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Laboratori Nazionali di Frascati

LNF-89/009(PT)
24 Febbraio 1989

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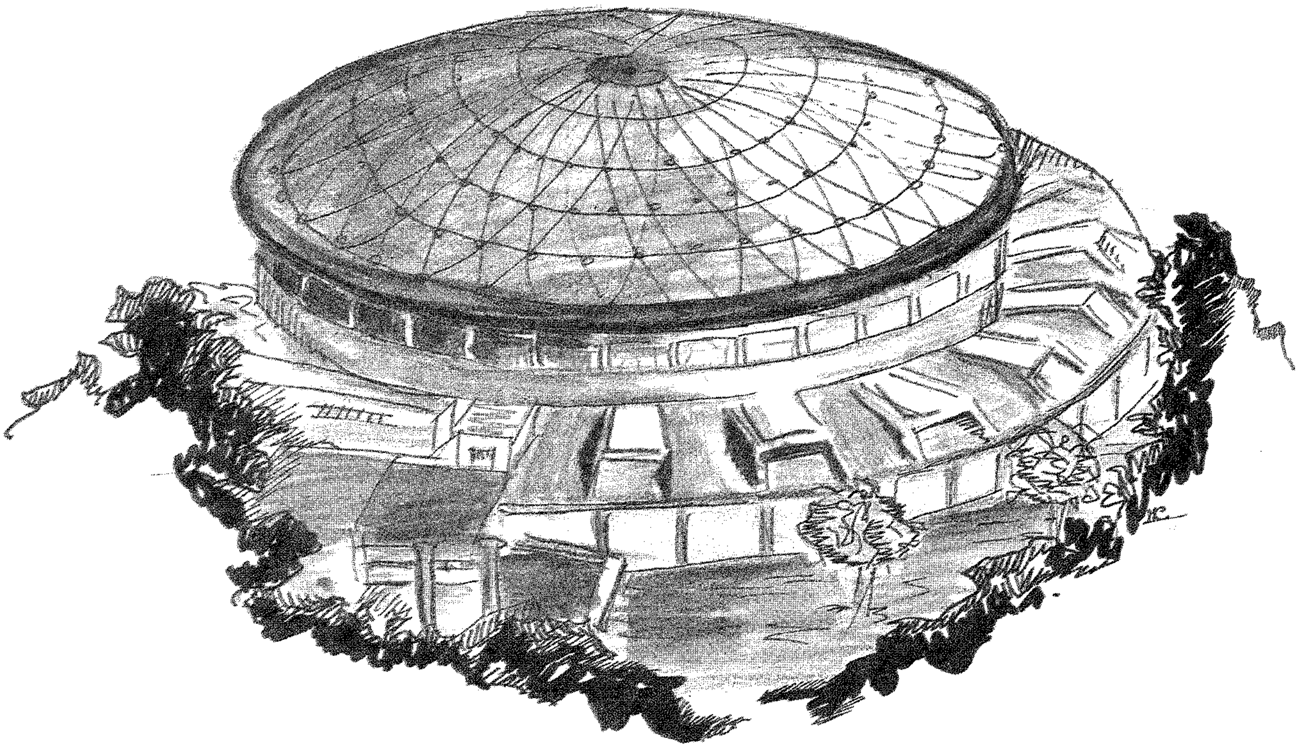
GEOMETRICAL INTERPRETATION OF BREMSSTRAHLUNG EFFECTS IN HADRON-HADRON COLLISIONS

Invited talk at the Conference on
Hadronic Matter in Collision,
Tucson, (Arizona), October6-12, 1988



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GEOMETRICAL INTERPRETATION OF BREMSSTRAHLUNG EFFECTS IN HADRON-HADRON COLLISIONS(*)

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1.Introduction

In this talk we investigate some geometrical aspects of hard parton parton scattering, in relation to the hypothesis that the appearance of copiously produced hard scattering events at the $Sp\bar{p}S$ Cern Collider [1]is at the origin of non-scaling phenomena observed by both UA1[2] and UA5 [3]Collaborations in low p_t physics.

While the relative frequency of hard events is reasonably reproduced by QCD calculations [4], the multiplicity features have, as yet, been only qualitatively understood. In the statistical model [5], the difference in shape between minimum bias and events con-

(*) This work was funded in part through a grant from the U.S. Department of Energy.

taining at least one jet (with $E_T^{jet} > 5\ GeV$) is attributed to purely statistical fluctuations introduced by the method of event selection. A different point of view is espoused by the proponents of a geometrical[6] picture of hadron- hadron collisions. In these descriptions, the increase in the average multiplicity of jet events , relative to minimum bias, is due to their higher "inelasticity", characterized on the average by small impact parameters. In some of these models the multiplicity distribution of charged tracks outside the jet is related to the spatial distributions of partons inside the proton. By studying as to whether various expressions for these distributions fit the data or not, one can infer information on the proton spatial structure[7]. In other models , both the increase in inelasticity as well as the change in shape are attributed to a difference in the number of parton chains produced in the reaction[8]. Our point of view is that a jet event (albeit a low-E_T one) should be studied as much as possible as the convolution of a hard scattering process, described by QCD techniques, with a background, whose shape and properties should reflect the degree of inelasticity of the process. Study of the background would then provide information on the breaking of the proton.

A first step to the pursuance of such a program consists in studying the hadronic properties of events in which production of a W-boson decaying into a leptonic channel is observed. At the CERN Collider this is a rather hard process and study of the hadronic background can throw light on the role played by bremsstrahlung in hard scattering processes. Next we shall discuss the theoretical framework in which this can accomplished.

2.Impact Parameter and Transverse Momentum Distribution for a Collinear Parton Pair

In this section we shall discuss the relationship between impact parameter and transverse momentum as it occurs in QCD bremsstrahlung processes. We shall show that, once bremsstrahlung is properly taken into account, the dependence of the underlying event upon energy, parton type and impact parameter variables is certainly very much reduced, if not altogether negligible.

The main observation here is that the energy dissipated in initial state bremsstrahlung depends upon energy and type of the emitting particles . By Fourier-transforming the distribution one can directly relate the particle's energy and type to the impact parameter.

Let us consider the transverse momentum acquired through QCD bremsstrahlung by a pair of collinear partons of total invariant mass $\sqrt{\hat{s}} = \sqrt{[(p_1^\mu + p_2^\mu)(p_{1\mu} + p_{2\mu})]}$ scattered into a final state of total invariant mass Q : the expression, for the soft part, can be written as

$$\frac{dP}{dp_t} = p_t \int bdbJ_0(p_tb)e^{-h(b;q_{max})} \tag{1}$$

with

$$h(b;q_{max}) = c_{ij}\int_0^{q_{max}} \frac{dk}{k}\frac{2\alpha_s}{\pi}(1 - J_0(kb))\, ln\left(\frac{q_{max} + \sqrt{q_{max}^2 - k^2}}{q_{max} - \sqrt{q_{max}^2 - k^2}}\right) \tag{2}$$

and $q_{max} = \frac{Q(1-z)}{2\sqrt{z}}$, $z = \frac{Q^2}{\hat{s}}$, $c_{ij} = \frac{4}{3}$ for quark-(anti)quark scattering,3 for gluon-gluon scattering and $\frac{13}{8}$ for (anti)quark-gluon scattering.For soft radiation, q_{max} is found to be proportional to Q. For $\alpha_s(k^2)$, we use the expression

$$\alpha_s(k^2) = \frac{12\pi}{27} \frac{1}{ln\left[\frac{k^2+a\Lambda^2}{\Lambda^2}\right]} \tag{3}$$

The parameter a regulates the behaviour of α_s as $k_t \to 0$.Following Altarelli et al.[9], one can use $a = 5$.

In fig.1 we show a plot of the function $e^{-h(b;Q)}$ for different Q values and for the two cases gg, and qq scattering. We see the following:

(i) gluon-gluon scattering is characterized by smaller b-values than quark-quark scattering;

(ii) for harder scattering, contribution to the b-integral comes from smaller b-regions;

(iii) the width of the curve $e^{-h(b;Q)}$ shrinks like $\frac{1}{Q}$for large Q.

Notice that the above correlation between impact parameter and hardness of the interaction comes into play through bremsstrahlung factors.To see directly the correlation between b and p_t , one can convolute the function $e^{-h(b;Q)}$ with the Bessel function $J_0(p_t b)$ for a set of p_t values. In figs.2a and b we show the relative curves for the case of qq scattering, for two different values of the fixed ratio $\frac{p_t}{Q}$ and $Q = 5, 20$and 80 GeV respectively.We see that, for the same Q value, higher transverse momentum corresponds to smaller impact parameter values.

What we have described in this section is the transverse momentum distribution of a pair of partons of total invariant mass Q, for the case of soft emission. A more complete expression is obtained by adding to this term all the contributions which would reproduce the full first order term. The soft term however gives the most important contribution and illustrates best the correlation between bremsstrahlung effects and impact parameter distribution. Clearly a Fourier transform of the hard term should give results similar to the one above.

Although this model suggests a relation between impact parameter distribution and bremsstrahlung probability, we notice that the transverse momentum distribution does not actually give any information on the so called Underlying Event, since it is assumed that, before interacting , the two partons are collinear. Thus we cannot explore the rest of the event . For this we need to go to the transverse energy distribution , which ceratinly gets contribution from the underlying event. To eliminate unwanted complications from the final state, it is again simpler to work with processes in which the two partons scatter into a colour singlet state, which is then seen to decay into lepton pairs, as in the case of J/Ψ or W-production. From fig.2, it has already been noticed that, at the same Q value, higher transverse momentum corresponds to smaller impact parameter regions. So in Drell-Yan type processes or resonance production, exploration of impact parameter dependence(hardness of the interaction for geometrical models) can be done by varying the observed p_t of the parton pair. As the next step, one needs to obtain the double distribution in p_t and E_T .This is what we shall discuss in the following section.

3. Transverse Energy and Momentum Distribution for W-boson Production

The transverse energy accompanying W-production can be seen as the sum of two terms, one which represents the energy radiated away by the two partons before the collision and the other which can be considered to come from the rest of the proton as it breaks up. Correspondingly the transverse energy distribution will be written as the convolution of two factors, one which describes initial state bremsstrahlung and for which there exists an exact expression to all orders in α_s (the strong coupling constant) and one which describes the transverse energy which appears from all other sources, in particular from the breaking up of the proton, what one can call the Hadronic Underlying Event (HUE). We write

$$\frac{1}{\sigma^{W+X}}\frac{d\sigma^{W+X}}{dE_T} = \int_0^{E_T} \Pi_{HUE}(E_T - E_T^*)\ B(E_T^*;s)\ dE_T^* \tag{4}$$

where Π_{HUE} is the (normalized) E_T-distribution of the HUE and

$$B(E_T^*;s) = \frac{1}{\sigma^W}\int dx_1 q(x_1)q(x_2)\frac{dP}{dE_T^*}(E_T^*,\hat{s})\hat{\sigma}_W(x_1,x_2) \tag{5}$$

where $\hat{\sigma}_W$ is the parton-parton cross section for W-production and σ^W is the hadronic W-production cross-section in $p\bar{p}$ collisions. With these definitions, we see that the distribution B(E;s)is normalized to 1.

From the above expressions one can calculate the average transverse energy as

$$< E_T >_{tot}=< E_T >_{HUE} + < E_T >_{bremss} \tag{6}$$

where $< E_T >_{bremss}$ is the average transverse energy calculated from initial state QCD bremsstrahlung.Geometrical models suggest that $< E_T >_{tot}$ should be larger than $< E_T >_{mb}$, the average transverse energy found in minimum bias events, since one is dealing here with a hard event, characterized by higher inelasticity. Since $< E_T >_{bremss}$ can in principle be calculated from QCD, the validity of this model can be tested by examining W-production and comparing with theoretical ansatzs. At present there exists a calculation by Altarelli and collaborators[10], in which the transverse energy distribution accompanying W-production is described by convoluting a minimum bias-type background with bremsstrahlung, i.e. in which the underlying event is taken to be described entirely by minimum bias, and independent of the hardness of the event. As mentioned earlier, the relationship between hardness of the event and scalar E_T can be best explored by studying transverse energy as a function of the W-transverse momentum p_t. Present data on W-production cover a range in transverse momentum from a few GeV until 40 Gev and beyond. This means that the bremsstrahlung expression must properly include soft contributions (i.e. summed to all orders in α_s)and at the same time reproduce at least the complete first order. For the soft contribution, one may use the expression for the double distribution function

$$\frac{d^2P^{soft}}{dp_t dE_T} = p_t\int\frac{dt}{2\pi}e^{iE_T t}\int bdbJ_0(p_t b)e^{-h(b,t;q_{max})} \tag{7}$$

which we have discussed in ref.[11]. In eq.(7)

$$h(b,t;q_{max}) = c_{ij}\int_0^{q_{max}} \frac{dk}{k}\frac{2\alpha_s}{\pi} ln\left(\frac{q_{max}+\sqrt{q_{max}^2-k^2}}{q_{max}-\sqrt{q_{max}^2-k^2}}\right)\left(1-J_0(kb)e^{-ikt}\right) \tag{8}$$

In eq.(7) the summation to all orders of soft gluons, results in a 2+1 dimensional distribution for which $E_T \geq |\mathbf{p}_t|$.This is due to the fact that at low p_t the number of gluons is not measurable and more than one gluon contributes to the observed distribution. On the other hand, for large p_t values, the emitted transverse energy is mostly carried by a single gluon and the relative double distribution is rather written as

$$\frac{d^3\sigma}{d^2\mathbf{p}_t dE_T} \approx \delta(E_T - p_t)\frac{d^2\sigma^{(1)}}{d^2\mathbf{p}_t} \tag{9}$$

The hard bremsstrahlung contribution can be obtained from the first order expression[12]

$$\frac{d^2\sigma^{(1)}}{dp_t dy} = \frac{d\sigma^{q\bar{q}}}{dp_t dy} + \frac{d\sigma^{qg}}{dp_t dy} \tag{10}$$

with

$$\frac{d\sigma^{q\bar{q}}}{dp_t dy} = \frac{4\pi\alpha}{9sin^2\theta_W}\frac{\alpha_s(m_w^2)}{s}\frac{1}{p_t}\int_{x_{min}}^1 dx_1$$

$$\sum_i \left[q_i(x_1,m_W^2)\bar{q}_i(x_2,m_W^2)+1\leftrightarrow 2\right]\left(\frac{1+\frac{\tau^2}{x_1^2x_2^2}-\frac{x_T^2}{2x_1x_2}}{x_1-\frac{1}{2}\bar{x}_Te^y}\right) \tag{11}$$

and

$$\frac{d\sigma^{qg}}{dp_t dy} = \frac{\pi\alpha}{3sin^2\theta_w}\alpha_s(m_W^2)\frac{p_t}{s^2}\int_{xmin}^1 \frac{dx_1}{x_1-\frac{1}{2}\bar{x}_Te^y}\frac{1}{x_1^2x_2^2}$$

$$\sum_i \left[q_i(x_1,m_W^2)G(x_2,m_W^2)\frac{(x_1x_2-\tau)^2+\frac{1}{4}(x_1x_2+\tau+V)^2}{x_1x_2-\tau+V}+(1\leftrightarrow 2, V\leftrightarrow -V)\right] \tag{12}$$

and with

$$\tau = \frac{m_W^2}{s} \quad x_T^2 = \frac{4p_T^2}{s} \quad \bar{x}_T^2 = x_T^2 + 4\tau$$

We also have the following definitions :

$$x_2 = \frac{\frac{1}{2}\bar{x}_Te^{-y}x_1-\tau}{x_1-\frac{1}{2}\bar{x}_Te^y} \quad x_{min} = \frac{\frac{1}{2}\bar{x}_Te^y-\tau}{1-\frac{1}{2}\bar{x}_Te{-}y} \quad V = x_1x_2+\tau-x_1\bar{x}_Te{-}y \tag{13}$$

where $\sqrt{s}$ is the proton-antiproton center of mass energy,G and q_i are the (singular) gluon and quark densities respectively. This expression is singular for $p_t \to 0$ (there is both an infrared as well a collinear singularity) and it should be regularized.. A complete two-loop expression has been presented in ref.(9)and in ref.(13).

We note that hard bremsstrahlung can only be measured if the transverse energy associated with it exceeds a certain threshold value, typically 5 GeV at the Cern Collider.This is the energy which allows the experimental definition of a hadronic jet, to be identified, in this case, with fragments from a hard bremsstrahlung gluon. To take into account this experimental lower limit in E_T, we propose the following phenomenological separation into soft and hard bremsstrahlung :

$$\frac{d^4\sigma}{d^2\mathbf{p}_t dE_T dy} = \frac{d^4\sigma^{soft}}{d^2\mathbf{p}_t dE_T dy} + \frac{d^4\sigma^{hard}}{d^2\mathbf{p}_t dE_T dy} \tag{14}$$

where

$$\frac{d^4\sigma^{soft}}{d^2\mathbf{p}_t dE_T dy} = \left(\frac{d\sigma^W}{dy}\right)\frac{d^3P^{soft}}{d^2\mathbf{p}_t dE_T} \tag{15}$$

In eq.(15), we have factorize the soft gluonh distribution from the double production cross-section (Born term). As indicated, this is an approximation, which should hold for the soft contribution. We differ from ref.(9), where the parton densities, while evaluated at $\sqrt{\hat{s}} = m_W$, are calculated at $Q^2 = \frac{1}{b^2}$, where b is the impact paramter variables of eq.(7). This b-dependence of the parton densities of course prevents complete factorization. On the other hand, this dependence is not mandatory (the choice $Q^2 = m_W^2$ cannot a priori be excluded) and the factorized expression given in eq.(15) allows for simpler calculation.If the hard cross-section is identified with the first order perturbative calculation, as in eq.(9), we obtain the following expression for the average transverse energy emitted through bremsstrahlung alone as a function of the W-transverse momentum

$$< E_T(p_t) >_{bremss} = [1 - N(p_t)] < E_(p_t) >_{soft} + N(p_t)p_t \tag{16}$$

where the relative weight $N(p_t)$ is given by

$$N(p_t) = \frac{\frac{d\sigma_{hard}}{dp_t dy}}{\frac{d\sigma_{hard}}{dp_t dy} + \frac{d\sigma_{soft}}{dp_t dy}} \tag{17}$$

and $< E_T(p_t) >_{soft}$ is given by

$$< E_T(p_t) >_{soft} = \frac{\int bdbJ_0(p_t b)S(b;q_{max})c_{ij}\int_0^{q_{max}} dk \frac{2\alpha_s}{\pi} ln\left(\frac{q_{max}+\sqrt{q_{max}^2-k^2}}{q_{max}-\sqrt{q_{max}^2-k^2}}\right) J_0(bk)}{bdbJ_0(p_t b)S(b;q_{max})} \tag{18}$$

with

$$S(b;q_{max}) = e^{-h(b;q_{max})} \tag{19}$$

In fig.3 we show a graph of $< E_T >_{soft}$ vs.p_t for two different values of the upper limit q_{max}.In the case $q_{max} = 10\ GeV$, the quantity $< E_T >_{soft}$ has been evaluated only for $p_t \leq 15\ GeV$ since the soft approximation needs to be implemented by the hard contribution when q_{max} starts exceeding p_t. A value of $q_{max} = 20.7\ GeV$ has been estimated for the W-case following the procedure of ref.(14.) This value is about 25% of the W-mass and hence not unreasonable for the maximum allowed value for the soft radiation.

We notice that as $p_t \rightarrow 0$,the average transverse energy emitted through soft bremsstrahlung does not go to zero, but it approaches a finite value, which is sensitive to the choice of α_s at small k^2. On the other hand, in the large p_t limit, $< E_t >$ should asymptote to p_t reflecting the intuitive result that W-production at reasonably large p_t values must principally involve one single hard bremsstrahlung.As p_t increases further, one may need to add terms from double hard bremsstrahlung. For events in which $p_t^W \approx m_W$ double bremsstrahlung may indeed have been observed [15]. Concerning the addition of first and eventually second order terms to the soft contribution, we point out the approach of refs. (9,10) in which the entire first order term (not just the leading double logarithms) has been exponentiated. We shall not discuss the precise form of the hard term in this paper.

Preliminary results from UA1 group indicate that one can write the following expression to fit the W-production data

$$< E_T >_{W+X} (p_t) = E_0 + A\ p_t^W$$

with the slope A of order unity.Comparing eqs.(6)and (20) one can write for the constant E_0

$$E_0 =< E_T >_{HUE} + < E_T >_{bremss} (p_t = 0) \tag{21}$$

A comparison with experimental data will determine an appropriate value for $< E_T >_{HUE}$ for W-production as well as for for other processes of interest, such as for direct photon production[16], two photons and J/Ψ production. Finally, averaging over p_t, one finds

$$< E_T >_{soft}= 15.44 GeV \qquad q_{max} = 20.7\ GeV \tag{22}$$

which is the transverse energy associated with initial state soft QCD bremsstrahlung in W-production. We have not yet introduced any rapidity dependence on the bremsstrahlung distribution so that comparison with data can only be done for and in the full rapidity range as far as particle emission is concerned.

4. Conclusions

In this paper the connection between impact parameter and event inelasticity in hadronic reactions is studied through the bremsstrahlung process. In physical terms, one can say that the degree of acollinearity between the two partons is the measure of their closeness of approach.In this sense one can talk of QCD interpretation of geometrical models.

We would like to make some further comments on the difference in b distribution between gluons and quarks. From the previous discussion, it should be apparent that, because of the color factor $\frac{c_a}{c_f}$ bremsstrahlung from gluon-gluon scattering is concentrated at smaller b-values, a fact which in some models has resulted in attributing to the gluons a different spatial distribution than that of the quarks. On the other hand, when examining bremsstrahlung factors from gluon-gluon scattering one should not ignore that hard gluons (those which do scatter with a finite order QCD cross-section) have been emitted from quarks of rather large energy, $E_{quark} \approx 10\ E_{gluon}$, and that QCD radiative corrections are very important for the quark - quark initial state. The full impact parameter dependence, in other words, for gluon-gluon scattering must go first through a complete evaluation of bremsstrahlung factors. These would be the corrections to be applied to study the impact parameter dependence of low-E_T jets, which are typically dominated by gluon-gluon scattering.

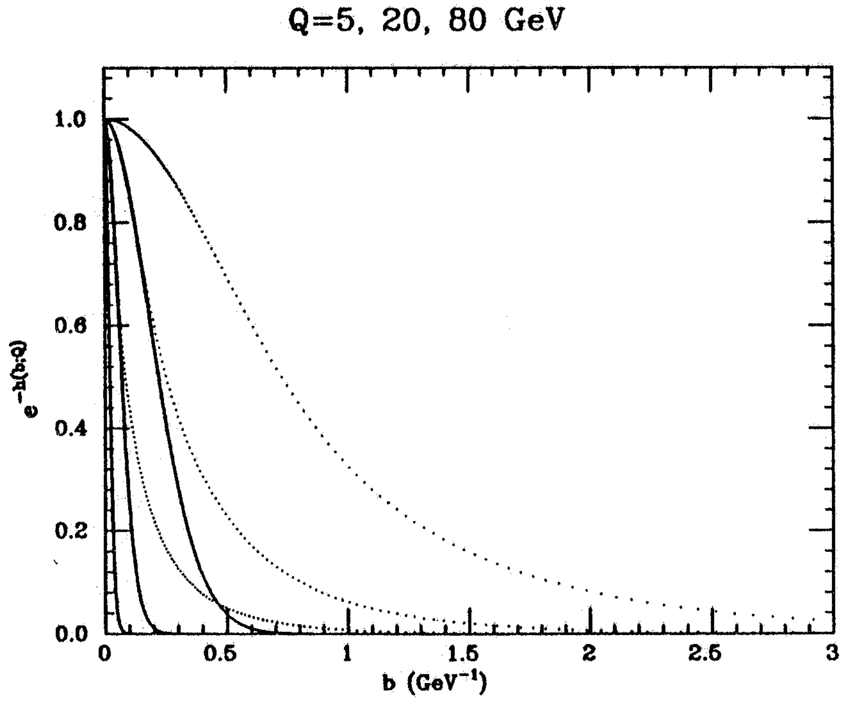
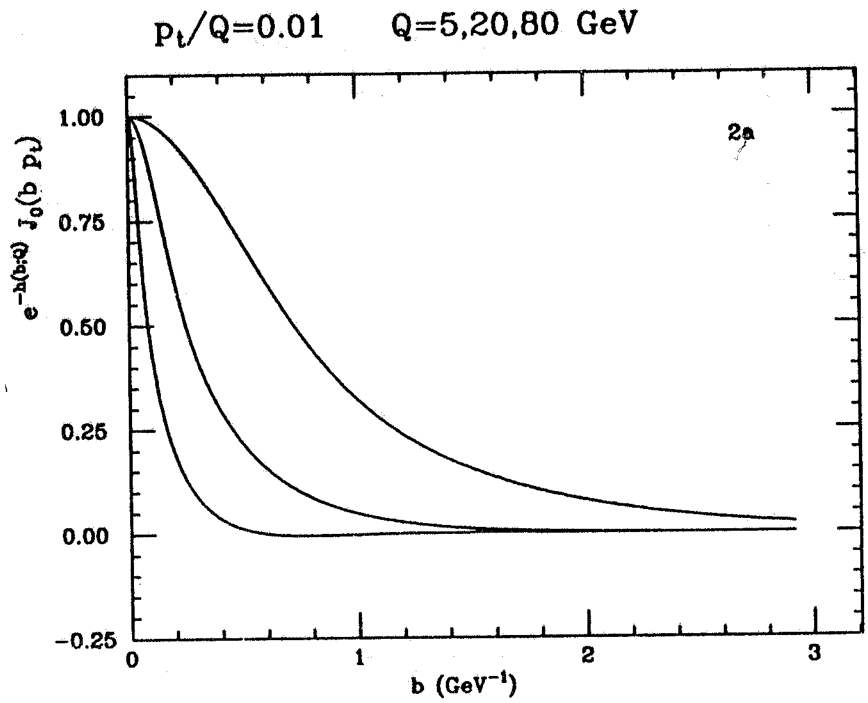


Fig.1 The function $e^{-h(b;Q)}$ is plotted for Q=5, 20 and 80 GeV for different Q values and for the two cases gg, and qq scattering. Full line corresponds to gluon-gluon scattering, dotted line to quark-quark scattering.



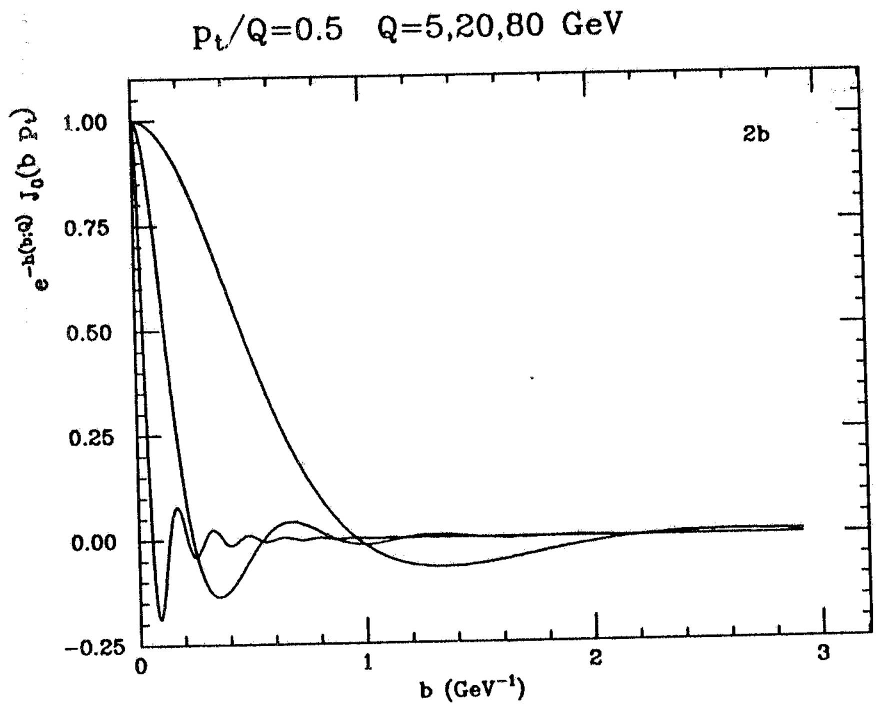


Fig.2 The function $e^{-h(b;Q)}J_0(p_t b)$ is plotted as a function of b for a set of p_t values,for the case of qq scattering.
(a) fixed ratio $\frac{p_t}{Q} = 0.01$ and $Q = 5,20$and 80 GeV respectively.
(b) fixed ratio $\frac{p_t}{Q} = 0.5$ and $Q = 5,20$ and 80GeV.

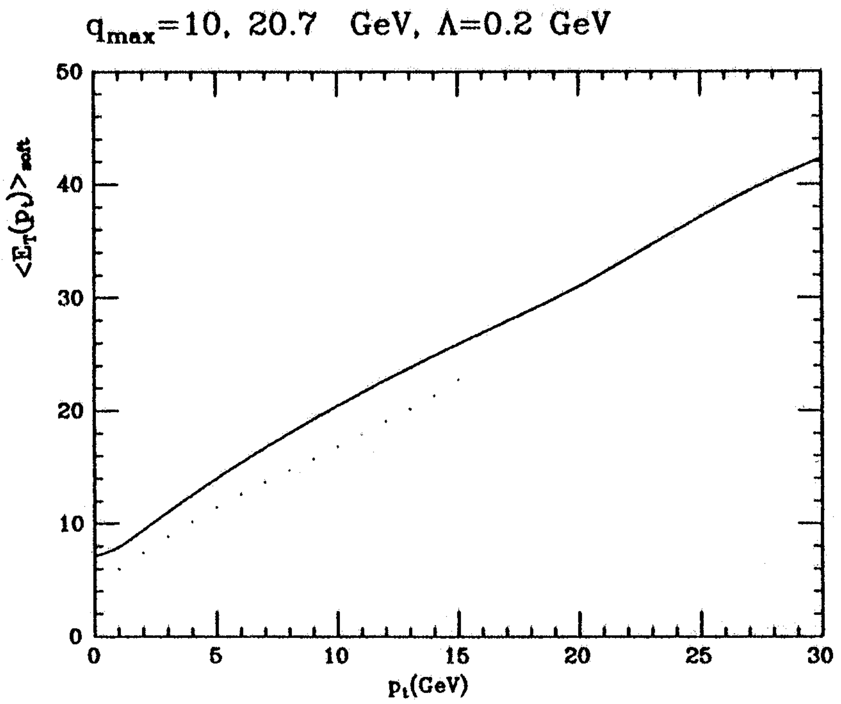


Fig.3 $< E_T >_{soft}$ vs.p_t for $q_{max} = 10$ (dotted line)and 20.7 GeV (W-production case as estimated in the text).

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