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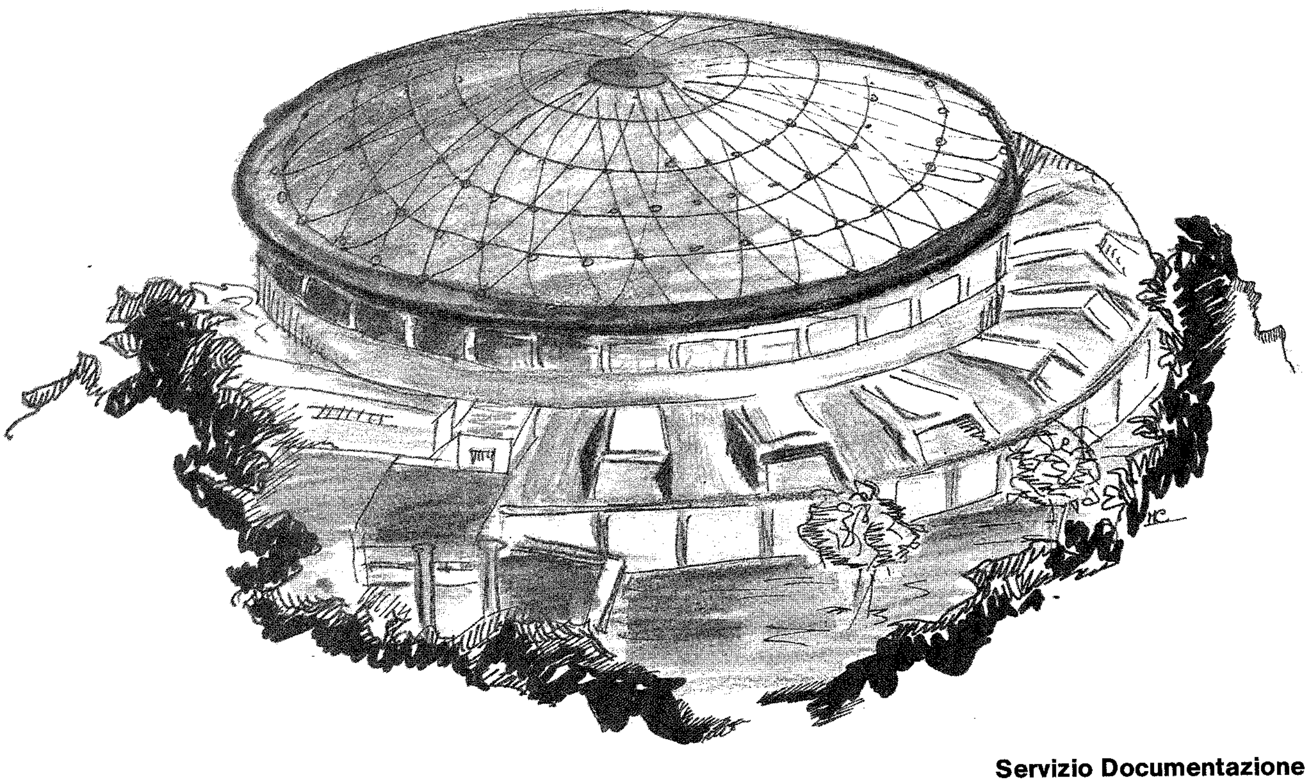

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STRUCTURE FUNCTIONS AND INITIAL-FINAL STATE INTERFERENCE IN QED

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ABSTRACT

Interference effects from initial and final state radiation are explicitly introduced within the formalism of structure functions in QED. The analytical results agree with those previously obtained in the approach of coherent states. The method is also suitable for Monte Carlo applications.

The structure functions approach in QED[1] has been proven[2] to be very convenient for an accurate determination of the radiative corrections at LEP/SLC energies[3] and also suitable for realistic Monte Carlo generators[4]. In a previous letter[5] an exact analytical solution has been given corresponding to initial and final state radiation, which coincides with the result obtained[6,7] long time ago in the framework of coherent states[8] for the exponentiated infrared factors and the remaining terms of o (α). In addition it provides the o (α^2) left over corrections necessary to get absolute control of the QED corrections.

The limitation of the method is represented so far by the lack of description of the interference effects between the initial and final state radiation, whose knowledge is essential for a correct description of forward-backward asymmetries, Bhabha scattering, etc. On the other hand such an information is available in the coherent states approach.

The aim of the present note is to provide a solution to this problem, which is also suitable to Monte Carlo applications. More in detail we will show that the initial-final state interference effects can be exactly calculated for QED and resonant processes in the structure functions method, following closely the usual techniques used to obtain the K-factors in QCD. Again our results are in excellent agreement with the previous treatment[6,7] of the problem.

In the reaction $e^+e^- \rightarrow \mu^+\mu^-$ the following formula[9]

$$d\sigma(s) = \int_0^\varepsilon dx\, d\sigma_0((1-x)s)\, H_e(x,s)\, F_\mu(\varepsilon - x, (1-x)s), \tag{1}$$

where $\varepsilon \equiv \Delta E/E$ is the energy resolution, and the initial and final state radiation kernels are given in terms of the electron and muon structure functions as

$$H_e(x,s) = \int_{1-x}^{1} \frac{dz}{z} D_e(z,s)\, D_e\left(\frac{1-x}{z}, s\right), \tag{2}$$

$$F_\mu(x,s) = \int_0^x dy\, H_\mu(y, (1-y)s), \tag{3}$$

with $H_\mu(x, s)$ defined as in eq. (2), describes the factorizable corrections only, corresponding to real and virtual photon emission from the initial and final legs, with no relative interference. By taking into account the effect of the soft radiation to all orders and of the hard one up to o (α^2) one obtains[9]

$$\begin{aligned} H_e(x,s) &= \Delta_e(s)\, \beta_e\, x^{\beta_e - 1} - \frac{1}{2}\beta_e(2-x) \\ &+ \frac{1}{2}\beta_e^2 \left\{ (2-x)[3\ln(1-x) - 4\ln x] - 4\frac{\ln(1-x)}{x} + x - 6 \right\}, \end{aligned} \tag{4}$$

with $\beta_e = \frac{2\alpha}{\pi}(L_e - 1)$, $L_e = \ln\left(\frac{s}{m_e^2}\right)$ and

$$\begin{aligned} \Delta_e(s) &= 1 + \frac{\alpha}{\pi}\left[\frac{3}{2}L_e + 2(\zeta(2) - 1)\right] + \left(\frac{\alpha}{\pi}\right)^2 \left\{ \left[\frac{9}{8} - 2\zeta(2)\right] L_e^2 \right. \\ &\left. + \left[3\zeta(3) + \frac{11}{2}\zeta(2) - \frac{45}{16}\right] L_e + \left[-\frac{6}{5}\zeta^2(2) - \frac{9}{2}\zeta(3) - 6\zeta(2)\ln 2 + \frac{3}{8}\zeta(2) + \frac{57}{12}\right] \right\} \\ &\equiv 1 + \frac{\alpha}{\pi}\Delta_e^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \Delta_e^{(2)} \end{aligned} \tag{5}$$

Now a full account of all radiative effects can be simply obtained by replacing the Born cross section $d\sigma_0$ [(1–x)s] in eq. (1) by $d\sigma_0$ [(1–x)s]·K(x), where the K-factor includes all notfactorizable corrections, which can be determined to all orders for the soft contribution and to o (α) for the nonleading terms. By splitting the Born cross section as $d\sigma_0 = d\sigma_0^{QED} + d\sigma_0^{INT} + d\sigma_0^{RES}$, with ($M_R^2 = M^2 - iM\Gamma$)

$$
\begin{aligned}
d\sigma_0^{QED}(s) &= A\frac{1}{s} \\
d\sigma_0^{INT}(s) &= B\cdot \mathrm{Re}\left\{\frac{1}{s-M_R^2}\right\} \\
d\sigma_0^{RES}(s) &= C\frac{s}{|s-M_R^2|^2},
\end{aligned}
\tag{6}
$$

and defining $\beta_{int} = (4\alpha/\pi)\ \ln \mathrm{tg}\ \theta/2$, with θ the muon scattering angle, we obtain for the soft contributions, corresponding to initial-final state interference, the following expressions for the $K^{(i)}(x)$ factors

$$
\begin{aligned}
K^{QED}(x) &= \frac{(\beta_e+\beta_{int})}{\beta_e}\,[x\,(\varepsilon-x)]^{\beta_{int}} \\
K^{INT}(x) &= \frac{(\beta_e+\beta_{int})}{\beta_e}\left[\frac{sx}{s\,(1-x)-M_R^2}(\varepsilon-x)\right]^{\beta_{int}} \\
K^{RES}(x) &= \frac{(\beta_e+2\beta_{int})}{\beta_e}\left|\frac{sx}{s\,(1-x)-M_R^2}\right|^{2\beta_{int}}.
\end{aligned}
\tag{7}
$$

The above result is based on the observation[6] that in the pure QED process the virtual matrix element M_V^{QED} scales as $\{\lambda^2/s\}^{(\beta_e+\beta_\mu+2\beta_{int})/4}$, while for a resonant process one has $M_V^{RES} \sim\{\lambda^2/s\}^{(\beta_e+\beta_\mu)/4}\cdot\{\lambda^2/(s-M_R^2)\}^{\beta_{int}/2}$, where λ is the minimum energy cutoff. Eqs. (7) allow us to obtain the complete analytical solution within the method of the structure functions, generalizing the results of ref. [5].

Indeed after the substitution $d\sigma_0^{(i)}[(1-x)s] \to d\sigma_0^{(i)}[(1-x)\,s]\cdot K^{(i)}(x)$ in eq. (1), using eqs. (7), and following ref. [5], one then finds for the leading terms - corresponding to the resummation of the soft contributions:

$$
d\sigma^{QED}(s) = d\sigma_0^{QED}(s)\ \Bigg\{\Delta_e(s)\,\Delta_\mu(s)\,\varepsilon^{\bar\beta_e+\bar\beta_\mu}
\cdot\frac{\Gamma(1+\bar\beta_e)\,\Gamma(1+\bar\beta_\mu)}{\Gamma(1+\bar\beta_e+\bar\beta_\mu)}\,{}_2F_1(1,\bar\beta_e;1+\bar\beta_e+\bar\beta_\mu;\varepsilon)+\dots\Bigg\},
\tag{8}
$$

$$d\sigma^{INT}(s) = d\sigma_0^{INT}(s)\,\Delta_e(s)\,\Delta_\mu(s)\,\varepsilon^{\bar\beta_\mu}\,\frac{\Gamma(1+\bar\beta_e)\,\Gamma(1+\bar\beta_\mu)}{\Gamma(1+\bar\beta_e+\bar\beta_\mu)}\,\frac{1}{\cos\delta_R}\,\cdot$$

$$\cdot\ \mathrm{Re}\left\{e^{i\delta_R}\left[\frac{\varepsilon}{1+(\frac{\varepsilon s}{M\Gamma})\sin\delta_R\, e^{i\delta_R}}\right]^{\beta_e}\left[\frac{\varepsilon}{\varepsilon+(\frac{M\Gamma}{s})\frac{e^{-i\delta_R}}{\sin\delta_R}}\right]^{\beta_{int}}\right\}+\ldots, \tag{9}$$

$$d\sigma^{RES}(s) = d\sigma_0^{RES}(s)\,\Delta_e(s)\,\Delta_\mu(s)\,\varepsilon^{\beta_\mu}\,\frac{\Gamma(1+\bar{\bar\beta}_e)\,\Gamma(1+\beta_\mu)}{\Gamma(1+\bar{\bar\beta}_\varepsilon+\beta_\mu)}\cdot\left|\frac{\varepsilon}{\varepsilon+(\frac{M\Gamma}{s})\frac{e^{-i\delta_R}}{\sin\delta_R}}\right|^{2\beta_{int}}\cdot$$

$$\left|\frac{\varepsilon}{1+(\frac{\varepsilon s}{M\Gamma})\sin\delta_R\, e^{i\delta_R}}\right|^{\beta_e}(\cos\beta_e\phi-\cot\delta_R\sin\beta_e\phi)+\ldots, \tag{10}$$

where $(M_R^2-s)^{-1} = \frac{\sin\delta_R\, e^{i\delta_R}}{M\Gamma}$, $\tan\delta_R = \frac{M\Gamma}{(M^2-s)}$, $\phi = \arctan\left[\frac{\varepsilon s+M^2-s}{M\Gamma}\right] -$

$- \arctan\left[\frac{M^2-s}{M\Gamma}\right]$ and $\bar\beta_{e,\mu} = \beta_{e,\mu}+\beta_{int}$, $\bar{\bar\beta}_e = \beta_e+2\beta_{int}$.

A few comments are in order here. First of all the main β_{int}-dependence in eqs. (8-10) appears through exponentiated factors, which coincide with those found in refs. [6, 7], using the method of coherent states. Of course they also reproduce the exact one-loop calculations[7, 10]. Furthermore the β_{int}-dependence drops out completely in the limiting case of narrow resonance production ($\Gamma \ll \Delta\omega$), as for example the J/ψ.

Physically this can be understood through the observation that the initial and final state can no longer interfere since the long time delay ($\tau \sim 1/\Gamma$) due to the resonance formation and decay.

Grouping together all non-infrared factors coming from $\Delta_e(s)$, $\Delta_\mu(s)$ and the Γ-functions in eqs. (8-10), as well as those coming from the non-soft terms of the electron and muon radiators, we then write, as in ref. [5],

$$\frac{d\sigma}{d\Omega} = \sum_i^{QED,\,INT,\,RES}\frac{d\sigma_0^{(i)}}{d\Omega}\left\{C_{infra}^{(i)}\,(1+\bar C_F^{(i)}) + C_F^{\prime(i)}\right\} \tag{11}$$

where the infrared factors $C^{(i)}_{\rm infra}$ are simply obtained from eqs. (8-10), and

$$\bar{C}_F^{(QED)} = \left(\frac{\alpha}{\pi}\right)\left[\Delta_e^{(1)} + \Delta_\mu^{(1)}\right] - \beta_\mu\,\varepsilon + \left(\frac{\alpha}{\pi}\right)^2\left[\Delta_e^{(2)} + \Delta_\mu^{(2)} + \Delta_e^{(1)}\Delta_\mu^{(1)}\right] - \frac{\pi^2}{6}\bar{\beta}_e\bar{\beta}_\mu - \frac{1}{4}\beta_e\bar{\beta}_e\,\varepsilon^{1-\bar{\beta}_e} \qquad (12)$$

$$\bar{C}_F^{(INT)} = \left(\frac{\alpha}{\pi}\right)\left[\Delta_e^{(1)} + \Delta_\mu^{(1)}\right] - \beta_\mu\,\varepsilon + \left(\frac{\alpha}{\pi}\right)^2\left[\Delta_e^{(2)} + \Delta_\mu^{(2)} + \Delta_e^{(1)}\Delta_\mu^{(1)}\right] - \frac{\pi^2}{6}\bar{\beta}_e\bar{\beta}_\mu$$

$$- \bar{\beta}_e\,\frac{\cos\phi\,(\beta_e+1) + \tan\delta_R\sin\phi\,(\beta_e+1)}{\cos\phi\beta_e + \tan\delta_R\sin\phi\beta_e}\left|\frac{\varepsilon}{1+\dfrac{\varepsilon s}{M_R^2 - s}}\right| \qquad (13)$$

$$-\frac{1}{4}\beta_e\bar{\beta}_e\,\frac{\cos\phi + \tan\delta_R\sin\phi}{\cos\phi\beta_e + \tan\delta_R\sin\phi\beta_e}\left|\frac{\varepsilon}{1+\dfrac{\varepsilon s}{M_R^2 - s}}\right|^{1-\beta_e}$$

$$\bar{C}_F^{(RES)} = \left(\frac{\alpha}{\pi}\right)\left[\Delta_e^{(1)} + \Delta_\mu^{(1)}\right] - \beta_\mu\,\varepsilon + \left(\frac{\alpha}{\pi}\right)^2\left[\Delta_e^{(2)} + \Delta_\mu^{(2)} + \Delta_e^{(1)}\Delta_\mu^{(1)}\right] - \frac{\pi^2}{6}\bar{\bar{\beta}}_e\beta_\mu$$

$$-\,2\bar{\bar{\beta}}_e\,\frac{\cos\phi\,(\beta_e+1) - \cot\delta_R\sin\phi\,(\beta_e+1)}{\cos\phi\,\beta_e - \cot\delta_R\sin\phi\,\beta_e}\left|\frac{\varepsilon}{1+\dfrac{\varepsilon s}{M_R^2 - s}}\right| \qquad (14)$$

$$-\frac{1}{4}\beta_e\bar{\bar{\beta}}_e\,\frac{\cos\phi - \cot\delta_R\sin\phi}{\cos\phi\,\beta_e - \cot\delta_R\sin\phi\,\beta_e}\left|\frac{\varepsilon}{1+\dfrac{\varepsilon s}{M_R^2 - s}}\right|^{1-\beta_e}.$$

Finally the factors $C_F'^{(i)}$ contain other $o(\alpha)$ finite terms, coming from bremsstrahlung and box diagrams, odd in the exchange $\theta \leftrightarrow \pi - \theta$, etc., and can be obtained from refs. [3, 5].

To conclude we have explicitly shown how to include the interference effects coming from initial and final state radiation within the approach of structure functions in QED. The resulting expression - which generalizes the previous analysis[5] limited to initial and final states only -

agrees the results from the coherent states approach, improving the theoretical accuracy to o(0.1%). The method is suitable for Monte Carlo applications.

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