{m}}, \theta^{\alpha}, \theta^{\dot\alpha}), \underline{M} = \underline{m}, \alpha, \dot\alpha$.

The superspace is described by a vielbein, Lorentz connection, a super one-form $A_{\underline{A}}$, two-form $B_{\underline{AB}}$ and a super three-form $\Gamma_{\underline{ABC}}$. Taking exterior derivatives of these forms, corresponding 2-,3- and 4-forms are defined: $F_{\underline{AB}}$, $G_{\underline{ABC}}$ and $F_{\underline{ABCD}}$. The vielbein and Lorentz connection give rise to torsion and curvature through the definition of supercovariant derivative and graded commutator of two such derivatives:

$$\begin{aligned} \nabla_{\underline{A}} &= E_{\underline{A}}{}^{\underline{M}} D_{\underline{M}} + \tfrac{1}{2}\omega_{\underline{A}\underline{c}}{}^{\underline{d}} \mathcal{M}_{\underline{d}}{}^{\underline{c}} \\ [\nabla_{\underline{A}}, \nabla_{\underline{B}}\} &= T_{\underline{AB}}{}^{\underline{C}} \nabla_{\underline{C}} + \tfrac{1}{2} R_{\underline{AB}\underline{c}}{}^{\underline{d}} \mathcal{M}_{\underline{d}}{}^{\underline{c}}. \end{aligned} \tag{2.1}$$

Being superfields, torsions and curvatures have too many x-dependent components to describe properly the dynamics of the superspace, so some of the components must be eliminated. This is done by imposing appropriate constraints and using Bianchi identities (BI):

$$[[\nabla_{[\underline{A}}, \nabla_{\underline{B}}\}, \nabla_{\underline{C})}\} = 0. \tag{2.2}$$

Using the definitions of torsions and curvatures we get:

$$\begin{aligned} \nabla_{[\underline{A}} T_{\underline{BC})}{}^{\underline{D}} - T_{[\underline{AB}|}{}^{\underline{E}} T_{\underline{E}|\underline{C})}{}^{\underline{D}} + R_{[\underline{ABC})}{}^{\underline{D}} &= 0 \\ \nabla_{[\underline{A}} R_{\underline{BC})\underline{D}}{}^{\underline{E}} + T_{[\underline{AB}|}{}^{\underline{G}} R_{\underline{G}|\underline{C})\underline{D}}{}^{\underline{E}} &= 0. \end{aligned} \tag{2.3}$$

If we now restrict any component of torsion or curvature, we have made a relation among vielbeins and connections and consequently, other components of torsions and curvatures are related through Bianchi identities. It was shown [6] that if the first set of Bianchi identities is satisfied, then the second one is also satisfied. All these identities are grouped according to engineering dimension. Vector indices have dimension 1 (-1) when they are down (up) and spinor indices carry dimension $\frac{1}{2}(-\frac{1}{2})$ under the same conditions. Gauge fields are of dimension 1 if they contain only vector indices. Replacement of a vector index by a spinor one lowers dimension by $\frac{1}{2}$. All field strengths with dimension less than zero vanish. Since the field strengths are exterior derivatives of their gauge fields, applying that derivative once more will give BIs for the field strengths:

$$\begin{aligned} \nabla_{[\underline{A}} F_{\underline{BC})} - T_{[\underline{AB}}{}^{\underline{D}} F_{\underline{D}|\underline{C})} &= 0, \\ \nabla_{[\underline{A}} G_{\underline{BCD})} - T_{[\underline{AB}}{}^{\underline{E}} G_{\underline{E}|\underline{BCD})} &= 0, \\ \nabla_{[\underline{A}} \tilde{F}_{\underline{BCDE})} - T_{[\underline{AB}}{}^{\underline{G}} \tilde{F}_{\underline{G}|\underline{CDE})} &= F_{[\underline{AB}} G_{\underline{CDE})} + G_{[\underline{ABC}} F_{\underline{DE})}. \end{aligned} \tag{2.4}$$

The field strength $\tilde{F}_{\underline{ABCDE}}$ differs from $F_{\underline{ABCD}}$ by the addition of two Chern-Simons forms whose exterior derivatives appear on the rhs of the final equation above. We also use conventions where our supersymmetrization symbol [) is normalized so that each term appears with wieght one.

The solution of the BIs up to level 1 is: a) supertorsions, level 0, $\frac{1}{2}$ and 1:

$$\begin{aligned}
&T_{\alpha\dot{\beta}}{}^{\underline{a}} = T_{\dot{\alpha}\dot{\beta}}{}^{\gamma} = T_{\alpha\dot{\beta}}{}^{\gamma} = T_{\dot{\alpha}\beta}{}^{\dot{\gamma}} = T_{\alpha\beta}{}^{\dot{\gamma}} = 0\,,\\
&T_{\alpha\underline{a}}{}^{\underline{b}} = T_{\dot{\alpha}\underline{a}}{}^{\underline{b}} = T_{\underline{ab}}{}^{\underline{c}} = 0\,,\\
&T_{\alpha\beta}{}^{\underline{a}} = i(\sigma^{\underline{a}})_{\alpha\beta} \qquad T_{\dot{\alpha}\dot{\beta}}{}^{\underline{a}} = i(\sigma^{\underline{a}})_{\dot{\alpha}\dot{\beta}}\,,\\
&T_{\alpha\beta}{}^{\gamma} = [\delta^{\gamma}_{(\alpha}\delta^{\delta}_{\beta)} + (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}})^{\gamma\delta}]\chi_{\delta}\,,\\
&T_{\dot{\alpha}\dot{\beta}}{}^{\dot{\gamma}} = [\delta^{\dot{\gamma}}_{(\dot{\alpha}}\delta^{\dot{\delta}}_{\dot{\beta})} + (\sigma^{\underline{a}})_{\dot{\alpha}\dot{\beta}}(\sigma_{\underline{a}})^{\dot{\gamma}\dot{\delta}}]\chi_{\dot{\delta}}\,,\\
&T_{\alpha\underline{a}}{}^{\beta} = -\tfrac{1}{8}(\sigma^{\underline{bc}})_{\alpha}{}^{\beta}G_{\underline{abc}}\,, \qquad C_{\alpha\dot{\alpha}}C^{\beta\dot{\beta}}T_{\dot{\beta}\underline{a}}{}^{\dot{\alpha}} = -\tfrac{1}{8}(\sigma^{\underline{bc}})_{\alpha}{}^{\beta}G_{\underline{abc}},
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
C_{\beta\dot{\beta}}T_{\alpha\underline{a}}{}^{\dot{\beta}} &= \tfrac{1}{16}(\sigma_{\underline{a}})_{\alpha\gamma}[(\sigma^{\underline{bc}})_{\beta}{}^{\gamma}B_{\underline{bc}} - \tfrac{1}{12}(\sigma^{[4]})_{\beta}{}^{\gamma}D_{[4]}]\,,\\
C^{\alpha\dot{\alpha}}T_{\dot{\alpha}\underline{a}}{}^{\beta} &= -\tfrac{1}{16}(\sigma_{\underline{a}})^{\alpha\gamma}[(\sigma^{\underline{bc}})_{\gamma}{}^{\beta}B_{\underline{bc}} - \tfrac{1}{12}(\sigma^{[4]})_{\gamma}{}^{\beta}D_{[4]}],
\end{aligned}$$

where

$$\begin{aligned}
B_{\underline{ab}} &= e^{-\Phi}F_{\underline{ab}} - \chi_{\beta}(\sigma_{\underline{ab}})_{\alpha}{}^{\beta}\chi^{\alpha}\,,\\
D_{[4]} &= 2e^{-\Phi}\tilde{F}_{[4]} + \chi_{\alpha}(\sigma_{[4]})_{\beta}{}^{\alpha}\chi^{\beta}.
\end{aligned}$$

(b) Gauge fields components are:

$$\begin{aligned}
&F_{\alpha\dot{\beta}} = C_{\alpha\dot{\beta}}e^{\Phi}, \qquad F_{\alpha\beta} = F_{\dot{\alpha}\dot{\beta}} = 0\,,\\
&F_{\underline{a}\alpha} = ie^{\Phi}(\sigma_{\underline{a}})_{\alpha\beta}\chi^{\beta}, \qquad F_{\underline{a}\alpha} = iC_{\alpha\dot{\alpha}}e^{\Phi}(\sigma_{\underline{a}})^{\alpha\beta}\chi_{\beta}\,,\\
&G_{\alpha\beta\gamma} = G_{\underline{a}\alpha\dot{\beta}} = G_{\underline{ab}\alpha} = G_{\underline{ab}\dot{\alpha}} = 0\,,\\
&G_{\underline{a}\alpha\beta} = i(\sigma_{\underline{a}})_{\alpha\beta}, \qquad G_{\underline{a}\dot{\alpha}\dot{\beta}} = -i(\sigma_{\underline{a}})_{\dot{\alpha}\dot{\beta}}\,,\\
&\tilde{F}_{\alpha\beta\gamma\underline{a}} = \tilde{F}_{\alpha\beta\underline{ab}} = \tilde{F}_{\dot{\alpha}\dot{\beta}\underline{ab}} = 0\,,\\
&\tilde{F}_{\alpha\dot{\beta}\underline{ab}} = e^{\Phi}(\sigma_{\underline{ab}})_{\alpha}{}^{\beta}C_{\beta\dot{\beta}}\,,\\
&\tilde{F}_{\alpha\underline{abc}} = -ie^{\Phi}(\sigma_{\underline{abc}})_{\alpha\beta}\chi^{\beta}, \qquad \tilde{F}_{\dot{\alpha}\underline{abc}} = iC_{\alpha\dot{\alpha}}e^{\Phi}(\sigma_{\underline{abc}})^{\alpha\beta}\chi_{\beta}.
\end{aligned} \tag{2.6}$$

Φ denotes a dilaton superfield, and χ_{α} is its partner dilatino. They satisfy:

$$\begin{aligned}
\nabla_{\alpha}\Phi &= \chi_{\alpha}, \qquad \nabla_{\dot{\alpha}}\Phi = \chi_{\dot{\alpha}},\\
\nabla_{\alpha}\chi_{\dot{\beta}} &= -\nabla_{\dot{\beta}}\chi_{\alpha}, \qquad \nabla_{\alpha}\chi_{\dot{\alpha}} = -8\chi_{\alpha}\chi_{\dot{\alpha}},\\
C^{\gamma\dot{\alpha}}(\sigma_{\underline{ab}})_{\alpha}{}^{\beta}\nabla_{\beta}\chi_{\dot{\alpha}} &= 6e^{-\Phi}F_{\underline{ab}} - 6C^{\gamma\dot{\alpha}}\chi_{\beta}(\sigma_{\underline{ab}})_{\alpha}{}^{\beta}\chi_{\dot{\alpha}}\,,\\
C^{\gamma\dot{\alpha}}(\sigma_{[4]})_{\alpha}{}^{\beta}\nabla_{\beta}\chi_{\dot{\alpha}} &= -4e^{-\Phi}\tilde{F}_{[4]} - 2C^{\gamma\dot{\alpha}}\chi_{\beta}(\sigma_{[4]})_{\alpha}{}^{\beta}\chi_{\alpha}\,,\\
\nabla_{\alpha}\chi_{\beta} &= -i\tfrac{1}{32}(\sigma^{\underline{a}})_{\alpha\beta}\nabla_{\underline{a}}\Phi - i\tfrac{1}{24}e^{-\Phi}(\sigma^{[3]})_{\alpha\beta}G_{[3]} - \chi_{\alpha}\chi_{\beta}\,,\\
\nabla_{\dot{\alpha}}\chi_{\dot{\beta}} &= -i\tfrac{1}{32}(\sigma^{\underline{a}})_{\dot{\alpha}\dot{\beta}}\nabla_{\underline{a}}\Phi + i\tfrac{1}{24}e^{-\Phi}(\sigma^{[3]})_{\dot{\alpha}\dot{\beta}}G_{[3]} - \chi_{\dot{\alpha}}\chi_{\dot{\beta}}.
\end{aligned} \tag{2.7}$$

3. Improved Type-IIB Supergeometry

Type-IIB theory is described by ten even coordinates $x^{\underline{a}}$, $\underline{a}=0,...,9$ and two Majorana-Weyl real spinors which have the same chirality. Unlike IIA model, which is vector-like, IIB is chiral. We can complexify these spinors to get the supercoordinate: $Z^{\underline{M}}=(x^{\underline{m}},\theta^{\alpha},\theta^{\bar\alpha})$, where $\theta^{\bar\alpha}\equiv\bar\theta^{\alpha}$. The content of the free IIB supergrávity is described by a complex scalar Φ, a complex Weyl spinor Λ_{α}, a complex antisymmetric tensor $A_{\underline{ab}}$, whose field strength is denoted by $G_{\underline{abc}}$, a complex Weyl gravitino $\psi_{\underline{a}}{}^{\alpha}$, a real graviton $h_{\underline{ab}}$ and a real forth rank antisymmetric tensor $A_{\underline{abcd}}$ whose field strength $F_{\underline{abcde}}$ is self-dual.

Using the constraints proposed by Grisaru et. al. [4] we find the complete set of torsions and gauge fields expressed in terms of scalar complex fields U and V and a spinor field Λ:

$$
\begin{aligned}
T_{\alpha\bar\beta}{}^{\underline{a}} &= i(\sigma^{\underline{a}})_{\alpha\beta}, \quad T_{\alpha\beta}{}^{\underline{a}} = T_{\bar\alpha\bar\beta}{}^{\underline{a}} = 0, \quad T_{\underline{Eb}}{}^{\underline{c}} = 0 ,\\
T_{\alpha\beta}{}^{\bar\gamma} &= [\delta^{\gamma}_{(\alpha}\delta^{\delta}_{\beta)} + (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}})^{\gamma\delta}]\bar\Lambda_{\delta} ,\\
T_{\bar\alpha\bar\beta}{}^{\bar\gamma} &= [\delta^{\gamma}_{(\alpha}\delta^{\delta}_{\beta)} + (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}})^{\gamma\delta}]\Lambda_{\delta} ,\\
T_{\alpha\beta}{}^{\gamma} &= -\tfrac{1}{2}\delta^{\gamma}_{(\alpha}\Lambda_{\beta)} \quad T_{\alpha\bar\beta}{}^{\bar\gamma} = \tfrac{1}{2}\delta^{\gamma}_{\beta}\Lambda_{\alpha} ,\\
T_{\bar\alpha\bar\beta}{}^{\gamma} &= -\tfrac{1}{2}\delta^{\gamma}_{(\alpha}\bar\Lambda_{\beta)} \quad T_{\bar\alpha\beta}{}^{\gamma} = \tfrac{1}{2}\delta^{\gamma}_{\beta}\bar\Lambda_{\alpha} ,
\end{aligned}
\tag{3.1}
$$

$$
\begin{aligned}
G_{\alpha\beta\gamma} &= 0, \quad G_{\underline{a}\alpha\bar\beta} = 0 ,\\
G_{\underline{a}\alpha\beta} &= iU(\sigma_{\underline{a}})_{\alpha\beta} , \quad G_{\underline{a}\bar\alpha\bar\beta} = iV(\sigma_{\underline{a}})_{\alpha\beta} ,\\
G_{\alpha\underline{ab}} &= -V(\sigma_{\underline{ab}})_{\alpha}{}^{\beta}\bar\Lambda_{\beta} , \quad G_{\bar\alpha\underline{ab}} = -U(\sigma_{\underline{ab}})_{\alpha}{}^{\beta}\Lambda_{\beta},
\end{aligned}
\tag{3.2}
$$

$$
\begin{aligned}
F_{\alpha\beta\underline{abc}} &= F_{\bar\alpha\bar\beta\underline{abc}} = 0 ,\\
F_{\bar\alpha\beta\underline{abc}} &= -\tfrac{3}{20}(\sigma_{\underline{abc}})_{\alpha\beta} ,\\
F_{\alpha\underline{abcd}} &= F_{\bar\alpha\underline{abcd}} = 0,
\end{aligned}
\tag{3.3}
$$

$$
\begin{aligned}
T_{\bar\alpha\underline{a}}{}^{\beta} &= \tfrac{1}{24}[(\sigma_{\underline{a}})_{\alpha\gamma}(\sigma^{[3]})^{\gamma\beta} + \tfrac{1}{2}(\sigma^{[3]})_{\alpha\gamma}(\sigma_{\underline{a}})^{\gamma\beta}][\bar U G_{[3]} - V\bar G_{[3]}] ,\\
T_{\alpha\underline{a}}{}^{\bar\beta} &= \tfrac{1}{24}[(\sigma_{\underline{a}})_{\alpha\gamma}(\sigma^{[3]})^{\gamma\beta} + \tfrac{1}{2}(\sigma^{[3]})_{\alpha\gamma}(\sigma_{a})^{\gamma\beta}][U\bar G_{[3]} - \bar V G_{[3]}],
\end{aligned}
\tag{3.4}
$$

$$
\begin{aligned}
T_{\alpha\underline{a}}{}^{\beta} &= \delta^{\beta}_{\alpha}A^{(1)}_{\underline{a}} + (\sigma_{\underline{a}}{}^{\underline{b}})_{\alpha}{}^{\beta}A^{(2)}_{\underline{b}} + \tfrac{1}{32}(\sigma^{\underline{bc}})_{\alpha}{}^{\beta}B_{\underline{abc}}\\
&\quad + \tfrac{1}{96}(\sigma_{\underline{a}}{}^{\underline{bcd}})_{\alpha}{}^{\beta}C_{\underline{bcd}} + \tfrac{1}{16\ 4!}(\sigma^{[4]})_{\alpha}{}^{\beta}D_{\underline{a}[4]} ,\\
T_{\bar\alpha\underline{a}}{}^{\bar\beta} &= \delta^{\beta}_{\alpha}\bar A^{(1)}_{\underline{a}} + (\sigma_{\underline{a}}\sigma^{\underline{b}})_{\alpha}{}^{\beta}\bar A^{(2)}_{\underline{b}} + \tfrac{1}{32}(\sigma^{\underline{bc}})_{\alpha}{}^{\beta}\bar B_{\underline{abc}}\\
&\quad + \tfrac{1}{96}(\sigma_{\underline{a}}{}^{\underline{bcd}})_{\alpha}{}^{\beta}\bar C_{\underline{bcd}} + \tfrac{1}{16\ 4!}(\sigma^{[4]})_{\alpha}{}^{\beta}\bar D_{\underline{a}[4]} ,\\
A^{(1)}_{\underline{a}} &= -\bar A^{(1)}_{\underline{a}} = -i\tfrac{21}{32}(\sigma_{\underline{a}})^{\alpha\beta}\bar\Lambda_{\alpha}\Lambda_{\beta} + \tfrac{1}{2}(L_{\underline{a}} - \bar L_{\underline{a}}) ,\\
A^{(2)}_{\underline{a}} &= -\bar A^{(2)}_{\underline{a}} = -i3(\sigma_{\underline{a}})^{\alpha\beta}\bar\Lambda_{\alpha}\Lambda_{\beta} ,\\
B_{\underline{abc}} &= \bar B_{\underline{abc}} = -i\tfrac{5}{2}(\sigma_{\underline{abc}})^{\alpha\beta}\bar\Lambda_{\alpha}\Lambda_{\beta} ,\\
C_{\underline{abc}} &= \bar C_{\underline{abc}} = -i\tfrac{3}{2}(\sigma_{\underline{abc}})^{\alpha\beta}\bar\Lambda_{\alpha}\Lambda_{\beta} ,\\
D_{[5]} &= -\bar D_{[5]} = i\tfrac{3}{2}(\sigma_{[5]})^{\alpha\beta}\bar\Lambda_{\alpha}\Lambda_{\beta} - i\tfrac{80}{3}\tilde F_{[5]} ,\\
D_{[5]} - \bar D_{[5]} &= -i\bar\Lambda_{\alpha}(\sigma_{[5]})^{\alpha\beta}\Lambda_{\beta}, \qquad \bar D_{[5]} \equiv \frac{1}{5!}\epsilon_{[5]}{}^{[\bar 5]}D_{[\bar 5]}.
\end{aligned}
\tag{3.5}
$$

Solving the G-Bianchi identity we obtain the following differential equations for the fields U and V:

$$\begin{aligned} \nabla_\alpha U &= -U\Lambda_\alpha, & \bar\nabla_\alpha V &= -V\bar\Lambda_\alpha\,, \\ \nabla_\alpha V &= (V-2U)\Lambda_\alpha, & \bar\nabla_\alpha U &= (U-2V)\bar\Lambda_\alpha\ . \end{aligned} \tag{3.6}$$

These equations give us the form of spinor derivatives of fields U and V in terms of U, V, Λ and $\bar\Lambda$. We can now try to express U and V as functions of a single chiral field, call it W, and still have the equations (3.6) satisfied. If we achieve that, it shows that the physical number of degrees of freedom is smaller than that which we started.

In order to solve these equations we impose two conditions:

$$a) \qquad |U|^2-|V|^2 = 1 \qquad and \qquad b) \qquad U+\bar V = \bar U + V\,. \tag{3.7}$$

These two conditions put very strong restriction on our system of equations. The first one can be imposed because the G field can be rescaled which follows directly from the Bianchi identity for this field. The right hand side still can not be zero because l.h.s appears in the denominators in our calculations. The second condition is the condition for κ-symmetry [4]. The solution of the given system of equations with imposed restrictions is given in the form:

$$U = \frac{1}{\sqrt2}\frac{1+\bar W}{\sqrt{W+\bar W}}\quad,\quad V = \frac{1}{\sqrt2}\frac{1-W}{\sqrt{W+\bar W}}\quad,\quad \nabla_\alpha W = 2(W+\bar W)\Lambda_\alpha, \tag{3.8}$$

where W is a chiral superfield $\bar\nabla_\alpha W = 0$.

The spinor supercovariant derivatives for spinor superfields Λ and $\bar\Lambda$ are readily found:

$$\begin{aligned} \nabla_\alpha\bar\Lambda_\beta &= -\tfrac{3}{2}\Lambda_\alpha\bar\Lambda_\beta + i\bar L_{\underline a}(\sigma^{\underline a})_{\alpha\beta}\,, \\ \bar\nabla_\alpha\Lambda_\beta &= -\tfrac{3}{2}\bar\Lambda_\alpha\Lambda_\beta + iL_{\underline a}(\sigma^{\underline a})_{\alpha\beta}\,, \\ \nabla_\alpha\Lambda_\beta &= \tfrac{3}{2}\Lambda_\alpha\Lambda_\beta - i\tfrac{1}{24}(\sigma^{[3]})_{\alpha\beta}[\bar U G_{[3]} - V\bar G_{[3]}]\,, \\ \bar\nabla_\alpha\bar\Lambda_\beta &= \tfrac{3}{2}\bar\Lambda_\alpha\bar\Lambda_\beta - i\tfrac{1}{24}(\sigma^{[3]})_{\alpha\beta}[U\bar G_{[3]} - \bar V G_{[3]}], \end{aligned} \tag{3.9}$$

where

$$\bar L_{\underline a} \equiv \frac{1}{2}\frac{\nabla_{\underline a}\bar W}{W+\bar W}\quad,\qquad L_{\underline a} \equiv \frac{1}{2}\frac{\nabla_{\underline a} W}{W+\bar W}.$$

As consequences for the fields U and V we now find:

$$\begin{aligned} \nabla_{\underline a} U &= (\bar L_{\underline a} - L_{\underline a})U - 2\bar L_{\underline a} V \\ \nabla_{\underline a} V &= (L_{\underline a} - \bar L_{\underline a})V - 2L_{\underline a} U\ . \end{aligned} \tag{3.10}$$

This set of torsions and gauge superfields represents the unique solution of Bianchi identities subject to the condition of Grisaru et. al. , i.e. the condition for κ-fermionic symmetry of corresponding IIB superstring propagating in the background of the massless sector of the string spectrum. But, our goal being covariant quantization of a string in the curved background, we are not satisfied with these results. In order to quantize string it is necessary to perform normal coordinate expansion [7] and treat the normal coordinates as quantum fields. One can then easily see that both bosonic and fermionic quantum fields contribute to one-loop divergencies [8]. The quantum calculation can be simplified if we use a generalized Weyl [9] transformation. This amounts only to field redefinitions; the new theory is completely equivalent to the one just derived. In order to see how the Weyl transformation works, let us recall that the basic objects of the theory are $\nabla_{\underline A}$, $\bar\nabla_{\underline A}$, and $\mathcal{M}_{\underline a}{}^{\underline b}$. We can rotate these generators to get equivalent set of primed generators:

$$\nabla'_\alpha = e^{d\Phi}\left[\nabla_\alpha + \tfrac{1}{2} f_{\alpha \underline{d}}{}^{\underline{e}} \mathcal{M}_{\underline{e}}{}^{\underline{d}}\right] ,$$
$$\bar{\nabla}_{\bar{\alpha}}{}' = e^{d\Phi}\left[\bar{\nabla}_{\bar{\alpha}} + \tfrac{1}{2} f_{\bar{\alpha} \underline{d}}{}^{\underline{e}} \mathcal{M}_{\underline{e}}{}^{\underline{d}}\right] , \qquad (3.11)$$
$$\nabla_{\underline{a}}{}' = e^{2d\Phi}\left[\nabla_{\underline{a}} + f_{\underline{a}}{}^{\alpha}\nabla_\alpha + f_{\underline{a}}{}^{\bar{\alpha}}\bar{\nabla}_{\bar{\alpha}} + \tfrac{1}{2} f_{\underline{a}\underline{d}}{}^{\underline{e}} \mathcal{M}_{\underline{e}}{}^{\underline{d}}\right] ,$$

whose inverse are:

$$\nabla_\alpha = e^{-d\Phi}\nabla'_\alpha - \tfrac{1}{2} f_{\alpha \underline{d}}{}^{\underline{e}} \mathcal{M}_{\underline{e}}{}^{\underline{d}} ,$$
$$\bar{\nabla}_{\bar{\alpha}} = e^{-d\Phi}\bar{\nabla}_{\bar{\alpha}}{}' - \tfrac{1}{2} f_{\bar{\alpha} \underline{d}}{}^{\underline{e}} \mathcal{M}_{\underline{e}}{}^{\underline{d}} ,$$
$$\nabla_{\underline{a}} = e^{-2d\Phi}\nabla'_{\underline{a}} - e^{-d\Phi}\left[f_{\underline{a}}{}^{\alpha}\nabla_\alpha{}' + f_{\underline{a}}{}^{\bar{\alpha}}\bar{\nabla}_{\bar{\alpha}}{}'\right] + \tfrac{1}{2}\left[f_{\underline{a}}{}^{\alpha} f_{\alpha\underline{d}}{}^{\underline{e}} + f_{\underline{a}}{}^{\bar{\alpha}} f_{\bar{\alpha}\underline{d}}{}^{\underline{e}} - f_{\underline{a}\underline{d}}{}^{\underline{e}}\right]\mathcal{M}_{\underline{e}}{}^{\underline{d}}$$

Relations (3.11) tell us that the components of the supervielbein are related to each other:

$$E'_\alpha{}^{\underline{M}} = e^{d\Phi} E_\alpha{}^{\underline{M}} ,$$
$$E'_{\bar{\alpha}}{}^{\underline{M}} = e^{d\Phi} E_{\bar{\alpha}}{}^{\underline{M}} , \qquad (3.12)$$
$$E'_{\underline{a}}{}^{\underline{M}} = e^{2d\Phi}\left[E_{\underline{a}}{}^{\underline{M}} + f_{\underline{a}}{}^{\alpha} E_\alpha{}^{\underline{M}} + f_{\underline{a}}{}^{\bar{\alpha}} E_{\bar{\alpha}}{}^{\underline{M}}\right].$$

These formulas deserve some explanation. The real f-superfunctions are completely arbitrary to begin. We can make them depend on the background fields. Φ, a superspace scale parameter can in general also be arbitrary, but we choose it to be dilaton in order to cancel factor $e^{-\Phi}$ in $G_{\underline{a}\alpha\beta}$ and $G_{\underline{a}\bar{\alpha}\bar{\beta}}$, so that the zero-dimension components of the field strength $H_{\underline{ABC}}$ (see formula (3.16)) which appears in the Wess-Zumino part of the string can be constant, i.e. independent of the dilaton. That is conditioned by the requirement that the effects of gravity should not be visible at the lowest dimensional level. The real constant d appearing in the exponent is the Weyl weight factor which is doubled for the vector index which follows from the flat case:

$$(D_\alpha, D_\beta) = i(\sigma^{\underline{a}})_{\alpha\beta}\partial_{\underline{a}}.$$

Multiplying vector derivative with e^{2d} means that spinor derivatives are multiplied with e^{d}. Also, it is important to notice the absence of factors in (3.11) which would relate unbarred spinor derivative with barred and vector ones. If we had such factors they would make $T_{\alpha\beta}{}^{\underline{a}}$ and $T_{\bar{\alpha}\bar{\beta}}{}^{\underline{a}}$ different from zero and that would mean that the effects of gravity are introduced already at the zero-dimensional level. Weyl transformations form a group and that makes possible to return to the original set of fields. The transformation is performed on the objects of the dimension zero and and $\frac{1}{2}$. With newly constructed torsions and gauge fields we then again use Bianchi identity to determine the components of the level one. The transformed components of the torsions and of the field G are:

$$T'_{\alpha\beta}{}^{\underline{a}} = T_{\alpha\beta}{}^{\underline{a}} = 0 ,$$
$$T'_{\alpha\beta}{}^{\gamma} = e^{d\Phi}\left[T_{\alpha\beta}{}^{\gamma} + d\Lambda_{(\alpha}\delta^{\gamma}_{\beta)} + \frac{1}{4} f_{(\alpha|\underline{d}}{}^{\underline{e}}(\sigma_{\underline{e}}{}^{\underline{d}})^{\gamma}_{\ \beta)} - T_{\alpha\beta}{}^{\underline{a}} f_{\underline{a}}{}^{\gamma}\right] ,$$
$$T'_{\alpha\beta}{}^{\bar{\gamma}} = e^{d\Phi}\left[T_{\alpha\beta}{}^{\bar{\gamma}} - T_{\alpha\beta}{}^{\underline{a}} f_{\underline{a}}{}^{\bar{\gamma}}\right] ,$$
$$T'_{\alpha\bar{\beta}}{}^{\underline{a}} = T_{\alpha\bar{\beta}}{}^{\underline{a}} = i(\sigma^{\underline{a}})_{\alpha\beta} ,$$
$$T'_{\alpha\bar{\beta}}{}^{\gamma} = e^{d\Phi}\left[T_{\alpha\bar{\beta}}{}^{\gamma} + d\bar{\Lambda}_{\beta}\delta^{\gamma}_{\alpha} + \frac{1}{4} f_{\bar{\beta}\underline{d}}{}^{\underline{e}}(\sigma_{\underline{e}}{}^{\underline{d}})^{\gamma}_{\ \alpha} - T_{\alpha\bar{\beta}}{}^{\underline{a}} f_{\underline{a}}{}^{\gamma}\right] , \qquad (3.13)$$
$$T'_{\alpha\bar{\beta}}{}^{\bar{\gamma}} = e^{d\Phi}\left[T_{\alpha\bar{\beta}}{}^{\bar{\gamma}} + d\Lambda_{\alpha}\delta^{\bar{\gamma}}_{\bar{\beta}} + \frac{1}{4} f_{\alpha\underline{d}}{}^{\underline{e}}(\sigma_{\underline{e}}{}^{\underline{d}})^{\bar{\gamma}}_{\ \bar{\beta}} - T_{\alpha\bar{\beta}}{}^{\underline{a}} f_{\underline{a}}{}^{\bar{\gamma}}\right] ,$$
$$G'_{\underline{a}\alpha\beta} = e^{4d\Phi}\left[G_{\underline{a}\alpha\beta} + f_{\underline{a}}{}^{\gamma} G_{\gamma\alpha\beta} + f_{\underline{a}}{}^{\bar{\gamma}} G_{\bar{\gamma}\alpha\beta}\right]$$
$$G'_{\alpha\underline{ab}} = e^{5d\Phi}\left[G_{\alpha\underline{ab}} + f_{[\underline{a}}{}^{\gamma} G_{\underline{b}]\alpha\gamma} + f_{[\underline{a}}{}^{\bar{\gamma}} G_{\underline{b}]\alpha\bar{\gamma}}\right].$$

The requirement that the gravitational effects will not be present at the lowest level of G fixes the constant d to be $\frac{1}{4}$. The f-functions are determined imposing that the zero dimension torsions should not be changed (the same as in the flat case) and that level $\frac{1}{2}$ torsions should be such that the quantum calculations give the smallest number of divergent graphs. We note that the improved constraints found above have led us to observe that there are some errors in reference [2]. These we be completely enumerated in an expanded discussion of type-II theories in preparation.

The complete information of the IIB model up to level one is then:

a) supertorsions, dimension $\frac{1}{2}$:

$$\begin{aligned} &T_{\alpha\bar{\beta}}{}^{\underline{a}} = i(\sigma^{\underline{a}})_{\alpha\beta}, \quad T_{\alpha\beta}{}^{\underline{a}} = T_{\bar{\alpha}\bar{\beta}}{}^{\underline{a}} = 0, \quad T_{\underline{Eb}}{}^{\underline{c}} = 0 \,, \\ &T_{\alpha\beta}{}^{\gamma} = T_{\bar{\alpha}\bar{\beta}}{}^{\gamma} = T_{\alpha\bar{\beta}}{}^{\bar{\gamma}} = [\delta^{\gamma}_{(\alpha}\delta^{\delta}_{\beta)} + (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}})^{\gamma\delta}]\Lambda_{\delta} \,, \\ &T_{\alpha\beta}{}^{\bar{\gamma}} = T_{\bar{\alpha}\beta}{}^{\gamma} = T_{\bar{\alpha}\bar{\beta}}{}^{\bar{\gamma}} = [\delta^{\gamma}_{(\alpha}\delta^{\delta}_{\beta)} + (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}})^{\gamma\delta}]\bar{\Lambda}_{\delta} \,, \end{aligned} \tag{3.14}$$

b) 3-index field strength, level 0 and $\frac{1}{2}$:

$$\begin{aligned} &G_{\underline{a}\alpha\beta} = i(1+\bar{W})(\sigma_{\underline{a}})_{\alpha\beta} \,, \qquad G_{\underline{a}\bar{\alpha}\bar{\beta}} = i(1-W)(\sigma_{\underline{a}})_{\alpha\beta} \,, \\ &G_{\underline{a}\bar{\alpha}\beta} = 0 \,, \\ &G_{\alpha\underline{ab}} = \tfrac{1}{2}e^{2\Phi}(\sigma_{\underline{ab}})_{\alpha}{}^{\beta}\bar{\Lambda}_{\beta} \,, \qquad G_{\bar{\alpha}\underline{ab}} = -\tfrac{1}{2}e^{2\Phi}(\sigma_{\underline{ab}})_{\alpha}{}^{\beta}\Lambda_{\beta} \,, \end{aligned} \tag{3.15}$$

c) real 3-form field strength which describes the Wess-Zumino term of the Green-Schwarz string necessary for ensuring $\kappa-$ symmetry.

$$\begin{aligned} H_{\underline{ABC}} &\equiv \tfrac{1}{2}(G_{\underline{ABC}} + \bar{G}_{\underline{ABC}}) \,, \\ H_{\underline{a}\alpha\beta} &= H_{\underline{a}\bar{\alpha}\bar{\beta}} = i(\sigma_{\underline{a}})_{\alpha\beta}, \quad H_{\underline{a}\alpha\bar{\beta}} = 0 \,, \end{aligned} \tag{3.16}$$

$$W + \bar{W} = \tfrac{1}{2}e^{2\Phi} \quad , \quad \nabla_{\alpha}W = e^{2\Phi}\Lambda_{\alpha} \quad , \quad \nabla_{\alpha}\Phi = \Lambda_{\alpha}, \tag{3.17}$$

where Φ is the dilaton superfield.

d) level one torsions:

$$\begin{aligned} T_{\alpha\underline{a}}{}^{\bar{\beta}} = {}& \tfrac{1}{24}(\sigma_{\underline{a}})_{\alpha\gamma}(\sigma^{[3]})^{\gamma\beta}\Big[e^{-2\Phi}(1+\bar{W})\bar{G}_{[3]} - e^{-2\Phi}(1-\bar{W})G_{[3]} - i(\sigma_{[3]})^{\alpha\beta}(\Lambda_{\alpha}\Lambda_{\beta} - \bar{\Lambda}_{\alpha}\bar{\Lambda}_{\beta})\Big] \\ & + \tfrac{1}{96}(\sigma^{[3]})_{\alpha\gamma}(\sigma_{\underline{a}})^{\gamma\beta}(G_{[3]} + \bar{G}_{[3]}) \,, \\ T_{\bar{\alpha}\underline{a}}{}^{\beta} = {}& \tfrac{1}{24}(\sigma_{\underline{a}})_{\alpha\gamma}(\sigma^{[3]})^{\gamma\beta}\Big[e^{-2\Phi}(1+W)G_{[3]} - e^{-2\Phi}(1-W)\bar{G}_{[3]} + i(\sigma_{[3]})^{\alpha\beta}(\Lambda_{\alpha}\Lambda_{\beta} - \bar{\Lambda}_{\alpha}\bar{\Lambda}_{\beta})\Big] \\ & + \tfrac{1}{96}(\sigma^{[3]})_{\alpha\gamma}(\sigma_{\underline{a}})^{\gamma\beta}(G_{[3]} + \bar{G}_{[3]}) \,, \\ T_{\alpha\underline{a}}{}^{\beta} = -T_{\bar{\alpha}\underline{a}}{}^{\bar{\beta}} = {}& \tfrac{1}{4}(\sigma_{\underline{a}})_{\alpha\gamma}(\sigma^{\underline{b}})^{\gamma\beta}\Big[e^{-2\Phi}\nabla_{\underline{b}}(W-\bar{W}) + i\tfrac{7}{4}(\sigma_{\underline{b}})^{\alpha\beta}\Lambda_{\alpha}\bar{\Lambda}_{\beta}\Big] \\ & + i\tfrac{1}{48}(\sigma_{[4]})_{\alpha}{}^{\beta}\Big[\tfrac{1}{8}(\sigma_{\underline{a}}{}^{[4]})^{\gamma\delta}\Lambda_{\gamma}\bar{\Lambda}_{\delta} - \tfrac{5}{3}e^{-2\Phi}\tilde{F}_{\underline{a}}{}^{[4]}\Big] \,, \end{aligned} \tag{3.18}$$

e) spinor derivatives of the dilatino:

$$\nabla_{\alpha}\bar{\Lambda}_{\beta} = (\sigma^{\underline{a}})_{\alpha\beta}\big[ie^{-2\Phi}\nabla_{\underline{a}}\bar{W} + (\sigma_{\underline{a}})^{\gamma\delta}\Lambda_{\gamma}\bar{\Lambda}_{\delta}\big] - \bar{\Lambda}_{[\alpha}\Lambda_{\beta]} \,,$$

$$\bar{\nabla}_{\alpha}\Lambda_{\beta} = (\sigma^{\underline{a}})_{\alpha\beta}\big[ie^{-2\Phi}\nabla_{\underline{a}}W + (\sigma_{\underline{a}})^{\gamma\delta}\bar{\Lambda}_{\gamma}\Lambda_{\delta}\big] + \bar{\Lambda}_{[\alpha}\Lambda_{\beta]} \,,$$

$$\nabla_{\alpha}\bar{\Lambda}_{\beta} + \bar{\nabla}_{\beta}\Lambda_{\alpha} = i(\sigma^{\underline{a}})_{\alpha\beta}\nabla_{\underline{a}}\Phi \,,$$

$$\nabla_{(\alpha}\Lambda_{\beta)} = \bar{\nabla}_{(\alpha}\bar{\Lambda}_{\beta)} = 0 ,$$

$$\nabla_\alpha \Lambda_\beta = i\tfrac{1}{24}(\sigma^{[3]})_{\alpha\beta}\Big\{e^{-2\Phi}\big[(\bar{G}_{[3]} - G_{[3]}) + (\bar{W} - W)H_{[3]}\big] - H_{[3]}\Big\} - 3\Lambda_\alpha\Lambda_\beta + \bar{\Lambda}_\alpha\bar{\Lambda}_\beta ,$$

$$\bar{\nabla}_\alpha \bar{\Lambda}_\beta = i\tfrac{1}{24}(\sigma^{[3]})_{\alpha\beta}\Big\{e^{-2\Phi}\big[(G_{[3]} - \bar{G}_{[3]}) + (W - \bar{W})H_{[3]}\big] - H_{[3]}\Big\} - 3\bar{\Lambda}_\alpha\bar{\Lambda}_\beta + \Lambda_\alpha\Lambda_\beta , \qquad (3.19)$$

f) and curvatures:

$$\begin{aligned} R_{\alpha\beta\underline{ab}} = & i\tfrac{1}{12}(\sigma_{\underline{ab}}{}^{[3]})_{\alpha\beta}\Big\{e^{-2\Phi}(1+\bar{W})\bar{G}_{[3]} - e^{-2\Phi}(1-\bar{W})G_{[3]} - i(\sigma_{[3]})^{\gamma\delta}\big[\Lambda_\gamma\Lambda_\delta - \bar{\Lambda}_\gamma\bar{\Lambda}_\delta\big] \\ & - \tfrac{1}{4}(G_{[3]} + \bar{G}_{[3]})\Big\} - i\tfrac{1}{2}(\sigma^{\underline{d}})_{\alpha\beta}\Big\{e^{-2\Phi}(1+\bar{W})\bar{G}_{\underline{abd}} - \\ & - e^{-2\Phi}(1-\bar{W})G_{\underline{abd}} - i(\sigma_{[3]})^{\gamma\delta}\big[\Lambda_\gamma\Lambda_\delta - \bar{\Lambda}_\gamma\bar{\Lambda}_\delta\big] + \tfrac{1}{4}(G_{\underline{abd}} + \bar{G}_{\underline{abd}})\Big\} , \end{aligned}$$

$$\begin{aligned} R_{\bar{\alpha}\bar{\beta}\underline{ab}} = & i\tfrac{1}{12}(\sigma_{\underline{ab}}{}^{[3]})_{\alpha\beta}\Big\{e^{-2\Phi}(1+W)G_{[3]} - e^{-2\Phi}(1-W)\bar{G}_{[3]} + i(\sigma_{[3]})^{\gamma\delta}\big[\Lambda_\gamma\Lambda_\delta - \bar{\Lambda}_\gamma\bar{\Lambda}_\delta\big] \\ & - \tfrac{1}{4}(G_{[3]} + \bar{G}_{[3]})\Big\} - i\tfrac{1}{2}(\sigma^{\underline{d}})_{\alpha\beta}\Big\{e^{-2\Phi}(1+W)G_{\underline{abd}} - \\ & - e^{-2\Phi}(1-W)\bar{G}_{\underline{abd}} + i(\sigma_{[3]})^{\gamma\delta}\big[\Lambda_\gamma\Lambda_\delta - \bar{\Lambda}_\gamma\bar{\Lambda}_\delta\big] + \tfrac{1}{4}(G_{\underline{abd}} + \bar{G}_{\underline{abd}})\Big\} . \end{aligned} \qquad (3.20)$$

4. Type II κ–supersymmetry Invariance and Green-Schwarz Actions.

Among other space-time and world-sheet symmetries, the classical Green-Schwarz superstring in the flat space possesses the so called fermionic κ-symmetry. This is a local, world-sheet gauge symmetry responsible for gauging away half of the fermionic degrees of freedom, which then makes equal the number of bosonic and fermionic degrees of freedom in the light cone. In the curved $D = 10$ background, κ-symmetry is maintained only if the Bianchi identities (and implicitly, the field equations) for the background are satisfied [4]. This means that the background is on-shell since the Bianchi identities of the engineering dimension 2 are the background equations of motion. Furthermore the requirements of κ-symmetry and supersymmetry combined give us the restriction that the space-time must be of dimension 3,4,6 or 10. At the quantum level κ-symmetry has even more profound role. For the reason stated above this symmetry must be present while acting on quantum fields which are obtained by the normal coordinate expansion. In order to quantize the model as in ref [8], we have to fix the gauge of this symmetry and this procedure breaks space-time global Lorentz invariance. In order to preserve the Lorentz invariance, infinite number of ghosts are introduced, showing that κ-symmetry has infinite degree of reducibility. The most general quantization procedure in the Lagrangian formulation of gauge theories with open gauge algebra is given by Batalin and Vilkovisky [10]. Some special truncation procedures were invented [5], with the goal of making the algebra irreducible. In this case the problem of quantization is reduced to the supergravity case without auxiliary fields, where the algebra is closed only on shell, which gives rise to four-ghost coupling. We shall now show that the Green-Schwarz superstring propagating in the curved background just derived possesses κ-symmetry.

The string action is given by:

$$S = \int d^2\sigma V^{-1}\Big[\Pi_{++}{}^{\underline{a}}\Pi_{--\,\underline{a}} + \int_0^1 dy \hat{\Pi}_y{}^{\underline{C}}\hat{\Pi}_{++}{}^{\underline{B}}\hat{\Pi}_{--}{}^{A}\hat{H}_{\underline{ABC}}\Big], \qquad (4.1)$$

where

$$\Pi_{++}{}^{\underline{A}} = V_{++}{}^{m}\partial_m Z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}} , \qquad \Pi_{--}{}^{\underline{A}} = V_{--}{}^{m}\partial_m Z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}},$$
$$\hat{Z}^{\underline{M}} = Z^{\underline{M}}(\sigma,\tau,y) , \qquad \hat{\Pi}_y^{\underline{A}} = \partial_y \hat{Z}^{\underline{M}} E_{\underline{M}}{}^{\underline{A}} , \qquad \hat{H}_{\underline{ABC}} = H_{\underline{ABC}}(\hat{Z}) .$$

(We have changed our convention for H from some of our works [10].) Variation of this action is given in [10]. We here only state that in the case of IIA theory this action is invariant under the following κ transformations:

$$\begin{aligned} \delta E^{\alpha} &= -i\kappa_{++\beta}(\sigma_{\underline{a}})^{\beta\alpha}\Pi_{--}^{\underline{a}} , & \delta E^{\dot\alpha} &= -i\kappa_{++\dot\beta}(\sigma_{a})^{\dot\beta\dot\alpha}\Pi_{++}^{\underline{a}} , \\ \delta V_{++}{}^{m} &= 2\kappa_{++\beta}\Pi_{++}^{\beta}V_{--}{}^{m} , & \delta V_{--}{}^{m} &= 2\kappa_{++\dot\beta}\Pi_{--}^{\dot\beta}V_{++}{}^{m} . \end{aligned} \tag{4.2}$$

Here $V_{++}{}^{m}, V_{--}{}^{m}$ are the components of the zweibein on the world-sheet and κ is a two dimensional vector, space-time spinor parameter defined on the world sheet. Also, $\delta E^{\underline{A}} \equiv \delta Z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}$.

For IIB theory there is also a set of variations of the fields $Z^{\underline{M}}, V_{++}{}^{m}$ and $V_{--}{}^{m}$ for which the action (4.1) is invariant. In order to check the invariance it is convenient to define:

$$2E_R^{\alpha} = E^{\alpha} + E^{\dot\alpha} , \qquad 2iE_L^{\alpha} = E^{\alpha} - E^{\dot\alpha} ,$$
$$2\Pi_{R++}^{\alpha} = \Pi_{++}^{\alpha} + \bar{\Pi}_{++}^{\alpha} , \qquad 2i\Pi_{L--}^{\alpha} = \Pi_{--}^{\alpha} + \bar{\Pi}_{--}^{\alpha} ,$$

with these definitions the action is invariant under:

$$\begin{aligned} \delta E_R^{\alpha} &= -i\kappa_{++\beta}(\sigma_a)^{\beta\alpha}\Pi_{--}^{\underline{a}} , & \delta E_L^{\alpha} &= -i\kappa_{--\beta}(\sigma_{\underline{a}})^{\beta\alpha}\Pi_{++}^{\underline{a}} , \\ \delta V_{++}{}^{m} &= 4\kappa_{++\beta}\Pi_{R++}^{\beta}V_{--}{}^{m} , & \delta V_{--}{}^{m} &= 4\kappa_{--\beta}\Pi_{L--}^{\beta}V_{++}{}^{m} . \end{aligned} \tag{4.3}$$

5 Conclusion

We have derived the complete set of torsions, curvatures and gauge fields for IIA and IIB supergravities in curved space up to dimension one relevant for the analysis of string propagation in the background made of its massless modes. The improved supergeometry is appropriate for quantum calculations and for the study of anomaly cancelation. The invariance of these models under κ -transformations is clear. In order to apply the method of Batalin and Vilkovisky [11], the complete algebra of all string symmetries in the curved space is needed. Besides the local Lorentz, reparametrization, conformal and κ symmetries, a bosonic λ-symmetry must be included for N=2 models [12]. This symmetry is not a gauge symmetry and its relevance and role in the covariant string quantization is not well understood.

References

[1] M. B. Green and J. H. Schwarz, Nucl.Phys. B243 (1984) 285.

[2] J. Carr, S. J. Gates,Jr. and R. Oerter, Phys.Lett. 189B (1987)68.

[3] M. T. Grisaru, P. Howe, L. Mezincescu, B. Nilsson and P. Townsend, Phys. Lett. 162B (1985) 116.

[4] M. T. Grisaru, H. Nishino and D. Zanon, Phys. Lett. 206B (1988) 625.

[5] R. Kallosh, Phys. Lett. 195B (1987) 369.

[6] N. Dragon,Z.Phys C 2,29 (1979).

[7] S. Mukhi, Nucl. Phys. B264 (1986) 640.

[8] M. T. Grisaru, H. Nishino and D. Zanon, Brandeis University Report BRX-TH-250 (1988) (to appear in Nucl. Phys. B), M. T. Grisaru, H. Nishino and D. Zanon, Brandeis University Report BRX-TH-251 (1988) (to appear in Nucl. Phys. B).

[9] S. J. Gates,Jr. and S. Vashakidze, Nucl.Phys.B291 (1987) 172; S. J. Gates,Jr.,in Supergravity, eds. P. van Nieuwenhuisen and D. Z. Freedmen, (North-Holand,1979),p.215; Nucl.Phys. B176 (1980).

[10] S.J. Gates Jr. and H. Nishino, Phys.Lett.B173 (1986) 96

[11] I. Batalin and G. Vilkovisky, Phys.Rev. D28 (1983) 2567.

[12] W. Siegel, Phys.Lett.128B (1983) 397; R. E. Kallosh, M .A. Rahmanov, Phys.Lett. B209 (1988) 133