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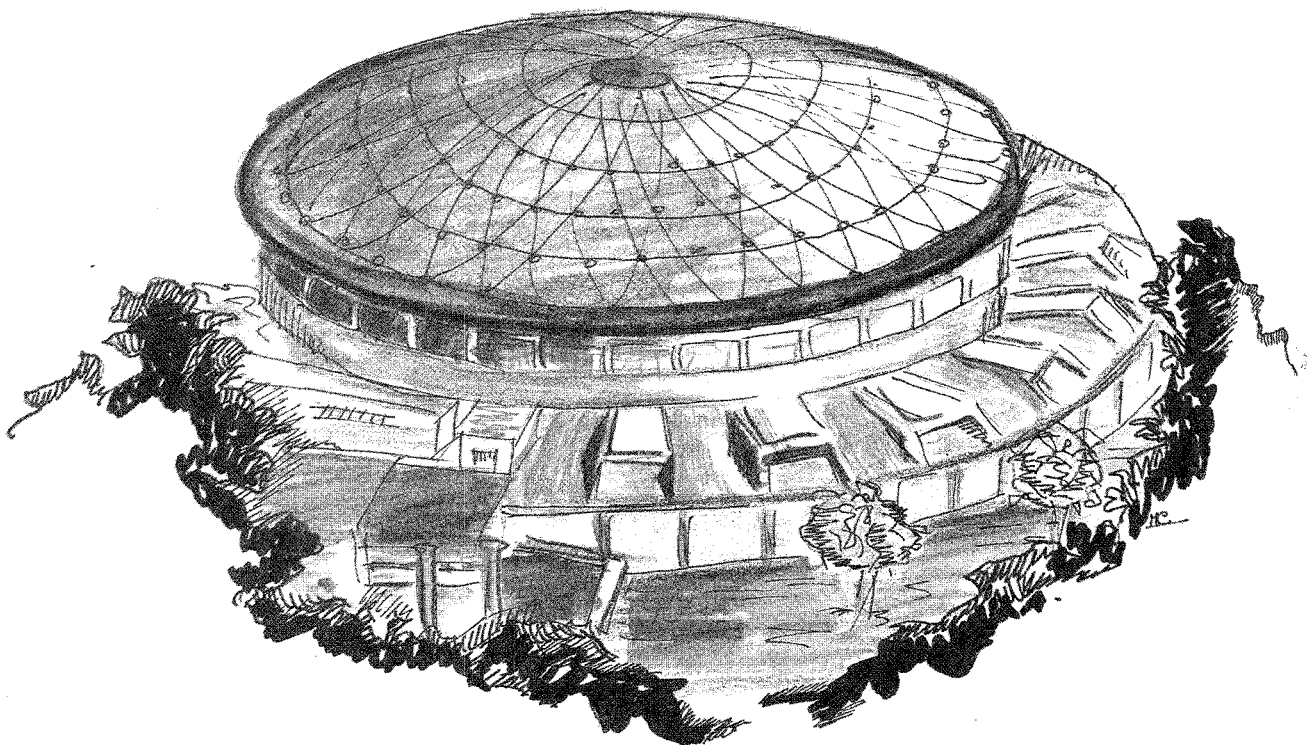
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**NUCLEON FORM FACTORS IN A VMD MODEL WITH BOTH Q^2 - AND - S
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ABSTRACT

VMD models fail in their descriptions of nucleon and nuclear form factors, not because they are wrong but because they are incomplete. The version of quark-hadron duality which they express, the so-called Q^2 - duality, refers only to the spectrum of vector mesons which couple to the photon. This version is incomplete. Quark-hadron duality includes also the s-channel or Bloom-Gilman duality which allows to reproduce asymptotic quark model behaviour e.g. in structure functions, by summing over the contributions of hadronic, not only vector meson, intermediate states. A VMD model which incorporates these two forms of duality is presented. It gives excellent fits to nucleon form factors in the space - like region. When the corresponding formulae are continued to the time - like region they yield absolute predictions. Those for the proton agree remarkably well with the available data. The proton and neutron form factors in this region turn out to be approximately of the same order for all values of the momentum transfer. We therefore predict approximately equal values for the cross-sections $\sigma(e^+ e^- \rightarrow p \bar{p})$ and $\sigma(e^+ e^- \rightarrow n \bar{n})$ for all values of the $e^+ e^-$ centre of mass energy.

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Can vector meson dominance (VDM) models⁽¹⁾ describe nucleon and nuclear form factors? The obvious answer is, no. In VMD models, all form factors fall-off like $1/q^2$ for large momentum transfers, $|q^2| \rightarrow \infty$, whereas experimentally⁽²⁾, nucleon form factors fall-off like $1/q^4$ and nuclear form factors even much faster. Quark parton model counting rules⁽³⁾ support the latter behaviour. QCD improves on it⁽⁴⁾. There exist generalised vector meson dominance models (GVMD)⁽⁵⁾ which reproduce leading QCD behaviour e.g. in structure functions⁽⁵⁾, in the ratio

$$R(s) := \sigma_{e^+e^- \rightarrow \text{hadrons}}(s) / \sigma_{e^+e^- \rightarrow \mu^+\mu^-}$$

and in photon-photon scattering, $\gamma\gamma \rightarrow \text{hadrons}$.

They fail however when applied to nucleon and nuclear form factors and in a way which undermines their consistency. The inconsistency arises from the fact that GVMD models would predict the correct asymptotic behaviour of these form factors if the generalised zeta function, which arises in these models, had a finite number of consecutive zeroes. The latter function has no such set of zeroes. One therefore concludes that, in their usual formulation VMD models (generalised or not) cannot describe consistently form factors of nucleons and nuclei.

Why do these models fail? It is proposed to answer this question in this paper. Based on this answer, we present a modified VMD model which incorporates more fully the idea of quark-hadron duality. It is this idea which underlies this class of models. In the usual formulation of these models, this idea is expressed only in part. It refers, on the one hand, to the hadronic constitution of the photon in terms of its couplings to vector mesons and, on the other, to the quark constitution of these mesons. This is the idea of Q^2 -duality. It describes, essentially, the structure of the hadronic electromagnetic current in terms of the couplings of the photon to vector mesons under the assumption that the current is proportional to a linear superposition of the fields of these vector mesons. This is an incomplete description of quark-hadron duality. A complete description must include the so-called s-channel or Bloom-Gilman duality⁽⁷⁾, which plays a very important role in relating asymptotic quark dynamics in structure functions to the behaviour of hadronic form factors^(7,8,9). S-channel duality allows to reconstruct asymptotic quark model behaviour from the contributions of hadronic intermediate states⁽¹⁰⁾, not only from the contributions of intermediate vector meson states. It is thus intimately connected with unitarity and analyticity. It describes the structure of a hadron, and describes it as a dual composite structure: firstly as a composite of other hadrons and secondly as a composite of quarks, its more elementary constituents. When the hadron in question is a vector meson, this dual composite structure is the same which is used as input in the Q^2 -duality description. When the photon, or Q^2 -leg, coincides with the s-channel, as in e^+e^- annihilation into hadrons, these two forms of duality do not coincide, except in the case of determining the structure of meson states. As pointed out above they refer to different aspects of the interaction. Combining them, therefore, does not lead to double counting. They are also not in conflict. Q^2 -

⁽⁵⁾ with an infinite number of vector mesons.

duality alone, even if extended to include an infinite number of vector meson states (GVMD), leads to inconsistencies in the description of form factors other than those of mesons. These inconsistencies are resolved if quark - hadron duality is understood to include also s - channel duality. We now proceed to do this. Our starting point⁽¹⁰⁾ is Fig. (1) which represents virtual Compton scattering off a hadron. VMD models picture this reaction exclusively from the point of view of the couplings along the photon or Q^2 - leg: the photon couples to the hadron h through the mediation of vector mesons (see Fig.(2)).

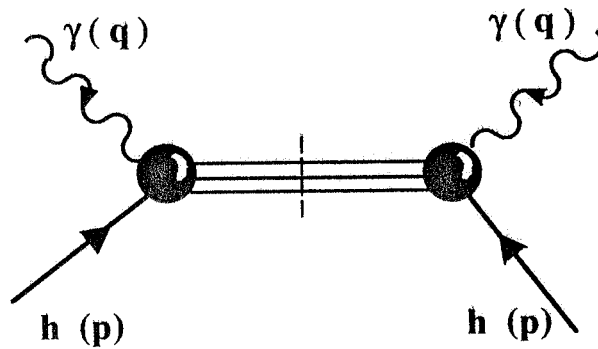


FIG. 1 - Virtual compton scattering of a photon γ against a hadron h .

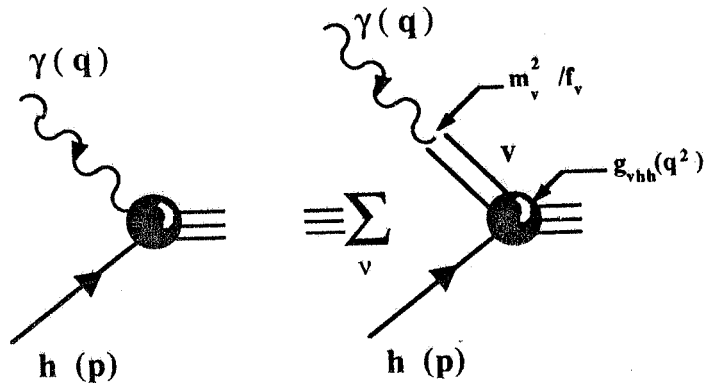


FIG. 2 - The VMD model for the photon coupling to the hadron h through vector meson v .

The coupling of the photon to the vector meson v of mass m_v is m_v^2/f_v . Considering, for simplicity, a scalar hadron, Figs. (1) and (2) lead to the following expression for the structure function of h :

$$F(q^2, \nu) \sim \sum_v \left(\frac{m_v^2}{f_v} \right) \frac{\sigma_{vh}(\nu)}{(m_v^2 + Q^2)^2} \quad (1)$$

where $Q^2 = -q^2$, $(m_v^2 - q^2)^{-1}$ the propagator of the vector meson v and $\sigma_{vh}(\nu)$ the total cross-section of $v - h$ scattering. The variable $\nu := p \cdot q/M$, with M the mass of h , is the photon

energy in the h rest frame. In the elastic limit, $2Mv \rightarrow Q^2$. In this limit, the structure function is related to the form factor $F(q^2)$ by^(7,8,9)

$$F(q^2, v) \xrightarrow{2Mv \rightarrow Q^2} \delta(2Mv - Q^2) |F(q^2)|^2 \quad (2)$$

Eqs (1) and (2) are consistent with the following Ansatz for $F(q^2)$, i.e.

$$F(q^2) = \sum_v \frac{m_v^2}{f_v} \frac{g_{vhh}(q^2)}{m_v^2 - q^2} \quad (3)$$

Eq. (3) is what Fig. (2) represents where $g_{vhh}(q^2)$ is the coupling constant at the three-particle vertex vhh . It is assumed in VMD models that $g_{vhh}(q^2)$ is independent of q^2 or, at least, for moderate q^2 that one may adopt the smooth extrapolation assumption $g_{vhh}(|q^2| = m_v^2) = g_{vhh}(q^2 = 0)$ so that, $g_{vhh}(q^2) = \text{const}$ for $0 \leq |q^2| \leq m_v^2$. The important point to note is that the q^2 -dependence of $g_{vhh}(q^2)$ cannot be determined from within these models. Other inputs are necessary. This means, quite simply, that Q^2 -duality alone cannot describe and relate properties of hadrons (e.g. form factors) to asymptotic quark dynamics except in very special cases (e.g. when $g_{vhh}(q^2)$ is a const, as happens for mesons $h = \pi^\pm, K^\pm$ etc). More to the point, this means that Q^2 -duality is an incomplete description of quark-hadron duality.

Actually the large q^2 behaviour of $g_{vhh}(q^2)$ is easily determined. The interpolation between this behaviour and the regular behaviour of $g_{vhh}(q^2)$ at $q^2 = 0$ (i.e. $g_{vhh}(q^2 = 0) < \infty$) is then carried out in the most straight forward manner. The additional input which makes this possible is the so called Bloom-Gilman or s -channel duality. In this duality scheme one looks at Fig. (1) in terms of the s -channel intermediate states h_n ($n = 1, 2, \dots$) which build up the structure function. In formulae

$$F(q^2, v) \sim \sum_{n=0}^{\infty} |F_n(q^2)|^2 \rho_n(s) \quad (4)$$

where $F_n(q^2)$ is the $h \rightarrow h_n$ transition form factor and $s = (p+q)^2 = 2Mv - Q^2 + M^2$. The spectral function $\rho_n(s)$ is the density of h_n states at invariant mass $s = M_n^2$. S -channel duality relates the asymptotic behaviour of $\rho_n(s)$ for large $s \rightarrow \infty$ to the asymptotic behaviour of $F(q^2, v)$ for $Q^2 \rightarrow \infty$ in the elastic limit $2Mv \rightarrow Q^2$ ^(7,8,9). The latter is usually expressed in this limit, in terms of the Bjorken variable⁽¹¹⁾, $\omega := 2Mv/Q^2$, as

$$F(Q^2, \omega) \xrightarrow{Q^2 \rightarrow \infty, \omega \rightarrow 1} (\omega - 1)^{2(N_q - 1)} \quad (5)$$

where N_q is the number of valence quarks in h . The power law in eq. (5) is determined by quark counting rule⁽³⁾. Comparing eqs. (4) and (5) and recalling that $s \sim Q^2(\omega - 1)$, we have the asymptotic relations⁽⁹⁾

$$\rho_n(s) \xrightarrow{s \rightarrow \infty} s^{2(N_q - 1)} \quad (6.1)$$

$$F_n(q^2) \xrightarrow{|q^2| \rightarrow \infty} \left(\frac{1}{q^2}\right)^{N_q - 1} \quad (6.2)$$

Eqs. (6.1) and (6.2) are duality relationships: on the left hand sides we have hadronic properties which are approximated by quark model expressions on the right hand sides. An important observation is that eqs. (6.1) and (6.2) can be used in any reaction in which $\rho_n(s)$ and $F_n(q^2)$ occur, not only in the reaction represented in Fig. (1). For instance, in $e^+ e^-$ annihilation into hadrons, the s - and Q^2 - channels coincide kinematically i.e. $s = Q^2$. Eqs. (6.1) and 6.2) then give rise to the constancy of the R-ratio

$$R(s) := \sigma_{e^+ e^- \rightarrow \text{hadrons}}(s) / \sigma_{e^+ e^- \rightarrow \mu^+ \mu^-}.$$

However note that, as observed previously, even in this case s - channel and Q^2 - dualities are not the same. As just pointed out above eq. (6.2) may be inserted in eq. (3) in the limit $|q^2| \rightarrow \infty$. This then allows to determine the asymptotic q^2 - dependence of the coupling constant $g_{vhh}(q^2)$. The result is

$$g_{vhh}(q^2) \xrightarrow{|q^2| \rightarrow \infty} \left(\frac{1}{q^2}\right)^{N_q - 2} \quad (7)$$

We now interpolate $g_{vhh}(q^2)$ between the behaviour in eq. (7) and its regular behaviour $g_{vhh}(q^2 = 0) < \infty$ at $q^2 = 0$ by means of the simple formula

$$g_{vhh}(q^2) = g_{vhh}^{(VMD)} \left(\frac{t_0}{|q^2| + t_1} \right)^{N_q - 2} \quad (8)$$

where $g_{vhh}^{(VMD)}$ is the VMD model coupling constant. t_0 and t_1 are two free mass parameters to be fitted to experiment. Note that

$$g_{vhh}(q^2 = 0) = g_{vhh}^{(VMD)} \left(\frac{t_0}{t_1} \right)^{N_q - 2} \quad (9)$$

If $t_0 \sim t_1$, eq.(9) satisfies the VMD smoothness assumption

$$g_{vhh}(q^2) \sim g_{vhh}^{(VMD)} := g_{vhh}(|q^2| = m_v^2)$$

Eq. (9) violates this assumption otherwise, unless $N_q = 2$, that is for mesons.

Inserting eq. (8) in (3) one gets

$$F(q^2) = \left(\frac{t_0}{|q^2| + t_1} \right)^{N_q - 2} F^{(VMD)}(q^2) \quad (10.1)$$

$$F^{(VMD)}(q^2) := \sum_v \frac{m_v^2}{f_v} \frac{g_{vhh}^{(VMD)}}{m_v^2 - q^2} \quad (10.2)$$

According to eq. (10.1), s - channel duality leads to a correction of the VMD model formula.

Note that the modified formula is not an analytic function of q^2 . The modulus, $|q^2|$, of q^2 appears in the interpolating formulae (8) and (10.1) so as to avoid a spurious pole at $q^2 = \pm t_1$ if

we used the expression $(q^2 \pm t_1)^{-(N_q - 2)}$.

For mesons (e.g. π^\pm , K^\pm , K^0 , \bar{K}^0 , ...), $N_q = 2$. For them $F(q^2) = F^{(VMD)}(q^2)$, in agreement with the fact that VMD models give a good description of meson form factors⁽¹²⁾.

These form factors are therefore not discussed in this paper. We have fitted eq. (10) to the

proton (p) and neutron (n) charge (E) and magnetic (M) form factors, $G_{E,M}^{p,n}(q^2)$ ⁽¹³⁾ in the space-like region ($q^2 = -Q^2 < 0$) to determine all the parameters involved. The resulting formula, with all the parameters fixed, is then extrapolated to the time - like region

($q^2 = Q^2 > 0$) where it fits the available data on $G_{E,M}^p(q^2)$ ⁽¹⁴⁾ and gives absolute predictions

for $G_{E,M}^n(q^2)$. The fits to the space - like form factors $G_{E,M}^{p,n}(q^2 < 0)$ are in Figs. (3a) - (3d).

The predictions for the time-like form factors $G_{E,M}^{p,n}(q^2 > 0)$ are in Figs. (4a)-(4d). Comparing

$G_E^{p,n}(q^2)$ with $G_M^{p,n}(q^2)$ in the time - like region one finds that the scaling assumption

$G_E^{p,n}(q^2) = G_M^{p,n}(q^2)$, used in the experimental fits⁽¹⁴⁾ is, approximately valid in the

threshold region, $q^2 \sim 4M^2$ where data presently exist for $G_{EM}^p(q^2)$. One can make a different

kind of comparison, namely $G_E^n(q^2)$ with $G_E^p(q^2)$ and $G_M^n(q^2)$ with $G_M^p(q^2)$. We have, to

a very good approximation and for a wide range of q^2 ,

$$\left| G_E^n(q^2) / G_E^p(q^2) \right| \sim 1 \quad \text{and} \quad \left| G_M^n(q^2) / G_M^p(q^2) \right| \sim 1 .$$

$$\left| G_E^n(q^2) / G_E^p(q^2) \right| \sim 1 \text{ and } \left| G_M^n(q^2) / G_M^p(q^2) \right| \sim 1 .$$

This comparison is important mainly in respect to a recent prediction⁽¹⁵⁾ which gives rather large ratios

$$\left| G_{E,M}^n(q^2) / G_{E,M}^p(q^2) \right| \sim 5 ,$$

thus promising very large cross sections for the colliding beam process $e^+ e^- \rightarrow n \bar{n}$ relative to $e^+ e^- \rightarrow p \bar{p}$. Experiments will soon be able to test these predictions⁽¹⁶⁾.

Finally let us explain briefly how eq. (10) was used in the fits. First, one expresses $G_{E,M}^{p,n}(q^2)$ in terms of the Dirac (F_1) and Pauli (F_2) form factors $F_{1,2}^{p,n}(q^2)$ i. e.

$$G_E^{p,n}(q^2) = F_1^{p,n}(q^2) + \frac{q^2}{4M^2} F_2^{p,n}(q^2) \quad (11.1)$$

$$G_M^{p,n}(q^2) = F_1^{p,n}(q^2) + F_2^{p,n}(q^2) \quad (11.2)$$

Next, one decomposes $F_{1,2}^{p,n}(q^2)$ into their isoscalar (s) and isovector (v) parts i. e.

$$F_{1,2}^p(q^2) = F_{1,2}^{(s)}(q^2) + F_{1,2}^{(v)}(q^2) \quad (12.1)$$

$$F_{1,2}^n(q^2) = F_{1,2}^{(s)}(q^2) - F_{1,2}^{(v)}(q^2) \quad (12.2)$$

We then apply eq. (10) only to $F_1^{(v)}(q^2)$ and $\frac{q^2}{4M^2} F_2^{(s,v)}(q^2)$ to get ($h \equiv \text{Nucleon, N}$)

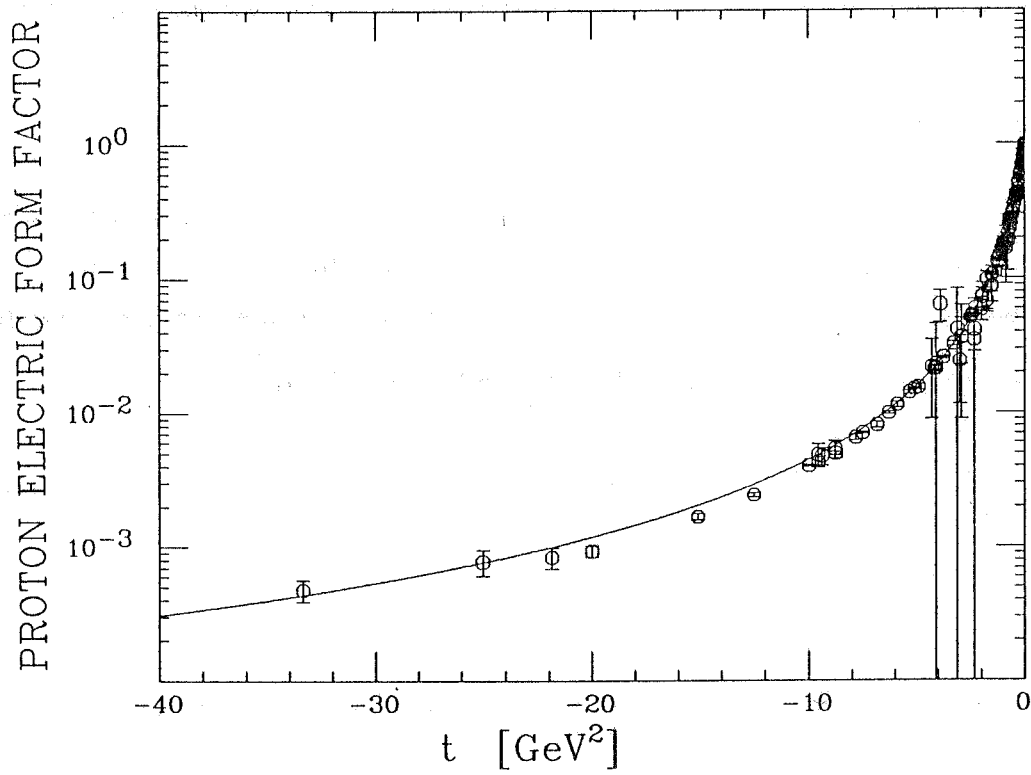
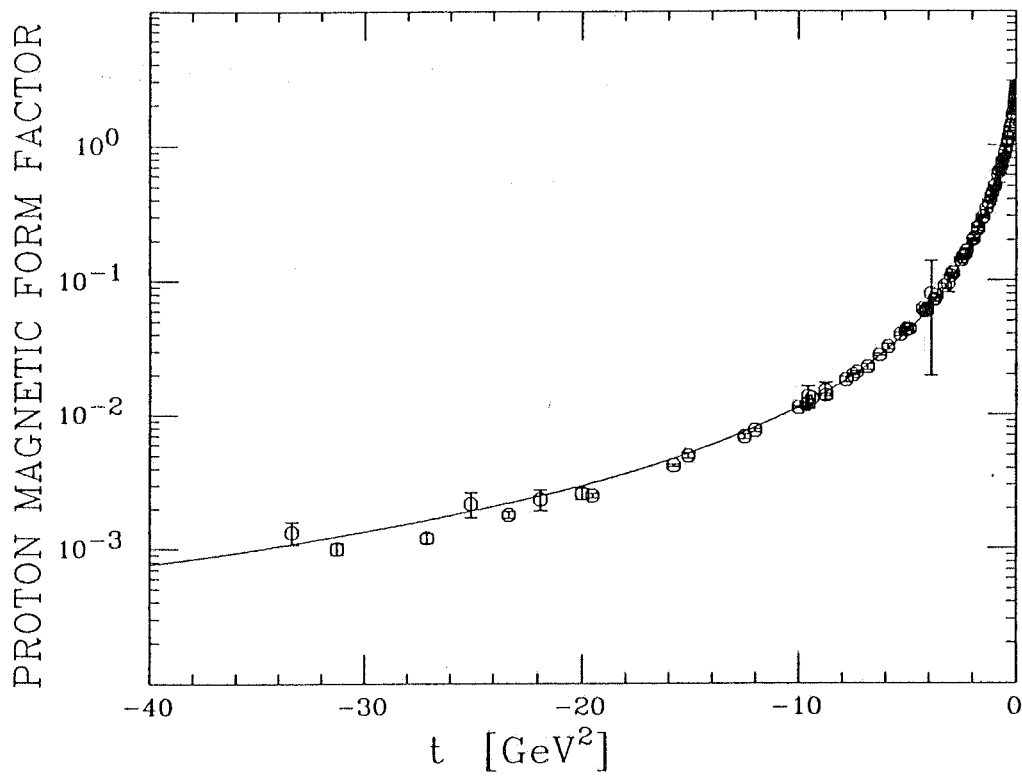
$$F_1^{(s)}(q^2) = \left(\frac{t_{01}}{|q^2| + t_{11}} \right)^{N_q^{-2}} \sum_{V \equiv \omega, \phi, \phi'} \frac{g_{vNN}^{(1)}}{f_v} \frac{m_v^2}{\left(m_v - \frac{i\Gamma_v}{2} \right)^2 - q^2} \quad (13.1)$$

$$F_1^{(v)}(q^2) = \left(\frac{t_{01}}{|q^2| + t_{11}} \right)^{N_q^{-2}} \sum_{V \equiv \rho, \rho'} \frac{g_{vNN}^{(1)}}{f_v} \frac{m_v^2}{\left(m_v - \frac{i\Gamma_v}{2} \right)^2 - q^2} \quad (13.2)$$

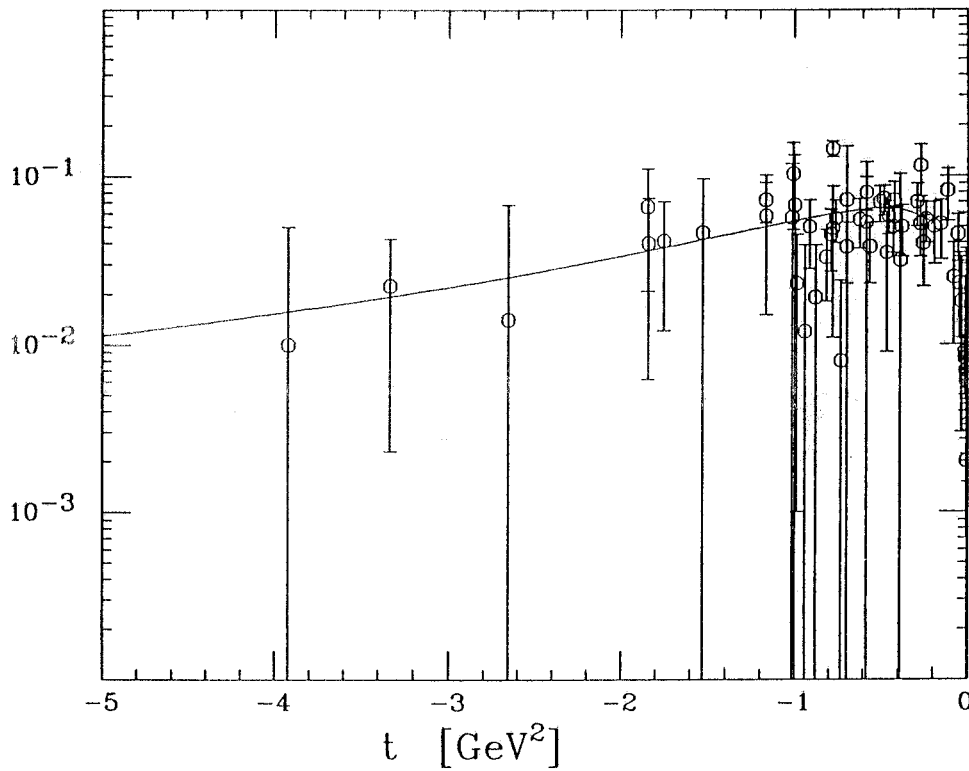
$$F_2^{(s)}(q^2) = \left(\frac{t_{02}}{|q^2| + t_{12}} \right)^{N_q^{-1}} \sum_{V \equiv \omega, \phi, \phi'} \frac{g_{vNN}^{(2)}}{f_v} \frac{m_v^2}{\left(m_v - \frac{i\Gamma_v}{2} \right)^2 - q^2} \quad (13.3)$$

$$F_2^{(v)}(q^2) = \left(\frac{t_{02}}{|q^2| + t_{12}} \right)^{N_q^{-2}} \sum_{V \equiv \rho, \rho'} \frac{g_{vNN}^{(2)}}{f_v} \frac{m_v^2}{\left(m_v - \frac{i\Gamma_v}{2} \right)^2 - q^2} \quad (13.4)$$

FIG. 3 - Fits to the nucleon space - like form factors:

a) $G_E^p(q^2)$;b) $G_M^p(q^2)$,

NEUTRON ELECTRIC FORM FACTOR

c) $G_E^n(q^2)$;

NEUTRON MAGNETIC FORM FACTOR

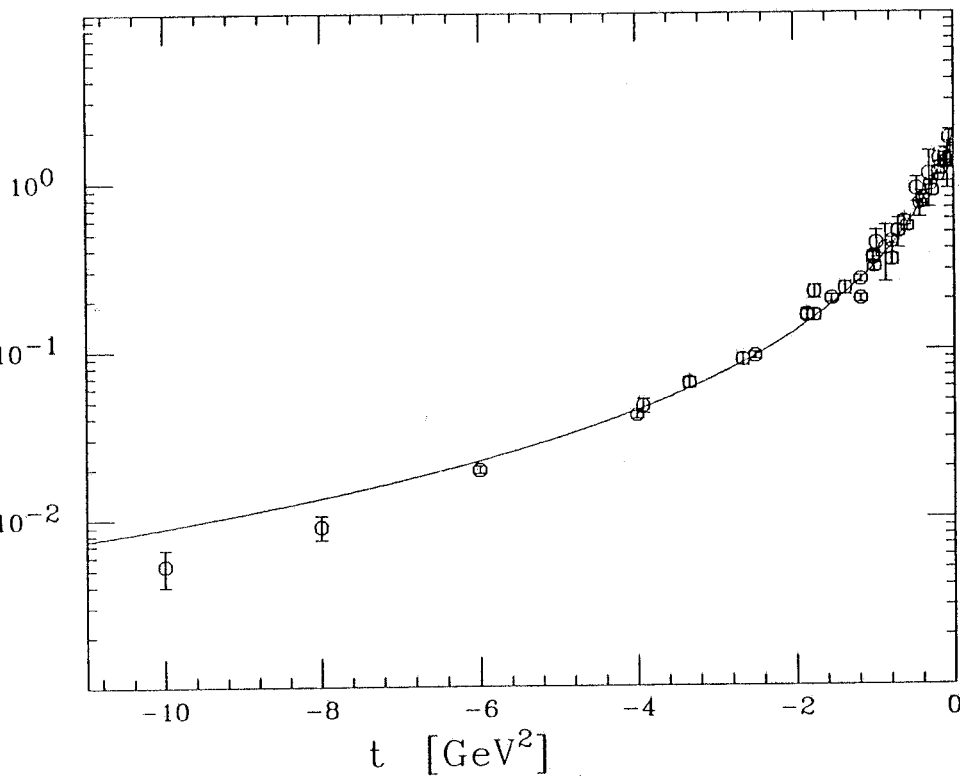
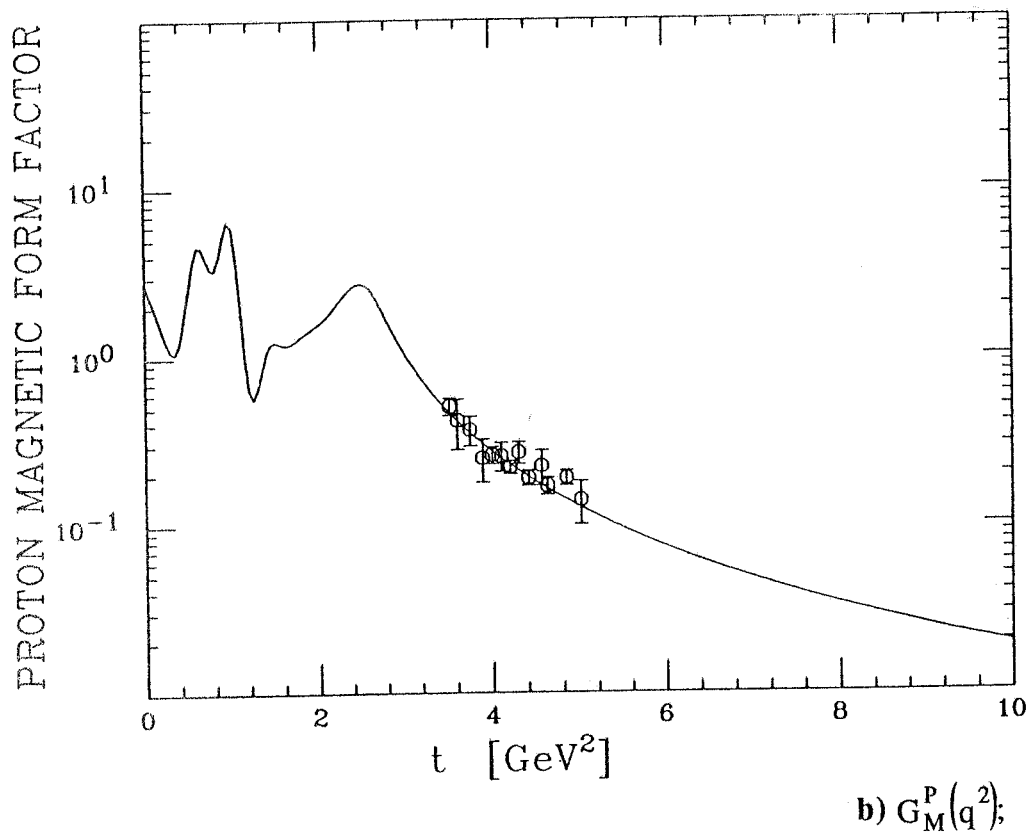
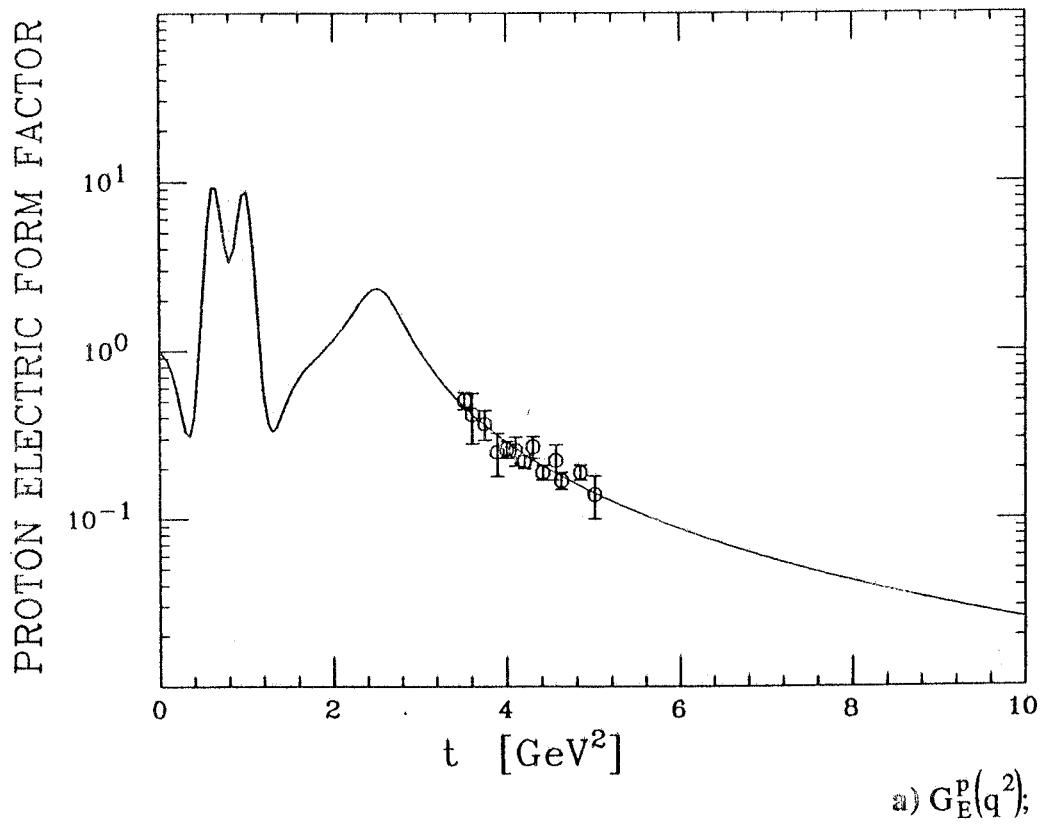
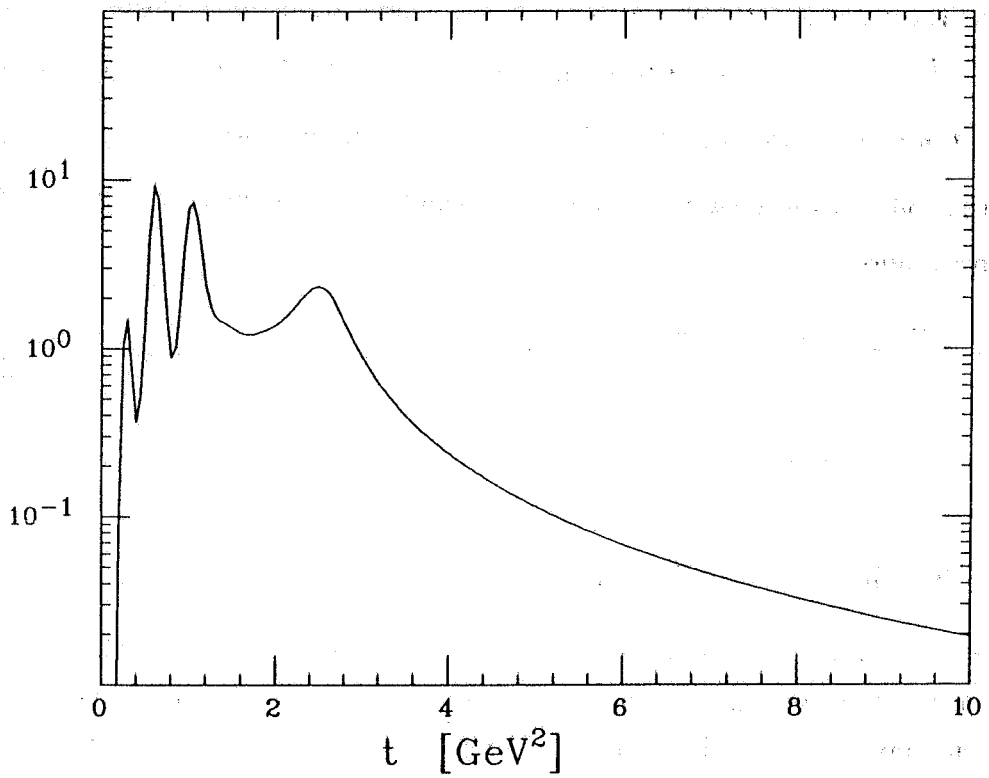
d) $G_m^n(q^2)$.

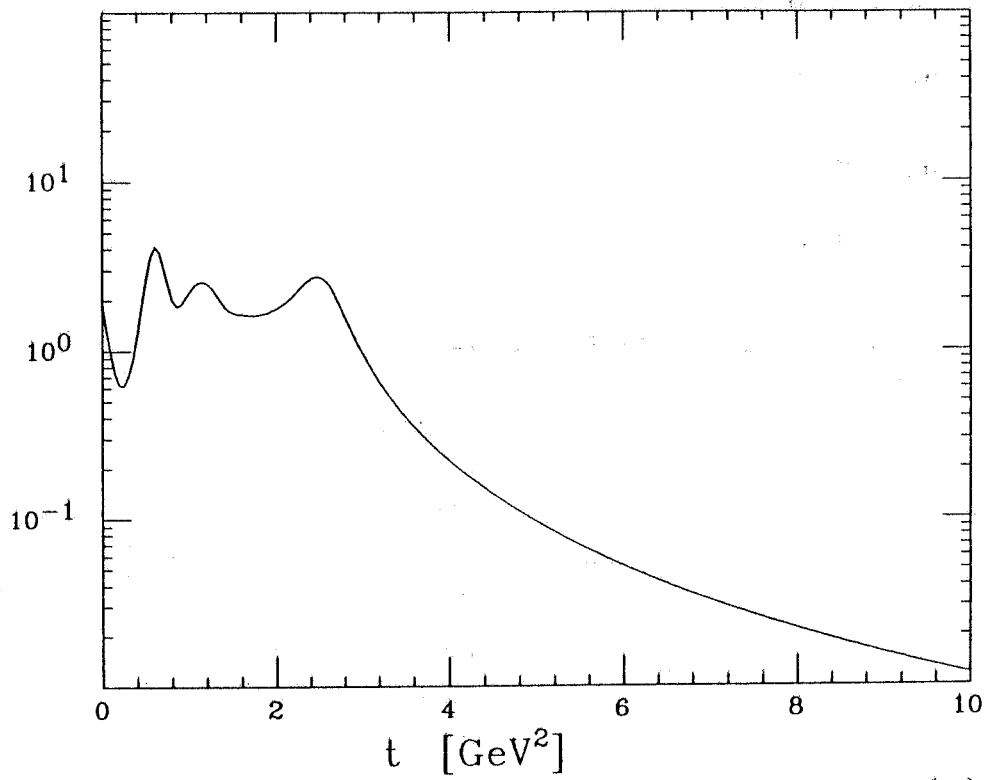
FIG. 4 - Predictions for the nucleon time-like form factors:



NEUTRON ELECTRIC FORM FACTOR

c) $G_E^n(q^2)$

NEUTRON MAGNETIC FORM FACTOR

d) $G_M^n(q^2)$

The widths Γ_v of the vector mesons have been included in eqs. (13). Furthermore, the mass parameters t_0, t_1 in $F_{1,2}^{(s)}(q^2)$ and $F_{1,2}^{(v)}(q^2)$ are assumed to be the same but those in $F_1^{(s,v)}(q^2)$ to be different from those in $F_2^{(s,v)}(q^2)$. The normalisations of $F_{1,2}^{(s,v)}(q^2)$ at $q^2 = 0$ give rise, not to four conditions for the coupling constants $g_{vNN}^{(1,2)}$ ($V = \rho, \rho', \omega, \phi, \phi'$) but to eight conditions since they involve both the real and imaginary parts of $F_{1,2}^{(s,v)}(q^2)$. The normalisations are

$$F_1^{(s)}(q^2 = 0) = F_1^{(v)}(q^2 = 0) = \frac{1}{2} \quad (14.1)$$

$$F_2^{(s)}(q^2 = 0) = \frac{1}{2} (\mu_p + \mu_n) \quad (14.2)$$

$$F_2^{(v)}(q^2 = 0) = \frac{1}{2} (\mu_p - \mu_n) \quad (14.3)$$

where μ_p, μ_n are the anomalous magnetic moments of the proton (p) and neutron (n), respectively. From eqs. (13) and (14) there are, thus, only two free coupling constants out of the ten involved. We choose these to be $g_{\omega NN}^{(1)}/f_\omega$ and $g_{\omega NN}^{(2)}/f_\omega$ which together with the mass parameters $t_{01}, t_{11}, t_{02}, t_{12}$ constitute our set of six free parameters. Using the masses (m_v) and widths (Γ_v) in Table I to fit $G_{E,M}^{p,n}(q^2)$ in the space-like region $q^2 < 0$, we find the values of these free parameters to be

$$\begin{aligned} t_{01} &= 0.303 \text{ GeV}^2 \\ t_{11} &= 0.384 \text{ GeV}^2 \end{aligned} \quad (15)$$

$$t_{02} = 0.580 \text{ GeV}^2$$

$$t_{12} = 0.614 \text{ GeV}^2$$

$$\frac{g_{\omega NN}^{(1)}}{f_\omega} = -1.055; \quad \frac{g_{\omega NN}^{(2)}}{f_\omega} = 1.717 \quad (16.1)$$

TABLE I

v	m_v (MeV)	Γ_v (MeV)
ρ	770	153
ρ'	1590	160
ω	782.6	9.8
ϕ	1019.5	4.22
ϕ'	1685	180

From eq. (15) one sees that $t_{0i}/t_{1i} \sim 1$ ($i = 1, 2$). Hence returning to eq. (9) and the comments on the smoothness assumption, one sees from eqs. (13.1) – (13.4) that this assumption holds better for the $g_{vNN}^{(2)}(q^2)$ than for the $g_{vNN}^{(1)}(q^2)$

Using these values and Table I we find, for the remaining eight coupling constants, from eqs. (13) and (14),

$$\frac{g_{\rho NN}^{(1)}}{f_\rho} = -1.661; \quad \frac{g_{\rho NN}^{(2)}}{f_\rho} = -2.167 \quad (16.2)$$

$$\frac{g_{\rho' NN}^{(1)}}{f_{\rho'}} = 1.286; \quad \frac{g_{\rho' NN}^{(2)}}{f_{\rho'}} = 4.217 \quad (16.3)$$

$$\frac{g_{\phi NN}^{(1)}}{f_{\phi'}} = 1.629; \quad \frac{g_{\phi NN}^{(2)}}{f_\phi} = -1.647 \quad (16.4)$$

$$\frac{g_{\phi' NN}^{(1)}}{f_{\phi'}} = 6.088; \quad \frac{g_{\phi' NN}^{(2)}}{f_{\phi'}} = 4.217 \quad (16.5)$$

The values of the coupling constants in eq. (16) are removed from SU(3) predictions. According to the latter, for instance, $g_{\phi NN}$ should be zero and $g_{\rho NN}/g_{\omega NN} = \sin\theta/\sqrt{3}$, where θ is the mixing angle. The fits to the form factors are quite stable over a wide range of variation of the parameters.

It should be mentioned, at this point, that there have been other attempts^(12,17) to fit the proton and neutron form factors with modified VMD models. These modifications are motivated in various ways but they all have in common the quark counting rule input of eq. (6.b). Our motivation to modify the usual VMD model so as to incorporate this input makes use of a general duality relationship in which this input is already present. Q^2 -duality, it may be said, refers to the hadronic structure of the photon while the s -channel duality incorporating the above input refers to the structure of the hadron itself. Our modification of the usual VMD model is therefore at the fundamental level. However, since we are using asymptotic arguments, the implementation of this idea is not rigorous. Our interpolation formula, eq. (8), is not unique. It also gives rise to a form factor $F(q^2)$ in eq. (10.1) which is not an analytic function of q^2 . The problem of analyticity of $F(q^2)$ will not be discussed here. We mention, in this regard, that the VMD model alone is not sufficient to address this problem. An attempt within the VMD scheme to do this leads, for instance, to the problem with the zeroes of generalised zeta function mentioned earlier. It is instructive to discuss this here briefly.

Consider the form factor as given by (10.2) and let the sum over V go over an infinite number of vector mesons. This gives rise to the generalised vector meson dominance model (GVMD)⁽⁵⁾. Letting n stand for the variable V , one gets

$$F(q^2) = \sum_{n=0}^{\infty} \frac{m_n^2}{f_n} \frac{g_{nhh}}{m_n^2 - q^2} = \sum_{N=0}^{\infty} A_{hN} z^N \quad (17)$$

where

$$z := q^2/m_0^2 \quad (18)$$

and

$$A_{hN} := \sum_{n=0}^{\infty} \frac{g_{nhh}}{f_n} \left(\frac{m_0^2}{m_n^2} \right)^N \quad (19)$$

We have dropped the label "VMD" on $F^{(VDM)}(q^2)$ and on $g_{nhh}^{(VDM)}$ for notational convenience. The coupling constants are assumed to be simple functions of m_n^2 , e.g. powers,

$$\frac{g_{nhh}}{f_n} = \frac{g_{ohh}}{f_0} \left(\frac{m_0^2}{m_n^2} \right)^{v_0} \quad (20)$$

Substituting (20) in (19) yields

$$A_{hN} := \frac{g_{ohh}}{f_0} \sum_{n=0}^{\infty} \left(\frac{m_0^2}{m_n^2} \right)^{v_0 + N} \quad (21)$$

The coefficients A_{hN} are then proportional to the values at $v = N + v_0$ ($N = 0, 1, 2, \dots$) of the zeta function

$$\zeta(v, \widehat{M}^2) := \sum_{n=0}^{\infty} \left(\frac{m_0^2}{m_n^2} \right)^v \quad (22)$$

associated with the mass squared operator \widehat{M}^2 having eigenvalues m_n^2 ($n = 0, 1, 2, \dots$). To find the asymptotic behaviour of the form factor from the Taylor series in eq. (17), we proceed in the standard manner: first we rewrite the power series as a contour integral

$$F(q^2) = \frac{i}{2} \oint \frac{dv}{\sin(\pi v)} A_h(v) (-z)^v \quad (23)$$

Eq. (23) is the so - called Sommerfeld Watson transform. The contour goes over the positive zeroes of $\sin(\pi v)$, i.e. $v = N$ ($N = 0, 1, 2, \dots$). Next we deform the contour to cover

only the negative zeroes of $\sin(\pi v)$, i.e. $v = -N$ ($N = 1, 2, \dots$). This gives rise to the asymptotic behaviour

$$F(q^2) \xrightarrow{|q^2| \rightarrow \infty} - \sum_{N=1}^{\infty} A_{h(-N)} z^{-N} \quad (24)$$

under the additional assumption that the coefficient function $A_h(v)$ has no singularities in the negative half - plane. Singularities of $A_h(v)$, e.g. poles, would modify eq. (24) through the presence of terms of the type $(\log(z))^{N_1} z^{-N_2}$ where N_1, N_2 are positive integers. We will not consider this more general situation. Now making use of eq. (6.2) in (24) one finds, firstly, that

$$F(q^2) \xrightarrow{|q^2| \rightarrow \infty} - \sum_{N=N_0}^{\infty} A_{h(-N)} z^{-N}; \quad N_0 = N_q - 1 \quad (25)$$

and, secondly, the set of zero coefficients

$$A_{h(-N)} = 0; \quad N = 1, 2, \dots, (N_0 - 1) = (N_q - 2) \quad (26)$$

For mesons $N_q = 2$ and eq. (26) says that there are no zero coefficients. The GVMD model gives therefore the same asymptotic behaviour of the form factors as simple VMD models. For a nucleon (with mass number $A = 1$) and nuclei of mass number $A \geq 2$, $N_q = 3A$. Eq. (26) then says that there are $3A - 2$ zero coefficients. The constraints on the parameters of the model which arise then are extremely difficult to satisfy. The difficulty arises as follows: with the usual simple Ansatz.

$$m_n^2 = m_0^2 (1 + bn)^\lambda; \quad n = 0, 1, 2, \dots \quad (27)$$

for the mass spectrum, the parameters b and λ are more or less defined within certain ranges; e.g. $0 < b < 2$ and $0 \cdot 1 < \lambda < 2$ ^(5,6,18,20). Substituting (27) in (22) one gets

$$\zeta\left(v, \widehat{M}^2\right) = \frac{1}{b^{\lambda v}} \zeta\left(\lambda v, \frac{1}{b}\right) \quad (28)$$

where

$$\zeta\left(v, \frac{1}{b}\right) := \sum_{n=0}^{\infty} \frac{1}{\left(n + \frac{1}{b}\right)^v} \quad (29)$$

is the standard generalised zeta function⁽²¹⁾. From eqs. (21), (22), (28) and (29) $A_{h(-N)}$ becomes

$$A_{h(-N)} = \frac{g_{ohh}}{f_0} b^{\lambda(N-v_0)} \zeta\left(-\lambda(N-v_0), \frac{1}{b}\right) \quad (30)$$

For nucleons ($A = 1$) and nuclei ($A > 2$), eqs. (26) and (30) imply that the generalised zeta function must have the following finite set of consecutive zeroes

$$\zeta\left(-\lambda(N-v_0), \frac{1}{b}\right) = 0; N = 1, 2, \dots (3A-2) \quad (31)$$

The point is that the function defined in eq. (29)⁽²¹⁾ has no such set of zeroes. The nucleon ($A = 1$) may be just barely accommodated if $b = 1$ ^(*). For this value of b eq. (29) defines the Riemann zeta function⁽²²⁾, $\zeta(v)$, i. e.

$$\zeta(-\lambda(N-v_0)) := \zeta(-\lambda(N-v_0), 1) \quad (32)$$

For $A = 1$ there is only one zero in eq. (31) for $N = 1$. For $b = 1$ and from properties of the Riemann zeta function⁽²³⁾ the zero occurs at

$$\lambda(1-v_0) = 2k_0 \quad (33)$$

where k_0 is a positive integer. This single constraint relates the coupling constant parameter v_0 to the mass spectrum parameter λ . The GVMD may thus still be useful for the description of nucleon form factors. Its inconsistency arises as soon as we try to apply it to nuclei ($A > 2$) quite apart from the unusual value of $b = 1$ required. The Riemann zeta function $\zeta(-\lambda(N-v_0))$ has an infinite number of zeroes at values of $\lambda(N-v_0)$ satisfying the conditions

$$\lambda(N-v_0) = 2k; k = k_0, (k_0+1), (k_0+2), \dots \quad (34)$$

Two consecutive values of N e.g. $N = 1$, and $N = 2$ corresponding to $k = k_0$ and $k_0 + 1$ determine the parameters λ and v_0 to be $\lambda = 2$ and $v_0 = 1 - k_0$. Consequently $\lambda(N-v_0) = 2(N+k_0-1)$ is an even integer for all values of N . That means that not only the $3A-2$ coefficients in eq. (26) are zero (for $N_q = 3A$), but also, all the coefficients $A_{h(-N)}$ ($N = 1, 2, \dots$) of eq. (30) which appear in the asymptotic expansion of eq. (25). This latter result is not surprising: it is the only way to match the set of zeroes of the two sides of eq. (30) for $b = 1$. An inconsistency arises because one gets a vanishing asymptotic expansion of the form factor $F(q^2)$ contrary to the model assumption that it is non-vanishing. Since, experimentally, $F(q^2)$ is different from zero for large q^2 one concludes that GVMD models cannot describe its behaviour consistently.

(*) Barely because this value of b will not fit other data. See also eq. (27) in this case and also refs. (18)-(20).

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- (15) Cf. Ref. (12): The formula for the form factor in this reference (eq. (7) is:

$$F(q^2) = \left(\frac{1 - u^2(q^2)}{1 - u_0^2} \right)^{2(N_q - 2)} F^{\text{VMD}}(q^2)$$

and hence basically the same as eq. (10) of the present paper.

The difference between them lies in the different factor functions correcting the pure VMD formula. This difference accounts for the most part for the difference between their predictions This correction factor is therefore crucial.

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