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Laboratori Nazionali di Frascati

To be submited to J. Geophy.Res. B

LNF-89/026(P)
16 Maggio 1989

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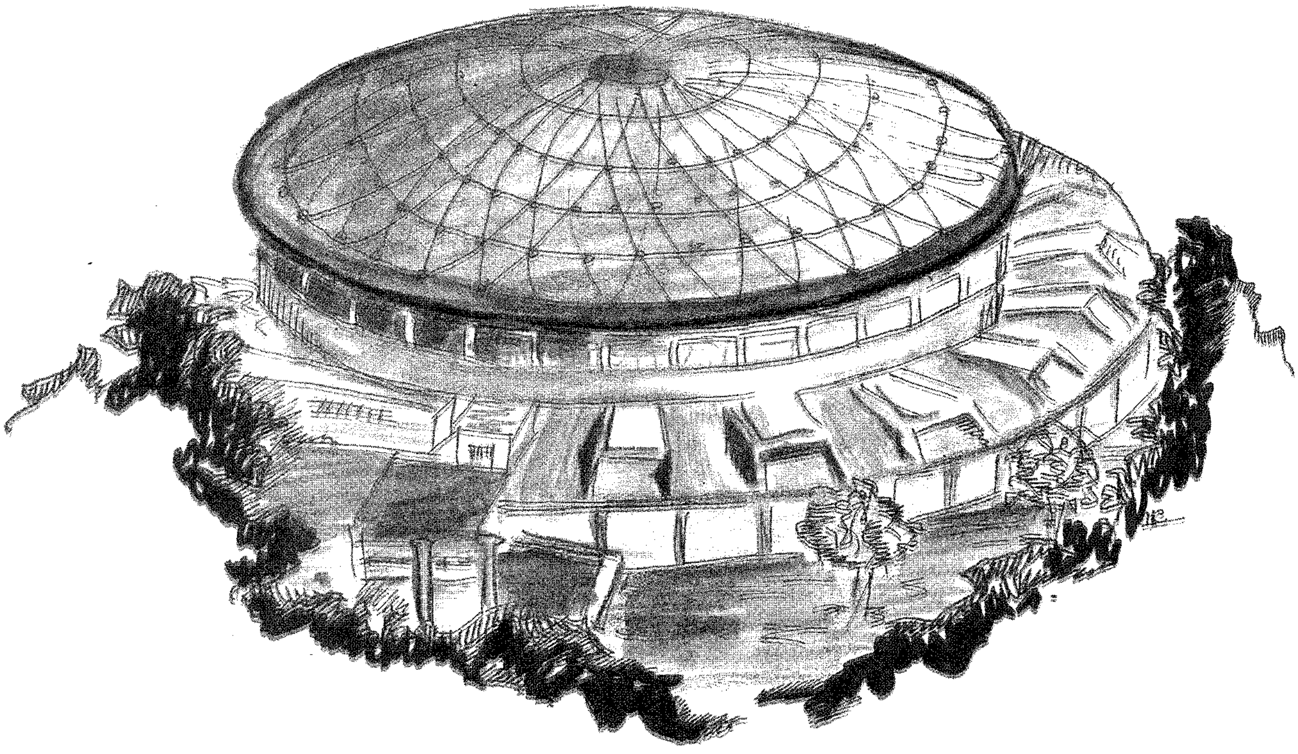

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THE FLUX OF THE VERTICAL NEGATIVE MUONS STOPPING AT DEPTHS 0.35 - 1000 hg/cm^2

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ABSTRACT

The vertical negative muon stopping rate at depths 0.35-1000 hg/cm^2 of standard rock has been evaluated by means of a Monte Carlo simulation of the muon transport underground. The muon energy losses take into account both continuous (ionization and pair production) and stochastic (bremmstrahlung and nulear interaction) processes. The dependence of the charge ratio μ^+/μ^- on the muon momentum is included. The effect of muon multiple scattering has been estimated.

Results are given both for high latitude and equatorial sites and they are compared to previous calculations and to experimental data.

1. - INTRODUCTION

Recent advances in Accelerator Mass Spectrometry (A.M.S.) have made it possible to measure the concentrations of a number of radioisotopes produced in-situ in rocks: ^{10}Be, ^{26}Al and ^{36}Cl (Lal and Arnold, 1985; Nishiizumi et al.,1986; Phillips et al., 1986).

These nuclides are mainly produced in nuclear spallations of target elements present in the rocks. However, in the case of ^{26}Al and ^{36}Cl a not inappreciable production comes from capture of μ^- mesons in ^{28}Si for ^{26}Al and in ^{39}K and ^{40}Ca for ^{36}Cl.

In the troposphere, production of ^{10}Be, ^{26}Al and ^{36}Cl is primarily due to the fast nucleonic component of the cosmic radiation. ^{10}Be is mainly produced in the spallation of N and O; ^{26}Al and ^{36}Cl are produced in spallation of ^{40}Ar, but ^{36}Cl also arises from μ^- capture by ^{40}Ar ($\mu^- + {}^{40}\mathrm{Ar} \rightarrow {}^{36}\mathrm{Cl} + 4\ \mathrm{n}$), Winsberg, 1956.

Isotopic transformations in the earth's surface are induced by stopped cosmic ray negative muons. This mechanism is the dominant cosmic ray production source for ^{36}Cl and ^{26}Al from depths of 2 m down to about 50 m. Below this depth, photonuclear reactions of the fast muon component gain importance, but μ^- capture processes continue to give a comparable contribution at least up to 500 m below surface. The ratio of the flux of stopping muons to the flux of througoing muons decreases with increasing depth from the value of $6.5\ 10^{-2}\ \mathrm{g}^{-1}\mathrm{cm}^2$ at 1 hg/cm^2 down to $1.5\ 10^{-2}\ \mathrm{g}^{-1}\mathrm{cm}^2$ at 100 hg/cm^2 (1 hg/cm^2 = 100 g/cm^2).

The effects of μ^- have been studied in targets exposed at sea level and underground by Rama and Honda, 1961, Tanaka et al., 1968 and later by Hampel et al., 1975. Definite evidence for μ-products has been found by Takagi et al., 1967, in tellurium and by Clayton and Morgan, 1977 in molibdenum.

The propagation of the cosmic ray secondaries in the atmosphere and in rocks and estimates of the rates of nuclear disintegrations due to neutrons, fast μ-mesons and capture of negative muons have been considered by Lal, 1988 in view of discussing the production of ^{3}He in terrestrial rocks.

Another interesting reason comes from being the μ-capture rate of importance in determining the ^{37}Ar background for the chlorine solar neutrino experiment at Homestake (Davis et al., 1987). The background is deduced by measuring the ^{37}Ar production in potassium detectors, due to both fast muons and negative stopped muons (Fireman et al., 1984) at different depths in the Homestake mine.

The two contributions are experimentally indistinguishable. According to the calculations of Zatsepin et al., 1981, the interactions of fast muons is the dominant effect at large depths, > 10^3 hg/cm^2. At shallow depths instead, the contribution of negative stopped muons is not negligible and it has to be evaluated in order to obtain the ^{37}Ar production rate by fast muons.

In view of the importance of both the nuclides produced in-situ as geophysical tracers and the background induced in solar neutrino experiments, it is important to know the stopping rate of negative cosmic ray muons in the lithosphere up to large depths.

Estimates were given before by Rossi 1952, Charalambus 1971, Short 1963 and some comparison will be made in the text.

The evaluation of this flux requires the knowledge of the negative muon differential momentum spectrum and of the interaction processes with matter responsible for the muon attenuation underground.

The measurements of the muon absolute vertical intensity at sea level and its charge ratio have been the subject of many investigations. Apart for a few exceptions, world data agree within the experimental uncertainties of the results.

Mainly, the discrepancies are originated by lack of statistics or systematic errors in the different experimental procedures, and by true variations of the muon flux which is sensitive to the changing of metereological conditions and to the solar activity.

Atmospheric effects and cyclic variations can be taken into account allowing one to define an average shape of the sea level differential muon momentum spectrum with an estimated error less than 10% up to several hundred GeV energies.

On the other hand, since the measurements of the muon charge ratio are largely unaffected by experimental biases, the cosmic ray negative muon fraction is obtained with an accuracy better than 4% in the momentum range 1-1000 GeV.

Hence, the negative muon differential spectrum can be derived with an uncertainty not exceeding 10% on average.

Muon interaction processes with matter are well understood in the energy range of several hundred GeV, important up to 10^3 hg/cm^2. Accurate cross sections are provided apart for the photonuclear interaction. However, the large uncertainties in describing the photonuclear interaction are not relevant in this energy range, its contribution to the total energy loss being less than 10%. Thus, muon absorption in standard rock is obtained with noticeable precision allowing the negative cosmic ray muon stopping rate to be calculated with good accuracy.

The present work was undertaken with the aim of producing the vertical negative muon stopping rate versus depth relation in the range 0.35-1000 hg/cm^2 of standard rock at different latitudes.

These results could be used as a basis for the calculation of the number of negative muons stopped per gram and per second at a given site underground once the surface profile is known.

The Monte Carlo method and the muon interaction cross sections used for underground muon transport are discussed in Sect.2, 3. Muon spectra assumed to derive the stopping rate versus depth relation are given in Sect.4. In the last section the results of these calculations are presented and discussed.

2. - MUON TRANSPORT UNDERGROUND

The absolute flux of vertical muons stopping in an absorber of thickness Δx at depth h is obtained as the difference of the survived muons at depth h and h + Δx, that is by differentiation of the depth-intensity curve.

However, underground measurements concern only the overall flux of both sign muons and no systematic investigation of the latitude dependence of the muon intensity underground has been carried on.

The lack of detailed experimental information and the uncertainty in comparing results obtained in different environmental conditions, in particular under different rock compositions, make it very difficult the use of underground data to evaluate the negative muon stopping rate.

For these reasons, it is more convenient to consider the muon differential spectrum at sea level in conjunction to the range energy curve.

In fact, the absolute muon intensity has been measured with good accuracy up to several hundred GeV and can be folded to the functions describing the latitude effect and charge ratio dependence on momentum. This procedure provides a realistic representation of the negative muon differential spectrum.

At low depths (< 50 hg/cm^2), where ionization losses dominate, the negative muon stopping rate $\Phi_s(h)$ is obtained as

$$\Phi_s(h) = \frac{I(p_1) - I(p_2)}{p_2 - p_1}$$

being I(p) the integral flux and p_1,p_2 the muon momenta corresponding to the ranges h and $h + \Delta x$ respectively.

In fact, at a depth of 60 hg/cm^2, which is the range of a muon with momentum of about 10 GeV/c, the contribution to the energy loss from pair production, bremmstrahlung and nuclear interactions amounts to 1% of the total energy loss. The contribution rises to 10% at 80 GeV/c, the average muon energy corresponding to a range of 350 hg/cm^2. The interaction processes other than ionization have an increasingly important effect at muon energies exceeding 100 GeV and their combined energy loss equals ionization losses at about 620 GeV. Thus, these contributions have to be included in order to derive a range-momentum relation valid also at energies greater than 50 GeV (at this energy, the range is decreased by about 2.5%).

Moreover, at high energies significant fluctuations in range arise, because the cross sections for bremmstrahlung and nuclear interaction fall down rather slowly $\left(\sim \frac{1}{v} \; ; \; v = \frac{E'}{E_\mu} \right)$ with increasing energy transfer E'. As a consequence, muons in the TeV region exhibit a broad "range straggling" distribution.

To account in a unique way for these different behaviours depending on the energy region, we have used a Monte Carlo simulation of the muon transport through an absorber. This technique allowed us to investigate also the effect of multiple scattering by using a three-dimensional version of the Monte Carlo program.

This code, Bilokon et al. 1989, is essentially the same one developed to analyze muon events recorded at Mt. Blanc both in the Nusex detector, Battistoni et al. 1985, and in a previous spark chamber experiment, Bergamasco et al. 1983. It has also been used to correlate the local muon spectrum at large depths of standard rock to the muon spectral index at sea level, Castellina et al. 1985.

The following procedure has been adopted:

a) for each depth h a muon energy is sampled from the appropriate muon vertical differential spectrum J(p) at sea level (p = muon momentum). The sampling of the distribution J(p) is performed between the limits p_{min} and p_{max}, where p_{min} is smaller than the momentum corresponding to a range h, and p_{max} is fixed at $5.\ 10^3$ GeV, the contribution from higher energies being irrelevant to the depths here considered.

b) the muon is transported underground dividing the total depth h in units of 0.02-0.4 hg/cm^2 (depending on the residual path) and calculating the energy loss in each element for both continuous (ionization and pair production) and stochastic (bremsstrahlung and nuclear interaction) processes.

Details of the procedure are given in Bilokon et al. 1989. Briefly, the stochastic energy loss in each depth unit is sampled from the appropriate function and added to the continuous one to give the total energy loss. Then the energy of the muon is reduced by this quantity and the process is repeated for the next unit. We have defined a limit energy of 5 GeV above which all processes are included.

At lower energies only the ionization loss is taken into account, the contribution of the other processes being more than 3 orders of magnitude smaller. When the muon energy falls below 5 GeV the residual range of the muon is calculated using the energy-range relation obtained integrating the expression for the muon energy loss. In such a way the number of survived muons and their residual energy can be obtained. The muon intensity I(h) is given by:

$$I\,(h) = \frac{N(h)}{N} \int_{p_{min}}^{p_{max}} J(p)\, dp$$

where N is the number of muons sampled between p_{min} and p_{max} according to the distribution J(p) and N(h) is the number of survived muons.

Thus the stopping muon rate Φ_s (muons/g sec sr) is simply obtained as:

$$\Phi_s\left(h + \frac{\Delta x}{2}\right) = \frac{I(h) - I(h + \Delta x)}{\Delta x} = \frac{1}{N} \int_{p_{min}}^{p_{max}} J(p)\, dp\ \frac{N(h) - N(h + \Delta x)}{\Delta x}$$

The values adopted for Δx range from 10 g/cm^2, for depths in the range 0.35-10 hg/cm^2, to 100 g/cm^2. This choice represents a compromise between the necessity of decreasing the statistical fluctuations and of minimizing any average effect on the muon spectrum. More than 50000 muons have been traced to each depth so that the resulting statistical error was less than 4%.

3. - ENERGY LOSS PROCESSES AND CROSS SECTIONS

Over the range of depths covered by our calculations the dominant mode of energy loss is that involving the ionization (and excitation) process. The average energy loss is given by the Bethe-Bloch formula (see Sternheimer, 1956)

$$\frac{dE_\mu}{dx}\left[\mathrm{MeV\ g^{-1}\ cm^{2}}\right] = 0.135\ \frac{1}{\beta_\mu^{2}}\ \frac{Z}{A}$$

$$\left\{ \ln\frac{m_e E_m}{I^2(Z)} + 0.693 + 2\ln\frac{p_\mu}{m_\mu} + \frac{1}{4}\left(\frac{E_m}{E_\mu}\right)^2 - 2\beta_\mu - \delta - U \right\}$$

where m_e is the electron mass, m_μ, β_μ, E_μ and p_μ are the mass, velocity, energy and momentum of the muon, E_m is the maximum energy the passing particle can transfer to an electron of the medium, given by

$$E_m = 2m_e \frac{p_\mu^2}{m_e^2 + m_\mu^2 + 2\,m_e \cdot E_\mu}$$

Z, A, I, are the average atomic number, atomic weight and ionization potential of the medium. δ is the density effect term and U is the shell correction term.

This last term is a very small quantity and can be neglected. For the density correction δ we have used the parametrization of Sternheimer et al., 1971:

$$\begin{aligned} \delta(x) &= 4.6052\,x + a\,(x_1 - x)^m + c \qquad && x_0 < x < x_1 \\ &= 4.6052\,x + c && x > x_1 \end{aligned}$$

where $x = \log_{10}\left(\frac{p_\mu}{m_\mu}\right)$. The values for Z, A, I, x_0, x_1, a, m, c relative to the standard rock are given by:

$$\begin{array}{lll} Z = 11, & A = 22, & I = 148\ \mathrm{eV} \\ x_0 = 0.2, & x_1 = 3.0 & \\ a = 0.14, & m = 3, & c = -3.99 \end{array}$$

The explicit form of the cross section $\frac{d\sigma}{dv}$ (v = fraction of the energy lost by the muon) for pair production, bremmstrahlung and nuclear interaction is given in Bilokon et al., 1989.

The adopted cross sections are those calculated in the works of Kokulin and Petrukhin, 1971, Petrukhin and Shestakov, 1968 and Minorikawa et al., 1981.

The inelastic muon-nucleus interaction contains many uncertainties. The description of this process involves the knowledge of the flux of virtual photons accompanying the muons and their interaction with the nucleus field.

Several models have been developed in which the results of direct measurements of the γ–A cross section up to about 200 GeV are extrapolated to higher energies. In calculating the nuclear energy loss, we use a photonuclear cross section with a small rising behaviour in the TeV region (case A of Minorikawa et al., 1981).

The uncertainty related to this choice does not affect the present results because of the restricted range of muon energies covered (differences in the muon nuclear interaction can be appreciated only at depths larger than 7000 hg/cm^2, see Bilokon et al., 1987.

The interaction probability per g/cm^2, that is the distribution sampled to simulate the stochastic energy loss by bremmstrahlung and nuclear interaction, is given by

$$\frac{dP}{dv} = \frac{N}{A}\frac{d\sigma}{dv}$$

while the average energy loss is obtained as

$$\frac{dE_\mu}{dx} = E_\mu \int_{v_{min}}^{v_{max}} v \, \frac{dP}{dv} \, dv = E_\mu \, b\,(E_\mu)$$

The contributions of the individual processes are shown in Fig. 1.

By accounting for all muon energy losses, the range momentum relation shown in Fig. 2 has been derived.

The curve obtained by only considering the ionization process is also drawn for comparison. The difference becomes appreciable at depths larger than 300 hg/cm^2.

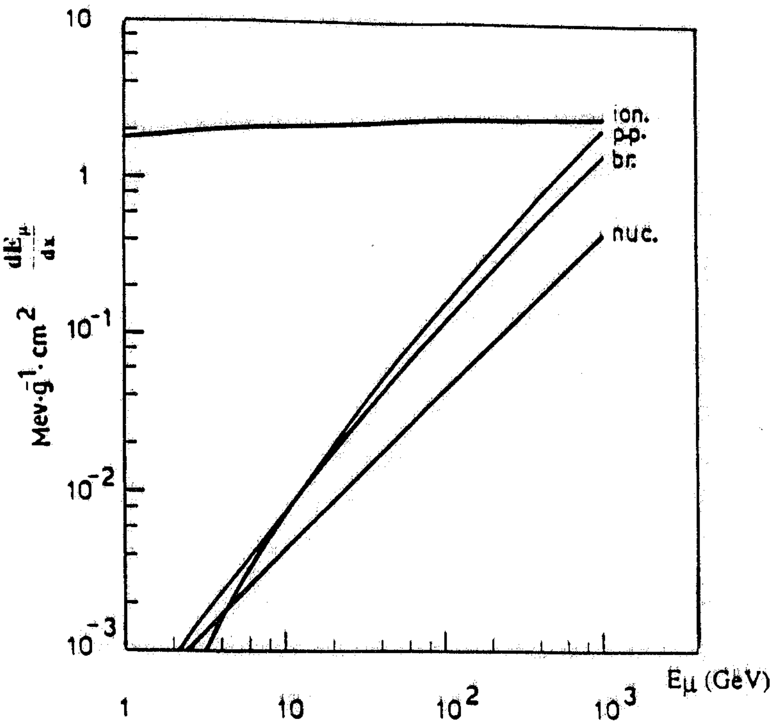

FIG. 1 - Contribution to the muon energy loss in standard rock from ionization (ion.), bremmstrahlung (br.), pair production (p.p.) and nuclear interaction (nuc.) as a function of the energy.

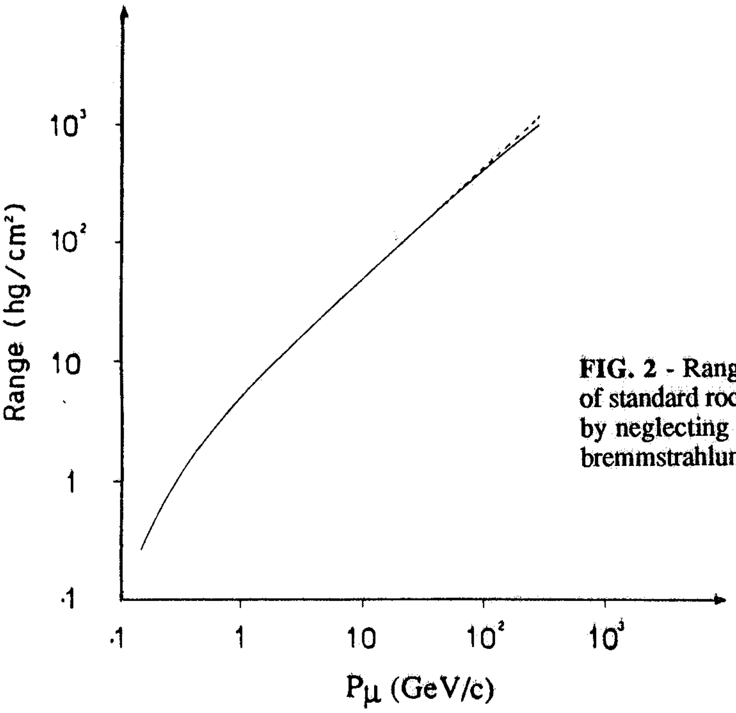

FIG. 2 - Range-momentum curve up to 1000 hg/cm^2 of standard rock (solid line). The dotted line is obtained by neglecting the energy losses from pair production, bremmstrahlung and nuclear interactions.

4. - ADOPTED MUON MOMENTUM SPECTRUM

We have derived the negative muon differential spectrum by folding the muon spectrum to the momentum dependence of the muon charge ratio.

Recent results on the muon spectrum at high geomagnetic latitudes (45^{0} N) come from the work of Baschiera et al., 1979, Shen Peiruo et al., 1981, Barbouti and Rastin, 1983, and Rastin, 1984.

The two last measurements have been performed using a solid iron magnet spectrometer and cover a large momentum range extending from 0.35 to 3000 GeV/c. They overlap to the muon spectrum obtained in the momentum range 20-500 GeV/c by the solid iron magnetic spectrograph located in Durham (Ayre et al., 1975),

Previous measurements of the absolute vertical intensity of muons between 0.1 and 10 GeV/c have been summarized in Rastin, 1984 and in the work of Allkofer and Jokisch, 1973, where the latitude dependence and modulation by the 11 year cycle of solar activity have been considered in order to compare experiments performed in different places and times.

When comparing all experimental data, we find a general agreement at a level of ± 10%. The discrepancies can be ascribed to different experimental conditions and methods of measurements, systematic apparatus errors combined to insufficient corrections and, for the low momentum region, to different methereological conditions and time variations.

In the present work, we have used the form fit of De et al., 1972, which has been estimated by Allkofer and Jokisch, 1973 to correctly reproduce, within the uncertainties, the experimental data up to 10 GeV/c.

This spectrum has been extrapolated up to 20 GeV/c to reach the differential intensities measured by Ayre et al., 1975. In doing this, a slight modification of the form fit parameters has been introduced to improve the consistency with the experimental data.

The parametrization quoted in Ayre et al., 1975 well reproduces the measured intensities even beyond the original momentum range, giving an intensity of 1.44 10^{-10} cm^2 sr^{-1} s^{-1} $(GeV/c)^{-1}$ at a momentum 10^3 GeV/c, to be compared with the experimental value of (1.42 ± 0.09) 10^{-10} cm^{-2} sr^{-1} s^{-1} $(GeV/c)^{-1}$ from the Mont Blanc experiment, Bergamasco et al., 1983, and in good agreement with the intensity (1.40 ± 0.50) 10^{-10} cm^{-2} sr^{-1} s^{-1} $(GeV/c)^{-1}$ from the world survey of Ng and Wolfendale, 1974 and Allkofer et al., 1979. Beyond 10^3 GeV/c, the spectrum has been assumed to follow a power law $E^{-3.71}$ (Bergamasco et al., 1983).

The adopted parametrizations provide a good representation of the experimental data in the whole momentum range 0.15-5000 GeV/c, the only remarkable exception being the low momentum (< 1 GeV/c) Nottingham data of Rastin 1984, which are up to 30% lower. The momentum spectrum is shown in Fig. 3.

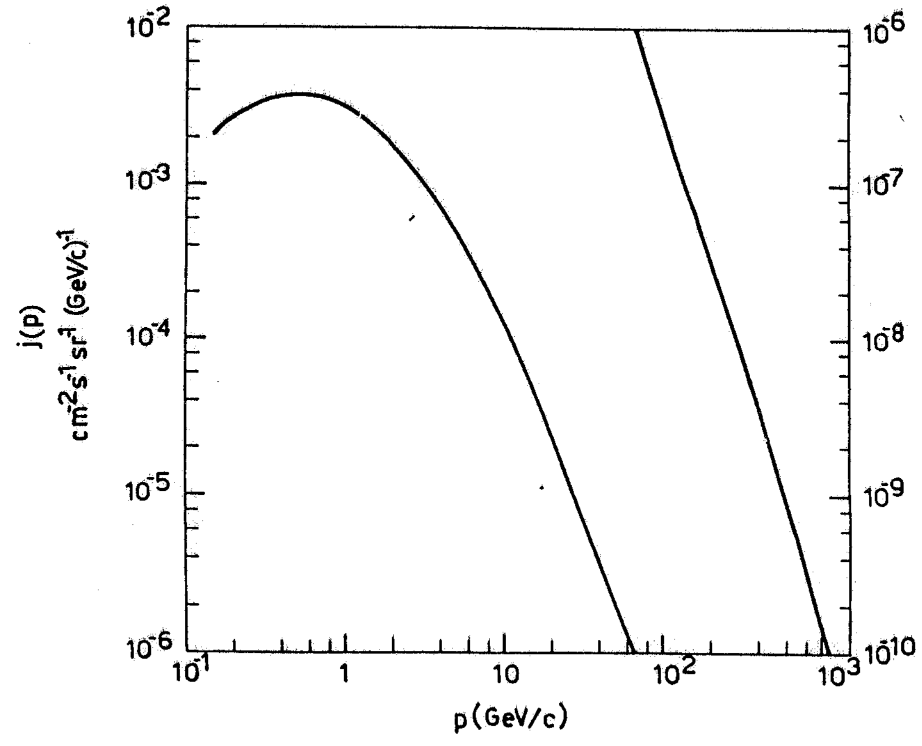

FIG. 3 - The differential absolute vertical muon spectrum J(p) at sea level used in the present work.

Only a few experiments have been performed at low latitudes near equator, showing the expected reduction of the muon intensity in the low energy region. The measured latitude effect includes the geomagnetic effect on the production spectrum of muons and the atmospheric latitude effect on their survival probability.

The latitude dependence of the sea level muon flux has been considered by Allkofer and Jokisch, 1973. Measurements performed at Kiel (high latitude) and at equator with the same spectrograph allow to obtain the effect without any experimental bias, Allkofer et al., 1975.

Fig. 4 gives the ratio of the muon intensity at low latitude (equator) to that at high latitude (~ 45° N). It shows that the latitude effect exists only for momenta below 4 GeV/c and reaches its maximum around 0.5 GeV/c.

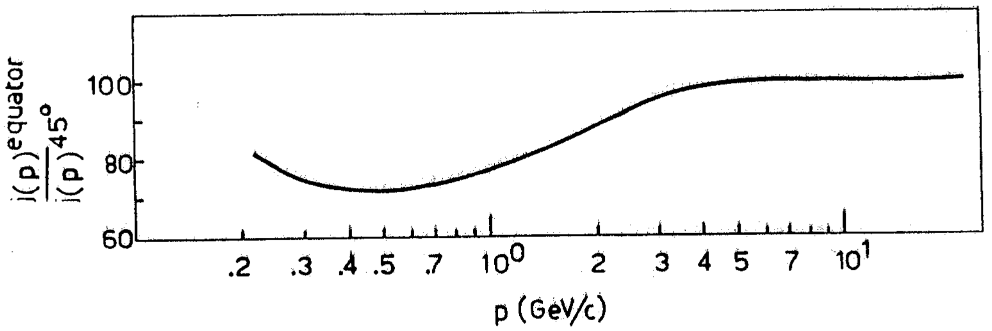

FIG. 4 - The latitude effect of the muon differential spectrum. The curve gives the momentum dependence of the ratio between the differential spectrum at the equator and that at high latitude (45° N).

The muon charge ratio μ^+/μ^- in vertical direction has been extensively measured in many experiments at high latitude. Since the primary cosmic ray particles are predominantly protons, an excess of positively charged muons results. A good, even if incomplete, summary above 2 GeV is given by Das and De, 1979. The charge ratio increases slowly passing from 1.25 to 1.37 in the momentum range 5-1000 GeV/c. Below 1 GeV/c the contamination from low energy electrons makes the measurements less reliable, so that conflicting results have been obtained by different authors, as pointed out by Singhal, 1983.

The charge ratio at low momenta seems to go down to nearly unit as expected from the diminished survival probability of muons from the first collisions and increased contribution from further generations in lower atmosphere, where the parent hadrons are more symmetrical in charge.

To allow for these experimental uncertainties two extrapolations below 2 GeV/c have been considered, corresponding to a limit charge ratio of 1.0 or 1.2 at 150 MeV/c, as shown in Fig. 5.

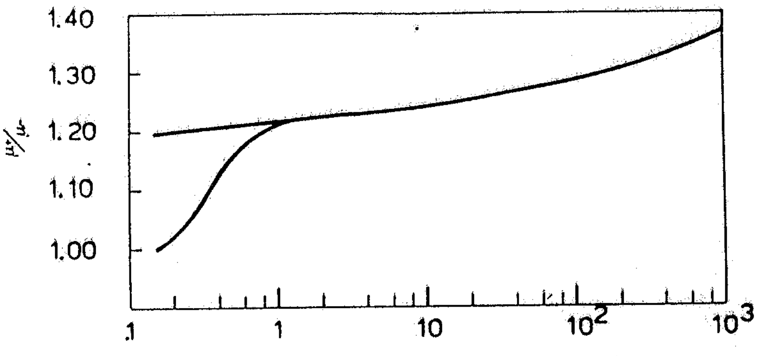

FIG. 5 - The charge ratio μ^+/μ^- as a function of the muon momentum. The branches below 1 GeV/c correspond to the two adopted extrapolations (see text).

They reasonably cover the true behaviour of the charge ratio at low momenta. This behaviour has been assumed independent from latitude as suggested by the data on muon charge ratio in equatorial regions.

The appropriate combination of the muon differential spectrum, latitude effect and charge ratio allows to determine the vertical negative muon momentum at high and low latitude sites with an average uncertainty ± 10%.

5. - RESULTS

The stopping rate $\Phi_S(h)$ $(g^{-1}s^{-1}sr^{-1})$ of vertical negative muons in the depth interval 0.35–1000 hg/cm^2 is shown in Fig. 6.

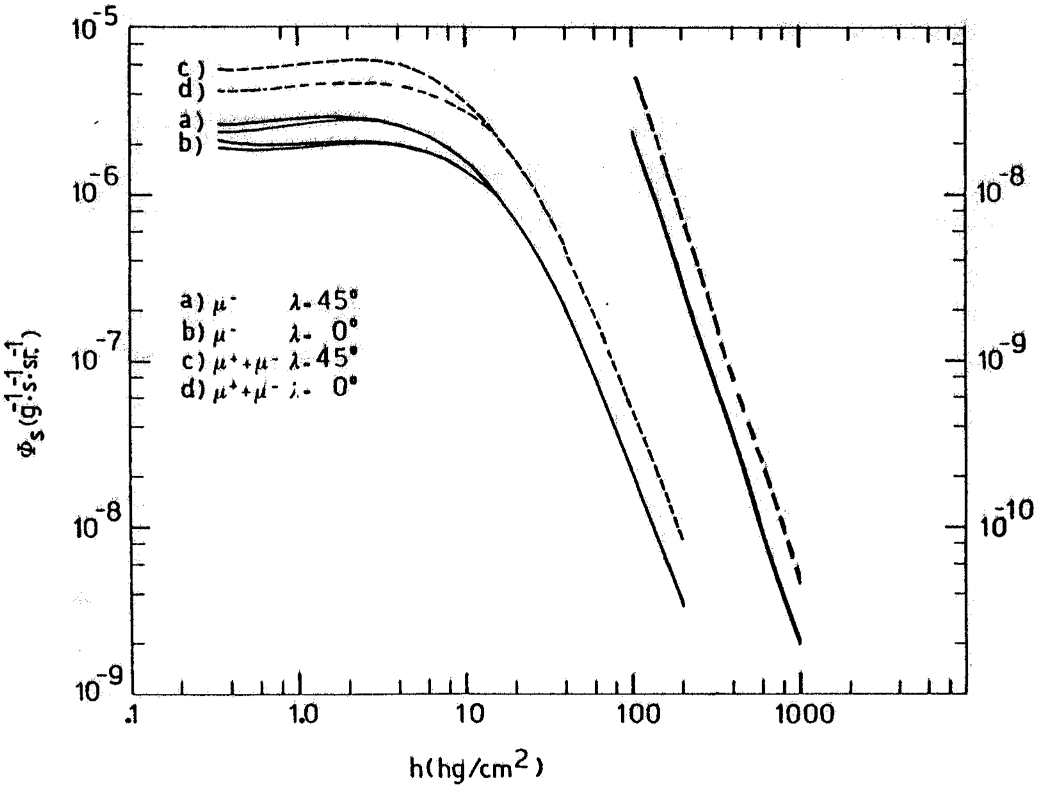

FIG. 6 - The stopping rate of vertical negative muons as a function of depth h (full curves). The upper curve (a) corresponds to high latitudes, the lower one (b) to the equator. The branches below 3 hg/cm^2 refer to the two extrapolations of the charge ratio: the upper one comes from the lower branch of Fig. 4 ($\mu^+/\mu^- = 1$ at 150 MeV/c), the lowest one from the upper branch in the same figure ($\mu^+/\mu^- = 1.2$ at 150 MeV/c). Note that h is the depth underground below sea level. The stopping rate of all muons is shown for comparison in the dashed curve.

The two branches below 20 hg/cm^2 give the stopping rate at both high latitude (45° N) and equatorial sites, while the branches below 3 hg/cm^2 correspond to the two extrapolations of the charge ratio at low momenta.

For comparison, the same results for all the muons are shown. An uncertainty of 12% (including the statistical errors of Monte Carlo procedure) is associated to both branches, which are therefore clearly distinct.

The slight increase of the stopping rate up to a depth of 2 hg/cm^2 reflects the shape of the muon spectrum, having a broad maximum in the interval 0.4-0.8 GeV/c; 1 GeV/c muons are stopped at a depth of 5 hg/cm^2.

Beyond this energy the stopping muon rate falls down following the behaviour of the momentum spectrum at sea level.

A check of the whole procedure has been performed by calculating the stopping rate as the derivative of the integral flux at a muon momentum related to the depth by means of the range energy relation of Fig. 2.

The resulting muon stopping rate is practically coincident up to about 600 hg/cm^2 to that obtained by means of the Monte Carlo simulation. This result confirms that the shape of the curves in Fig. 6 depends on the assumed muon momentum spectrum and that the use of the range-momentum relation is well suitable to calculate the muon stopping rate at low and shallow depths.

At depths greater that 600 hg/cm^2 an increasing difference is found, the numerical calculation giving a muon stopping rate larger than the one found by means of the Monte Carlo.

This difference reflects the effect of the range fluctuations which are not accounted for in a calculation based on the range-momentum relation.

However, the effect is rather small in the range of depths here considered, being of the order of 15% at 1000 hg/cm^2.

A further check has been carried on by calculating the vertical muon intensity over the same range of depths from 0.35 to 1000 hg/cm^2 of standard rock below sea level. The resulting depth-intensity variation is in excellent agreement with the experimental data (see for instance Rastin, 1984).

A comparison of the negative muon stopping rate with the calculations of Charalambus, 1971 has been performed. The muon stopping rates given there are integrated over the solid angle. Therefore, this result has been converted by applying a correction factor of $3/2\pi$ which accounts for the integration on the muon angular distribution.

The results coincide within a few percent at low depths less than 10 hg/cm^2.

Discrepancies up to about 50% exist at higher depths below 100 hg/cm^2, due to an incorrect depth-intensity curve used in the range 10–100 hg/cm^2 in Charalambus, 1971. In fact, the negative muon stopping rate is there obtained by the differentiation of the depth-intensity curve scaled for a constant factor 2.27 to account for the fraction of negative muons.

In Table I the results of the present calculation (third column) are compared to the negative muon stopping rates evaluated by Fireman et al., 1984 (second column). (We have converted our rates in terms of events per day and per ton).

These authors obtain the stopped muon rate at depths in the range 177-820 hg/cm^2, from the product R(h) I(h), being R(h) the ratio between stopping and crossing muons calculated by Grupen et al., 1972, and I(h) the muon intensity underground according to Miyake, 1973, and scaling the result by a factor 2.27 to account for a constant positive to negative muon ratio of 1.27. This is a reasonable approximation since the energy threshold for muons to reach these depths ranges between 40 GeV and 200 GeV, where the positive to negative muon ratio increases slowly from about 1.26 to 1.30 (Fig. 5). However the calculation of the stopped muon rate R(h) I(h) can be improved by using recent underground intensity data and differentiating the depth-intensity curve I(h). An updated parametrization of the depth-intensity curve I(h) covering the range 10-10000 hg/cm^2, has been given by Barbouti and Rastin, 1983.

TABLE I - Comparison of the calculated negative muon stopping rate (negative muons/day ton) - third column - with the calculations by Fireman et al., 1984 and the results obtained by differentiating the depth-intensity curve parametrization of Barbouti and Rastin, 1983 (see text). Depths are measured from the sea level.

Depth (hg/cm^2)	μ-stopping rate (ev/ day . ton)	Present work	$\frac{dI}{dh}\frac{\mu^-}{\mu_{tot}}$
177	1200	860	910
254	286	326	308
327	127	122	139
473	52.6	36.2	41.0
620	19.0	15.4	16.0
820	6.8	6.3	5.9

In this way, the fourth column of Table I has been obtained. All results of these calculations are in excellent agreement with each other, differences not exceeding 15%, apart for the stopping rate at 177 hg/cm^2, quoted by Fireman et al., 1984, which appears to be too high.

It also appears that in this depth interval a straightforward use of the underground intensities with the assumption of a constant positive to negative muon ratio produces sufficiently accurate results .

The latitude effects and the dependence of the charge ratio on the muon momentum can be accounted for only by means of a Monte Carlo simulation of the muon transport underground.

This technique allowed us to investigate the effect of multiple scattering.

In fact this process causes the effective thickness of the absorber to be greater than the actual geometrical path. The lenghtening of the path originates practically only at the end of the range, so that the corresponding increase of the muon momentum is of the order of a few percent only at very low depths and vanishes with increasing absorber thickness. On the other hand, the muon spectrum exhibits a flat shape at low energies ($p < 1$ GeV/c) so that a slight momentum shift is not expected to modify substantially the stopping muon flux.

A three dimensional version of the Monte Carlo, allowing for multiple scattering and zig-zag motion through matter has been used to evaluate this effect (Bilokon et al., 1987). Negligible differences have been found up to about 10 hg/cm^2. For larger depths the results from the three dimensional transport indicate a muon stopping rate about 4% lower in the interval 40-100 hg/cm^2. Then this effect decreases with increasing depths, being 1% at 1000 hg/cm^2.

In Table II, the present calculations concerning the all-muon stopping rate are compared to the experimental results of Barton and Slade, 1965, Bhat and Ramana Murthy, 1973 and Bakatanov et al., 1979.

Data at the highest depths have been corrected for the locally produced muon flux following the prescriptions of the authors. Except for the point by Bakatanov et al., 1979, the agreement is excellent, particularly with the data from the Kolar Gold Field experiment, in which a narrow and vertically oriented telescope has been used.

This consistency confirms that the present one dimensional model for muon transport is well adequate to reproduce the muon intensities underground.

TABLE II - Comparison of the calculated muon stopping rate to the experimental results (all-muons/g s sr). Depths are measured from the sea level.

Ref.	Depth (hg/cm^2)	Experimental rates	Present calculations
a	60	$(1.5 \pm 0.2)\ 10^{-7}$	$1.8\ 10^{-7}$
b	86	$(7.7 \pm 0.4)\ 10^{-8}$	$7.5\ 10^{-8}$
b	407	$(7.9 \pm 0.5)\ 10^{-10}$	$8.2\ 10^{-10}$
c	850	$(4.7 \pm 0.2)\ 10^{-11}$	$7.0\ 10^{-11}$
b	975	$(4.4 \pm 0.4)\ 10^{-11}$	$4.8\ 10^{-11}$

Experimental values from :
a) J.Barton and M.Slade, 1965.
b) P.N.Bhat and P.V.Ramana Murthy, 1973.
c) V.N.Bakatanov et al., 1979.

CONCLUSIONS

A Monte Carlo technique has been used to simulate the propagation of muons in standard rock in the depth range 0.35-1000 hg/cm^2. In this way, the contribution to the energy loss from stochastic processes (bremmstrahlung and nuclear interaction) has been accounted for.

Up to about 600 hg/cm^2, the muon stopping rate can be calculated with good accuracy as the derivative of the integral flux at a muon momentum related to the depth h by means of the range-momentum relation. Interaction processes other than ionization can be neglected at depths less than 300 hg/cm^2.

The curves plotted in Fig. 6 give an accurate stimate of the stopping rate of negative vertical muons within the uncertainties on the sea level muon spectrum and charge ratio.

They can be used to calculate the negative muon stopping rate at both high latitude or equatorial underground sites once the pattern of the surrounding rock is known.

ACKNOWLEDGEMENTS

We are grateful to Prof. D. Lal for stimulating discussions.

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