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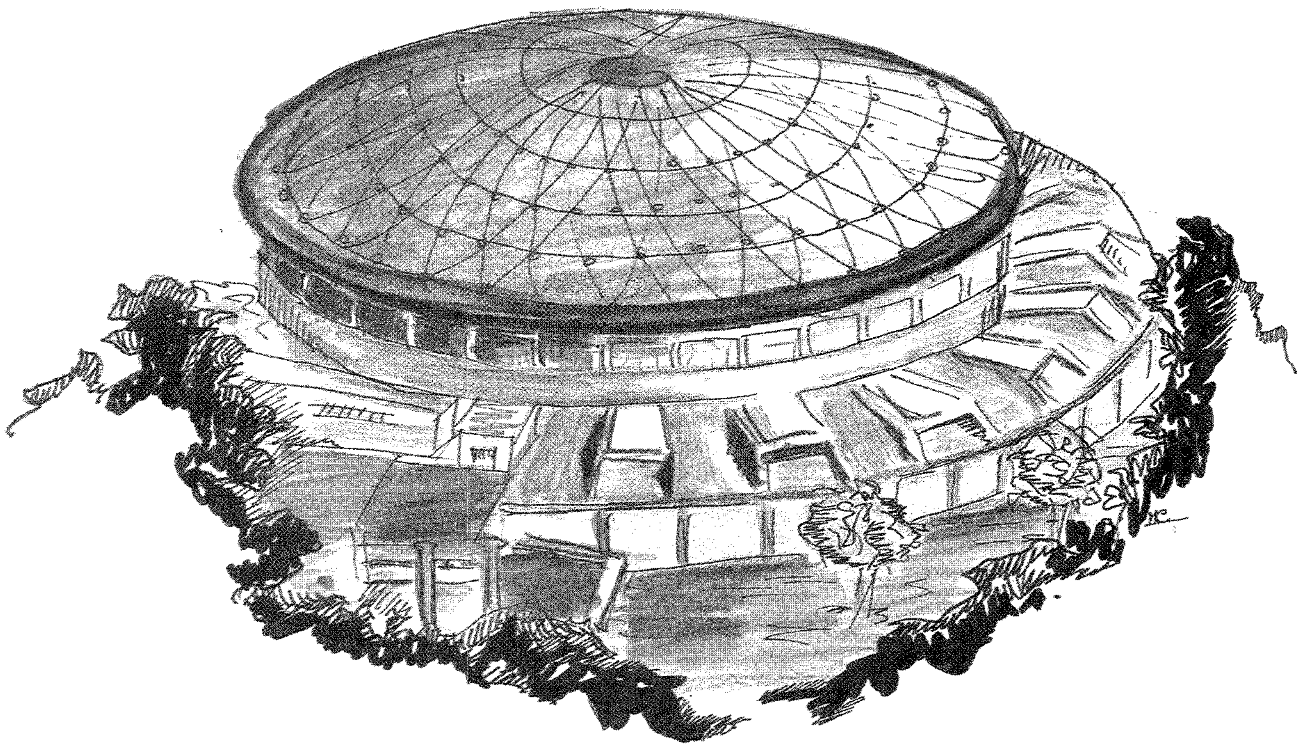

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COHERENT STATES AND STRUCTURE FUNCTIONS IN QED

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ABSTRACT

The methods of coherent states and of structure functions in QED are considered in detail for a precise evaluation of the radiative effects at LEP / SLC. They are explicitly shown to give identical results for the exponentiated infrared factors and the $o(\alpha)$ terms corresponding to the initial and final state radiation. Then the combined use of both techniques further improves the theoretical predictions to an accuracy better than 1% - including interference effects - and provides simple analytical formulae of immediate phenomenological application.

The role played by QED radiative corrections in e^+e^- reactions at LEP / SLC energies for the precise determination of electroweak parameters is well known[1][2]. The resummation of the leading double and single logarithms of soft origin[3], together with the exact determination of the left over $o(\alpha)$ terms[4][5], is essential to reach a degree of accuracy of $o(1\%)$ in the theoretical predictions. The calculation of all logarithmic $o(\alpha^2)$ corrections, as recently done[6][7] for the reaction $e^+e^- \to \mu^+\mu^-$, is the further necessary tool to get absolute control of the QED corrections, as required for precision tests of the theory.

To this aim the method of the structure functions[8], extended to both initial and final states [6] in the reaction $e^+e^- \to \mu^+\mu^-$, has been shown to be quite powerful, suggesting a systematic approach to other processes as, for example, Bhabha scattering. However, only numerical solutions have been obtained so far in the case of a resonant cross section, leaving unclear the connection to other analytical approaches to the problem. The method of coherent states[9], for example, has been proved to be very successful in describing the main features of multiphoton emission with good accuracy, giving explicit analytical results for resonant processes and interference effects with pure QED background [3,4]. The determination of the left over $o(\alpha^2)$ logarithms is needed only for the evaluation of the radiative effects beyond the 1% level.

A close connection between the two above methods of resummation has been explicitly shown[10] to exist in the non-singlet case for QED non resonant processes, as well as for QCD. The aim of the present paper is to further explore this relation for e^+e^- reactions when they proceed via Z exchange also. More in detail we will explicitly show that the method of structure functions applied to initial and final state radiation coincides with the coherent state approach with an accuracy of $o(1\%)$, giving explicitly in addition the desired $o(\alpha^2)$ left over corrections.

The basic formula which describes the reaction $e^+e^- \to \mu^+\mu^-$ in the approach of structure functions is the following [6]

$$\sigma(s) = \int_0^{\epsilon} dx \quad \sigma_0((1-x)s) H_e(x,s) F_\mu(\epsilon - x, (1-x)s) \tag{1}$$

where $\epsilon = \frac{\Delta E}{E}$ and the initial and final state radiation kernels are expressed in terms of the electron and muon structure functions as

$$H_e(x,s) = \int\limits_{1-x}^{1} \frac{dz}{z} D_e(z,s) D_e\left(\frac{1-x}{z}, s\right) \tag{2}$$

$$F_\mu(x,s) = \int\limits_{0}^{x} dy H_\mu(y,(1-y)s) \tag{3}$$

and $H_\mu(x,s)$ is defined as in eq. (2). By taking into account the effect of the soft radiation to all orders and of the hard one up to $o(\alpha^2)$ one obtains [6] ⋆

$$\begin{aligned} H_e(x,s) = \Delta_e(s)\beta_e x^{\beta_e - 1} - \frac{1}{2}\beta_e(2-x) \\ +\frac{1}{2}\beta_e^2 \left\{(2-x)\left[3\ln(1-x) - 4\ln x\right] - 4\frac{\ln(1-x)}{x} + x - 6\right\} \end{aligned} \tag{4}$$

with $\beta_e = \frac{2\alpha}{\pi}(L_e - 1), L_e = \ln\left(\frac{s}{m_e^2}\right)$ and

$$\begin{aligned} \Delta_e(s) = 1 + \frac{\alpha}{\pi}\left[\frac{3}{2}L_e + 2\left(\varsigma(2) - 1\right)\right] + \left(\frac{\alpha}{\pi}\right)^2 \left\{ \left[\frac{9}{8} - 2\varsigma(2)\right] L_e^2 \right. \\ + \left[3\varsigma(3) + \frac{11}{2}\varsigma(2) - \frac{45}{16}\right] L_e + \left[-\frac{6}{5}\varsigma^2(2) - \frac{9}{2}\varsigma(3) - 6\varsigma(2)\ln 2 \right. \\ \left.\left. + \frac{3}{8}\varsigma(2) + \frac{57}{12}\right]\right\} \equiv 1 + \frac{\alpha}{\pi}\Delta_e^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \Delta_e^{(2)} \end{aligned} \tag{5}$$

By insertion of (4) in eqs. (2) and (1) one easily obtains

$$\sigma(s) = \int\limits_0^{\epsilon} dx \quad \sigma_0(s(1-x)) \left\{\Delta_e(s)\Delta_\mu(s)\beta_e x^{\beta_e - 1}(\epsilon - x)^{\beta_\mu} + R(x, ...)\right\} \tag{6}$$

where the first term in the r.h.s. of eq. (6) is proportional to the leading soft contribution, while R(x,...) give further correction terms of order (β^2) and $\beta\epsilon$ in

⋆ Eq. (4) agrees with the complete second order result of [7] up to hard terms of $o(\alpha^2 L_e)$. In the resonant region this is not of numerical importance. The generalization of our results using the more complete but longer expression of [7] is straightforward.

the final cross section. We will assume the fractional energy resolution $\epsilon \equiv \frac{\Delta E}{E}$ of order $10^{-1} - 10^{-2}$.

By splitting the Born cross section $\sigma_0(s)$ as $\sigma_0 = \sigma_0^{QED} + \sigma_0^{INT} + \sigma_0^{RES}$, with†

$$\sigma_0^{QED}(s) = A\frac{1}{s}$$

$$\sigma_0^{INT}(s) = B Re\left\{\frac{1}{s - M^2 + i\Gamma M}\right\} \tag{7}$$

$$\sigma_0^{RES}(s) = C\frac{s}{(s - M^2)^2 + \Gamma^2 M^2}$$

the corresponding radiatively corrected cross sections are obtained as follows

$$\sigma^{QED}(s) = \frac{A}{s}\left\{\Delta_e(s)\Delta_\mu(s)\epsilon^{\beta_e+\beta_\mu} \cdot \frac{\Gamma(1+\beta_e)\Gamma(1+\beta_\mu)}{\Gamma(1+\beta_e+\beta_\mu)} \; {}_2F_1(1, \beta_e; 1+\beta_e+\beta_\mu; \epsilon) + \dots\right\}, \tag{8}$$

$$\sigma^{INT}(s) = B\left\{\Delta_e(s)\Delta_\mu(s)\epsilon^{\beta_e+\beta_\mu}\frac{\Gamma(1+\beta_e)\Gamma(1+\beta_\mu)}{\Gamma(1+\beta_e+\beta_\mu)} \cdot Re\left[\frac{1}{s - M_R^2}(1-z)^{-\beta_e} \; {}_2F_1\left(\beta_e+\beta_\mu, \beta_e; 1+\beta_e+\beta_\mu; \frac{z}{z-1}\right)\right] + \dots\right\}, \tag{9}$$

$$\sigma^{RES}(s) = -C\left\{\frac{s}{M\Gamma}\Delta_e(s)\Delta_\mu(s)\epsilon^{\beta_e+\beta_\mu}\frac{\Gamma(1+\beta_e)\Gamma(1+\beta_\mu)}{\Gamma(1+\beta_e+\beta_\mu)} \cdot Im\left[\frac{1}{s - M_R^2}(1-z)^{-\beta_e} \; {}_2F_1\left(\beta_e+\beta_\mu, \beta_e; 1+\beta_e+\beta_\mu; \frac{z}{z-1}\right)\right] + \dots\right\} \tag{10}$$

where $M_R^2 = M^2 - i\Gamma M, z = \frac{\epsilon s}{s - M_R^2}$ and the dots indicate next to leading terms corresponding to R(x,..) in eq. (6). By expanding in β_e the hypergeometric functions one then finds :

† The modifications of the Born cross sections (7) due to electroweak corrections, with the usual replacement in M and Γ, can be correspondingly introduced in our final expressions.

$$\sigma^{QED}(s) = \sigma_0^{QED}(s)\Delta_e(s)\Delta_\mu(s)\epsilon^{\beta_e+\beta_\mu} + \ldots \tag{11}$$

$$\begin{aligned}\sigma^{INT}(s) = \sigma_0^{INT}(s)\Delta_e(s)\Delta_\mu(s)\epsilon^{\beta_\mu}\frac{\Gamma(1+\beta_e)\Gamma(1+\beta_\mu)}{\Gamma(1+\beta_e+\beta_\mu)}\frac{1}{\cos\delta_R} \\ \cdot Re\left\{e^{i\delta_R}\left[\frac{\epsilon}{1+(\frac{\epsilon s}{M\Gamma})\sin\delta_R e^{i\delta_R}}\right]^{\beta_e}\right\} + \ldots\end{aligned} \tag{12}$$

$$\begin{aligned}\sigma^{RES}(s) = \sigma_0^{RES}(s)\Delta_e(s)\Delta_\mu(s)\epsilon^{\beta_\mu}\frac{\Gamma(1+\beta_e)\Gamma(1+\beta_\mu)}{\Gamma(1+\beta_e+\beta_\mu)} \\ \cdot\left|\frac{\epsilon}{1+(\frac{\epsilon s}{M\Gamma})\sin\delta_R e^{i\delta_R}}\right|^{\beta_e}\left(\cos\beta_e\phi - \cot\delta_R\sin\beta_e\phi\right) + \ldots\end{aligned} \tag{13}$$

where $(M_R^2 - s)^{-1} = \frac{\sin\delta_R e^{i\delta_R}}{M\Gamma}$, $\tan\delta_R = \frac{M\Gamma}{(M^2-s)}$ and $\phi = \arctan\left[\frac{\epsilon s+M^2-s}{M\Gamma}\right] - \arctan\left[\frac{M^2-s}{M\Gamma}\right]$.

We now wish to comment briefly on the various factors appearing in the r.h.s. of eqs. (11-13) and compare them with the analogous expressions obtained in the framework of coherent states, where, to leading order‡ , the differential cross section is given by [2,4]

$$\frac{d\sigma}{d\Omega} = \sum_i \frac{d\sigma_0^{(i)}}{d\Omega}\left(C_{infra}^{(i)} + C_F^{(i)}\right), \qquad (i = \text{QED,RES,INT}) \tag{14}$$

with

$$C_{infra}^{(QED)} = \epsilon^{\beta_e+\beta_\mu+2\beta_{int}} \tag{15}$$

$$C_{infra}^{(INT)} = \frac{\epsilon^{\beta_\mu+\beta_{int}}}{\cos\delta_R}Re\left\{e^{i\delta_R}\left[\frac{\epsilon}{1+(\frac{\epsilon s}{M\Gamma})\sin\delta_R e^{i\delta_R}}\right]^{\beta_e}\left[\frac{\epsilon}{\epsilon+\left(\frac{M\Gamma}{s}\right)\frac{e^{-i\delta_R}}{\sin\delta_R}}\right]^{\beta_{int}}\right\} \tag{16}$$

$$\begin{aligned}C_{infra}^{(RES)} = \epsilon^{\beta_\mu}\left|\frac{\epsilon}{1+(\frac{\epsilon s}{M\Gamma})\sin\delta_R e^{i\delta_R}}\right|^{\beta_e} \\ \cdot\left|\frac{\epsilon}{\epsilon+\left(\frac{M\Gamma}{s}\right)\frac{e^{-i\delta_R}}{\sin\delta_R}}\right|^{2\beta_{int}}\left(1+\beta_e\frac{s-M^2}{M\Gamma}\phi\right)\end{aligned} \tag{17}$$

‡ This corresponds to keep $C_F^{(i)}$ to $o(\alpha)$ only.

and $\beta_{int} = \frac{4\alpha}{\pi} \ln \tan \frac{\theta}{2}$.

Furthermore the finite terms $C_F^{(i)}$ can be written as

$$C_F^{(i)} = \frac{\alpha}{\pi} \left[\frac{3}{2} (L_e + L_\mu) + 4 (\varsigma(2) - 1) \right] + C'^{(i)}_F, \tag{18}$$

with $C'^{(i)}_F$ containing other $o(\alpha)$ finite terms, coming from bremsstrahlung and box diagrams, odd in the exchange $\theta \leftrightarrow \pi - \theta$, etc.

Then, after putting $\beta_{int} = 0$ in eqs.(15-17), the exponentiated factors coincide with those in eqs.(11-13). Furthermore the finite terms $C_F^{(i)}$ contain the $o(\beta_e, \beta_\mu)$ expansion in $\Delta_e(s)\Delta_\mu(s)$ in eqs.(11-13). The remaining non factorizable real and virtual terms can also be included in the approach of structure functions [6]. On the other hand the extra terms contained in eqs.(11-13) are of $o(\beta^2)$- in particular the factor $-\beta_e^2 \frac{\phi^2}{2}$ in eq(13) arising from the expansion of $\cos \phi\beta_e$ - and $o(\epsilon\beta)$, not written explicitly.

From the above discussion it follows that the two formalisms give identical answers for both the exponentiated and $o(\beta)$ finite terms relative to initial and final state radiation. On the other hand they add complementary informations for the β_{int} dependence and $o(\beta^2, \epsilon\beta)$ terms, respectively.

Then, from the combined informations of eqs. (11-13) and (15-17) one can transform eq. (14) into the following form

$$\begin{aligned} \frac{d\sigma}{d\Omega} = \frac{d\sigma_0^{(QED)}}{d\Omega} & \left[C_{infra}^{(QED)} (1 + \overline{C}_F^{(QED)}) + C'^{(QED)}_F \right] \\ + \frac{d\sigma_0^{(INT)}}{d\Omega} & \left[C_{infra}^{(INT)} (1 + \overline{C}_F^{(INT)}) + C'^{(INT)}_F \right] \\ + \frac{d\sigma_0^{(RES)}}{d\Omega} & \left[C_{infra}^{(RES)} \frac{\cos(\beta_e\phi) - \cot\delta_R \sin(\beta_e\phi)}{1 + \beta_e \frac{s-M^2}{M\Gamma}\phi} (1 + \overline{C}_F^{(RES)}) + C'^{(RES)}_F \right] \end{aligned} \tag{19}$$

with the definitions (15-17) of $C_{infra}^{(i)}$ and

$$\begin{aligned} \overline{C}_F^{(QED)} = & \left(\frac{\alpha}{\pi}\right) \left[\Delta_e^{(1)} + \Delta_\mu^{(1)} \right] - \beta_\mu \epsilon \\ + \left(\frac{\alpha}{\pi}\right)^2 & \left[\Delta_e^{(2)} + \Delta_\mu^{(2)} + \Delta_e^{(1)}\Delta_\mu^{(1)} \right] - \frac{\pi^2}{6}\beta_e\beta_\mu - \frac{1}{4}\beta_e^2 \epsilon^{1-\beta_e} \end{aligned} \tag{20}$$

$$\overline{C}_F^{(INT)} = \left(\frac{\alpha}{\pi}\right)\left[\Delta_e^{(1)} + \Delta_\mu^{(1)}\right] - \beta_\mu\epsilon + \left(\frac{\alpha}{\pi}\right)^2\left[\Delta_e^{(2)} + \Delta_\mu^{(2)} + \Delta_e^{(1)}\Delta_\mu^{(1)}\right] - \frac{\pi^2}{6}\beta_e\beta_\mu$$
$$-\beta_e\frac{\cos\phi(\beta_e+1) + \tan\delta_R\sin\phi(\beta_e+1)}{\cos\phi\beta_e + \tan\delta_R\sin\phi\beta_e}\left|\frac{\epsilon}{1+\frac{\epsilon s}{M_R^2-s}}\right|$$
$$-\frac{1}{4}\beta_e^2\frac{\cos\phi + \tan\delta_R\sin\phi}{\cos\phi\beta_e + \tan\delta_R\sin\phi\beta_e}\left|\frac{\epsilon}{1+\frac{\epsilon s}{M_R^2-s}}\right|^{1-\beta_e} \tag{21}$$

$$\overline{C}_F^{(RES)} = \left(\frac{\alpha}{\pi}\right)\left[\Delta_e^{(1)} + \Delta_\mu^{(1)}\right] - \beta_\mu\epsilon + \left(\frac{\alpha}{\pi}\right)^2\left[\Delta_e^{(2)} + \Delta_\mu^{(2)} + \Delta_e^{(1)}\Delta_\mu^{(1)}\right] - \frac{\pi^2}{6}\beta_e\beta_\mu$$
$$-2\beta_e\frac{\cos\phi(\beta_e+1) - \cot\delta_R\sin\phi(\beta_e+1)}{\cos\phi\beta_e - \cot\delta_R\sin\phi\beta_e}\left|\frac{\epsilon}{1+\frac{\epsilon s}{M_R^2-s}}\right|$$
$$-\frac{1}{4}\beta_e^2\frac{\cos\phi - \cot\delta_R\sin\phi}{\cos\phi\beta_e - \cot\delta_R\sin\phi\beta_e}\left|\frac{\epsilon}{1+\frac{\epsilon s}{M_R^2-s}}\right|^{1-\beta_e} \tag{22}$$

The above equations represent our final result, which describes the radiative correction factors to an accuracy better than (1%). The effect of the new terms of $o(\beta^2, \epsilon\beta)$ in eqs.(18) is shown in figs. (1,2), where we plot the ratios $R^{(i)} = \frac{d\sigma^{(i)}[1+o(\alpha)+o(\beta\epsilon)+o(\alpha^2)]}{d\sigma^{(i)}[1+o(\alpha)]}$, for i=QED, INT, RES, for $\epsilon = 0.01$ and $\epsilon = 0.05$. Notice that the factors $C_{infra}^{(i)}$ of eqs. (15,17) do not appear in the ratios $R^{(i)}$. We have taken the scattering angle $\theta = \frac{\pi}{2}$ in the factors $C'^{(i)}_F$ to authomatically cancel the box diagram and other non factorizable contributions.

As is clear from figs.(1,2) the QED and INT corrections are practically constant in the resonant region to a value of about -0.01, while the RES correction is modulated essentially by the term $1 - \frac{\beta_e^2\phi^2}{2}$, with an extra factor of o(0.01 - 0.02).

The extension of our results to the case of the Z line shape is straightforward. It simply corresponds to take the limit $\beta_\mu = 0, \Delta_\mu = 1$ and $\epsilon = 1 - \frac{4\mu^2}{s}$ in eq. (10). Then one simply obtains for $s \simeq M_R^2$

$$\sigma(s) \simeq \sigma_0(s)\left|\frac{M_R^2 - s}{M_R^2}\right|^{\beta_e}\frac{\sin(1-\beta_e)\delta_R}{\sin\delta_R}\frac{\pi\beta_e}{\sin\pi\beta_e}\Delta_e(s) \tag{23}$$

which agrees with refs. [3] [4] [11] up to constant factors of $o(\beta^2)$.

To conclude, we have explicitly shown that the approach of the coherent states and that of the structure functions offer two complementary methods in QED to

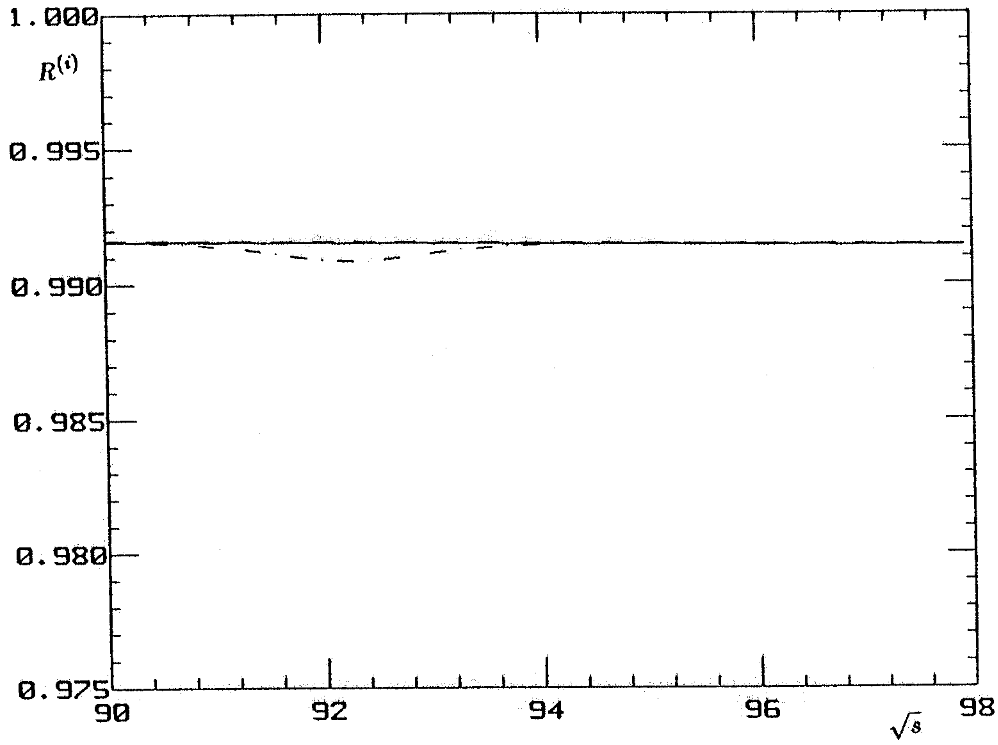

Fig. 1 Ratio $R^{(i)} = \frac{d\sigma^{(i)}[1+o(\alpha)+o(\beta\epsilon)+o(\alpha^2)]}{d\sigma^{(i)}[1+o(\alpha)]}$ for $e^+e^- \rightarrow \mu^+\mu^-$, with i=QED (solid line), INT (dashed line), RES (dotted-dashed line) for $\epsilon = 0.01$. The values of M and Γ are taken to be 92 Gev and 2.6 Gev respectively.

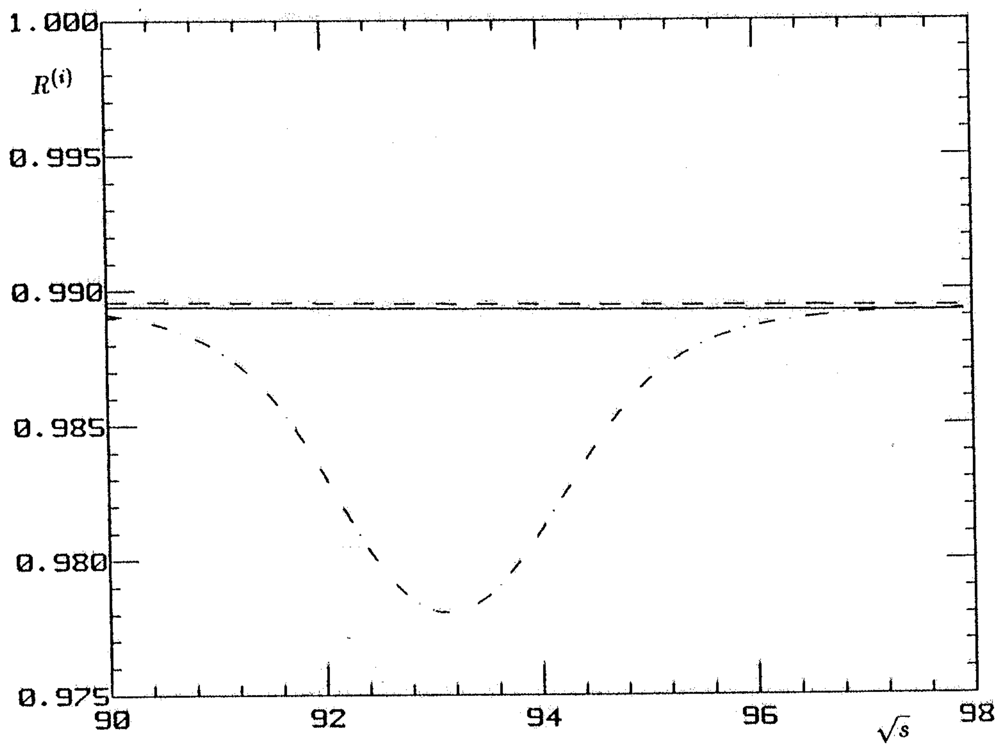

Fig. 2 Same as in fig. (1) for $\epsilon = 0.05$.

improve the theoretical accuracy required for precision measurements at LEP / SLC energies. They give identical results for the exponentiated and finite $o(\alpha)$ factors relative to the initial and final states radiation. Moreover the remaining logarithmic $o(\alpha^2)$ corrections, obtained with the method of structure functions complete the information on interference effects provided by the approach of coherent states. The overall picture provides simple analytical formulae which can be easily extended to e^+e^- reactions other than $e^+e^- \to \mu^+\mu^-$.

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